SHORT COMMUNICATIONS

A GENERALIZED LOGISTIC MODEL FOR ANALYSIS OF DOSE-RESPONSE DATA

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IN order to study the relationship between a binary zero-one response variable and one or more explanatory variables, the logistic model has been a very useful tool because of its closeness to normal, simplicity of its form, ease of interpretation of results and its importance as a response curve. The probability density function (p.d.f.) and the corresponding cumulative distribution function (c.d.f.) of a logistic random variable X are given respectively by

$$f(x) = F_0(x)[1-F_0(x)] \quad -\infty < X < \infty, \tag{1}$$
$$F_0(x) = [1+e^{-x}]^{-1} \quad -\infty < X < \infty. \tag{2}$$

In particular, the logit model has many useful and practical applications in the analysis of quantal response data, epidemiologic data, dose-response curves, population growth studies and the analysis of bioassay data used by Berkson[1], Brown[2], George and Ojo[3] and others.

In this paper we embed the logit model into a more general parametric family of models, the family of generalized logistic, in which the logistic is its special case. The generalized logistic distribution is derived as follows. Let X be a random variable with a p.d.f. of an extreme-value distribution of exponential type given by

$$f(x|\alpha) = \frac{\alpha^n \exp(-nx)\exp(-\alpha\exp(-\alpha))}{\Gamma n} \qquad -\infty < x < \infty \qquad 0<\alpha<\infty,\quad 0<n<\infty. \tag{3}$$

Assume that the parameter α in (3) has a gamma distribution with parameter m. Then the p.d.f. of α is given by

$$g(\alpha) = \exp(-\alpha)\alpha^{m-1}/\Gamma m. \tag{4}$$

The p.d.f. of the compounding distribution based on expressions (3) and (4) is obtained as follows:

$$f(x) = \int_0^\infty f(x|\alpha)\, g(\alpha)\, d\alpha,$$
$$= \frac{\exp(-nx)[1+\exp(-x)]^{-(m+n)}}{B(m,n)},$$
$$= \frac{[F_o(x)]^m[1-F_o(x)]^n}{B(m,n)}, \tag{5}$$

where

$F_0(x)$ is the logistic function given in (2) and $B(m,n)$ is the complete beta function with

$$(m+n-1)B(m,n) = (n-1)B(m,n-1). \tag{6}$$

Using (5) the c.d.f. of X is given by

$$F(x) = \frac{1}{B(m,n)} \int_0^{F_0(x)} w^{m-1}(1-w)^{n-1} dw. \tag{7}$$

We call $F(x)$ the generalized logistic distribution with parameters m and n and denote it by $GL(m,n)$. This family of the standard logistic is quite rich and contains many of the well-known distributions as its special or limiting distributions. For example, if $m=n=1$, (7) becomes the logistic distribution; if $(m,n)\to(\infty,\infty)$, the $GL(m,n)$ converges to the normal distribution; it converges to an extreme minimum value if $(m,n)\to(1,\infty)$ and to an extreme maximum value if $(m,n)\to(\infty,1)$; it also converges to a double exponential if $(m,n)\to(0,0)$, to an exponential if $m\neq 0$ and $n\to 0$ or if $m\to 0$ and $n\neq 0$. It should also be noticed here that $GL(m,n)$ is symmetric if $m=n$, positively skewed if $m>n$ and negatively skewed if $m<n$.

The three forms of the standard generalized logistic derived by Balakrishan and Leung[4] are the special cases of our generalization. For $n=1$, (7) becomes $GL(m,1)$ and is given by

$$F(x) = [F_0(x)]^m \tag{8}$$

and p.d.f. of X as

$$f(x) = m[F_0(x)]^m[1-F_0(x)]. \tag{9}$$

$GL(m,1)$ is a family of positively skewed distributions with its coefficient of kurtosis greater than that of the logistic when $m>1$. For $m=1$, (7) is $GL(1,n)$ and is given by

$$F(x) = 1-[1-F_0(x)]^n. \tag{10}$$

Table 1 *Mortality of adult flour beetles after 5 h exposure to gaseous carbon disulphide*

