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# Geometric Phase Experiments in Optics – A Unified Description*

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## INTRODUCTORY BACKGROUND

A few years ago, M. V. Berry made a curious observation[1] about the evolution of a quantum system in an energy eigenstate under the action of a hamiltonian $H = H[\underline{R}(t)]$, which is a function of periodic abiabatically varying parameters $\underline{R}(t)$, with period $T$, describing a circuit $C$ in the parameter space. Berry noted that at the end of a cycle the system returns to the original state with a circuit-dependent factor $\exp\{i\gamma(C)\}$ in addition to the familiar dynamical phase factor $\exp\{-i/\hbar \int E\, \mathrm{d}t\}$, where $E$ is the energy of the instantaneous eigenstate. Berry noted that $\gamma(C)$ has interesting properties in that it depends only on the circuit $C$ in parameter space and does not depend upon how the circuit is traversed, provided, the evolution remains adiabatic. Further, $\gamma(C)$ is non-integrable and is not single-valued, i.e., $\gamma(T) \neq \gamma(0)$. Under repeated traversals of a circuit $\gamma$ builds up. In a simple example of a particle in a state with spin quantum number $\sigma$ along the direction of a slowly rotating magnetic field $B$, Berry showed that the wave-function of the particle picks up, during each cycle of rotation, a geometric phase factor $\exp\{-i\sigma\, \Omega(C)\}$ where $\Omega(C)$ is the solid angle subtended by the circuit traced out by the magnetic field in the 'magnetic field space', at the origin of the space. $\sigma$ can be an integer or a half integer.

For a magnetic field of magnitude $B$ rotating about an axis making an angle $\theta$ with the field direction, $\Omega(C)$ is equal to $2\pi(1 - \cos\theta)$. Berry's original observation, referred to in literature as the 'Berry Phase' was soon placed in much more general framework by a series of authors. B. Simon[2] gave a simple geometric interpretation of Berry's phase as a parallel transport in a curved space appropriate to a quantum system.

Aharanov and Anandan[3] pointed out that it is not necessary to tie the notion of geometric phase to adiabatic evolution. A geometric phase is defined by them for any cyclic evolution on the 'projective Hilbert space' of the system under consideration. The latter, also called the ray space, is defined as the space obtained by assigning all quantum states differing only in phase to a single point. Aharanov and Anandan made the important observation that out of the total phase, if the dynamical phase is identified with the quantity $-1/\hbar \int \langle H \rangle\, \mathrm{d}t$, the remaining phase is a geometric phase which has the same interesting properties observed by Berry, namely, invariance of this phase w.r.t. details of the hamiltonian, which need not even be adiabatic. Geometric phase is completely defined by the circuit on the projective Hilbert space. In a yet further generalisation, my colleague Joseph Samuel and I showed that, using ideas proposed by Pancharatnam more than thirty years ago in the context of interference of polarized light, a geometric phase can be defined for non-cyclic as well as non-unitary evolution[4]. The important idea in this work is that open circuits in the projective Hilbert space can be closed by geodesics.

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I shall next describe in brief the contribution of S. Pancharatnam[5] to the subject. To do this I need to introduce the Poincaré sphere which is a very useful geometric representation[6] for the polarization states and for the polarization transformations of light and was used extensively by Pancharatnam in his work. A general state of a plane monochromatic wave of light is represented by a set of two complex numbers (four real numbers) which represent the amplitudes and phases of the two orthogonal components of the electric field in the plane transverse to the propagation direction. If we are dealing with light of fixed intensity, say 1, we are left with three independent real numbers. Further, if we seek a representation in which all states of a wave differing only by an overall phase are represented by the same point, we are left with two independent real numbers. These can be mapped onto the surface of a sphere, called the Poincaré sphere, henceforth denoted by PS, such that all states of linear polarization are represented by points on the equator, a rotation by an angle $\theta$ of the polarization in real space amounting to a rotation by $2\theta$ on the PS. The two poles represent the right-hand circular (RHC) and the left-hand circular (LHC) states and points on the rest of the sphere represent all the elliptically polarized states (see figure 1). A general intensity-preserving