CS_2 mg l	Dosage in $\log_{10} CS_2$ mg l	Number of insects exposed		Number of insects killed	
		Data set I	Data set II	Data set I	Data set II
49 06	1 6907	29	30	2	4
52.99	1.7242	30	30	7	6
56.91	1.7552	28	24	9	9
60 84	1.7842	27	29	14	14
64 76	1 8113	30	33	23	29
68 69	1.8369	31	28	29	27
72.61	1.8610	30	32	29	32
76.54	1.8839	29	31	29	31

It is a family of negatively skewed distributions with its coefficient of kurtosis greater than that of the logistic when $n>1$. For $m=n=P$, (7) is $GL(P,P)$ and is given by

$$F(x)=\frac{1}{B(P,P)}\int_0^{F_0(x)}[w(1-w)]^{P-1}\,dw. \qquad (11)$$

It is a family of symmetric distributions with a coefficient of kurtosis smaller than that of the logistic and decreasing with increasing values of P. This family of distributions is very close to the normal distribution.

We expect that this family of generalized logistic, in which the logistic and probit models are special cases, should provide a better fit. In the next section, we illustrate the application of the generalized logistic model, $GL(m,1)$, to two data sets derived from the experiments of Strand[5] with Tribolium confu...um.

Applications to dose-response data

The mortality of the adult flour beetles after 5 h exposure to known concentrations of gaseous carbon disulphide for two experimental series, designated as I and II, selected from the experiments of Strand[5] is given in table 1. A plot of percentage killed, in each data set, against dosage suggests that the dose-response curve may be skewed. Therefore the generalized logistic $GL(m,1)$ model is considered a good candidate to fit these data sets.

For comparison we also fit the logit model, and the results of the probit analysis obtained by Bliss[6] are included. The following three models are used:

Logit model

$$F_0(\alpha+\beta x_i)=\{1+\exp[-(\alpha+\beta x_i)]^{-1}\}.$$

GL(m, 1) model

$$F(\alpha+\beta x_i)=[F_0(\alpha+\beta x_i)]^{-m}.$$

Probit model

$$F(\alpha+\beta x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\alpha+\beta x}\exp(-z^2/2)\,dz,$$

where α, β and m are unknown parameters to be estimated from the observed data and x_i is the ith dosage (in $\log_{10} CS_2$).

Analysis of data set I: The logit model fit yields $\hat\alpha=-60.6156$ and $\hat\beta=34.2129$, while the $GL(m,1)$ model provides $\hat\alpha=-107.90$, $\hat\beta=59.43$ and $\hat m=0.3081$. Bliss[6] treated the data by fitting separate probit models to the lowest two concentrations yielding $\hat\alpha=-15.4336$, $\hat\beta=11.35$, and for the highest 3rd–8th concentrations the estimates are $\hat\alpha=-40.3838$ and $\hat\beta=25.51$.

Analysis of data set II: The maximum likelihood estimates of the unknown parameters obtained by the logit model are $\hat\alpha=-66.6508$ and $\hat\beta=37.6683$. The $GL(m,1)$ model fit yields $\hat\alpha=-195.60$, $\hat\beta=107.60$ and $\hat m=0.1695$. Bliss's estimates for the lowest two concentrations are $\hat\alpha=-15.1941$, $\hat\beta=11.35$, and for the highest six concentrations are $\hat\alpha=-39.7413$ and $\hat\beta=25.15$.

The expected number of insects killed, computed

Table 2 *Comparison of observed and expected mortality of adult flour beetles (data set I)*

Obs number killed	Exp number killed Logit	Probit	GL (0.3081, 1 0)
2	1.7072	3.1900	2.9133
7	4.9327	5.8140	5.5589
9	10.1465	7.5992	9.0901
14	16.3399	14.9094	14 3968
23	23 8448	23.8380	23 0922
29	27.9906	28 8300	28.6452
29	28.6495	29.4510	29.3829
29	28.3887	28.8985	28 8420

Table 3 *Comparison of observed and expected mortality of adult flour beetles (data set II)*

Obs number killed	Exp number killed Logit	Probit	GL (0.1695, 1.0)
4	1.4709	4.7310	2.9511
6	4.6217	8.0130	5.4372
9	12.5547	6.5976	10.8452
14	18.4368	16.0138	15.6331
29	27.3534	26.1327	27.3642
27	25.9571	25.9672	27.4325
32	31.0153	31.3728	31.9482
31	30.5901	30.8698	30.9957

by the three models as well as the observed number killed, are given in tables 2 and 3 for data sets I and II, respectively. Table 2 suggests that the *GL*(0.3081, 1.0) model is a more appropriate response function for data set I. On the other hand, table 3 shows that the *GL*(0.1695, 1.0) model fits data set II much better compared to the logit and probit models.

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1. Berkson, J., *J. Am. Stat. Assoc.*, 1944, **39**, 357.
2. Brown, C. C., *Commun. Stat.*, 1982, **A11**, 1087.
3. George, E. O. and Ojo, M. O., *Ann. Inst. Stat. Math.*, 1980, **32**, 161.
4. Balakrishnan, N. and Leung, M. Y., *Commun. Stat.*, 1988, **B17**, 25.
5. Strand, A. L., *Ind. Eng. Chem.* (*Anal. ed.*), 1930, **2**, 4.
6. Bliss, C. I., *Ann. Appl. Biol.*, 1935, **22**, 134.

NEW RECORDS OF FUNGI FROM INDIA

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DURING studies on curd-rot and its relationship with aeromycoflora at Agra, several fungal species were isolated, out of which the two species briefly described below were found to be new records for India.

Phoma medicaginis Malbr. & Roum. (original reference– *Rev. Mycologique Toulouse*, 1886, **8**, 91): Isolated from the aeromycoflora of a cauliflower field at Agra (Near Namner) in July 1986. Colonies on Czapek's agar dark-brown to black; hyphae septate, branched, blackish-brown; pycnidia developing in 15-day-old cultures, numerous, round to ovoid, black, ostiolated, 57–104 μm × 80–172 μm; conidia hyaline, single-celled, ovoid, 7 × 5 μm. Specimen deposited at CAB International Mycological Institute, Kew, England (IMI 313255).

Tritirachium oryzae (Vincens) de Hogg. (original reference—*CBS Studies in Mycology*, 1972, **1**, 22): Isolated from the aeromycoflora of cauliflower field at Agra (Near Namner) in October 1986. Colonies of Czapek's agar whitish, restricted, actinomycetous type, reverse pale yellowish; mycelium hyaline, tortuous, septate, sparingly branched, branches usually appearing near septa; conidiophore upright, long, slender, triverticillately to pentaverticillately branched; sporogenous branches tapering, 42–70 μm in length, fertile portion zig-zag; conidia apical on sympodially formed growing region, hyaline, unicelled, globose to ovate, 1.25–2.0 μm × 2.0–2.5 μm. Specimen deposited at CAB International Mycological Institute, Kew, England (IMI 319328).

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HERMAPHRODITE FLOWERS IN DIOECIOUS *MOMORDICA DIOICA* ROXB.

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MOMORDICA DIOICA is a semi-wild, perennial, tuberous and distinctly dioecious species of the Cucurbitaceae[1,2]. Polyploidization was induced in this species by colchicine treatment. Artificial triploids were also raised by crossing the diploid female with induced tetraploid male and reciprocal crosses[3]. All the cytotypes (diploid, triploid and tetraploid) were dioecious. But one of the hybrid males showed monoecious character and rare occurrence of hermaphrodite flowers on the branch bearing female flowers. This communication deals with morphology and pollen behaviour of hermaphrodite flowers.

Seeds of the hybrid between tetraploid female and diploid male were sown. During the first season one plant in the F_1 generation grew vigorously and produced only male flowers. In the following season the same tuber produced some female twigs and