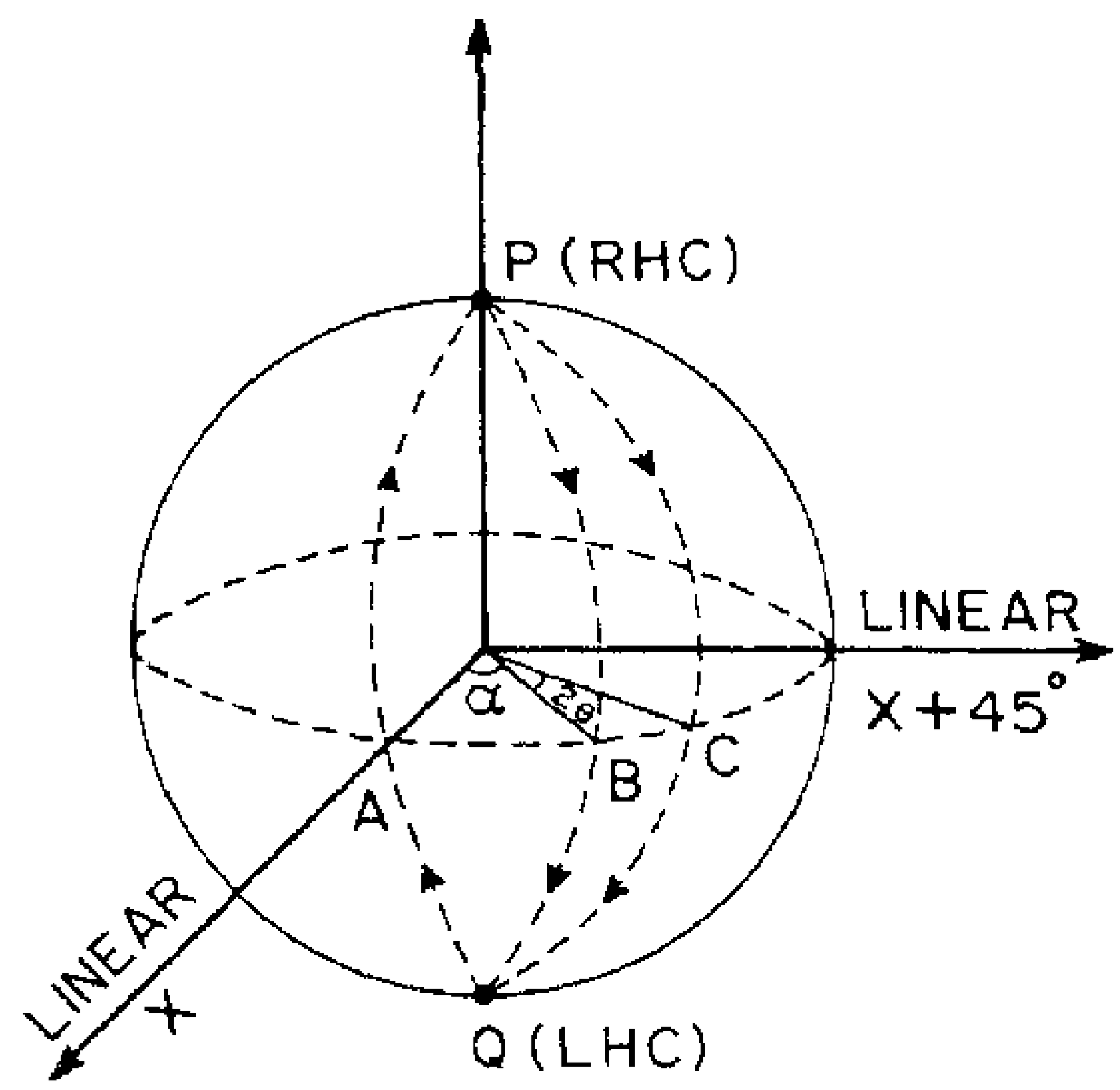


**Figure 1.** Circuits on the Poincaré Sphere corresponding to the experiment of ref. 15. An angle $\theta$ of rotation of the HWP in real space corresponds to a rotation through $2\theta$ on the Poincaré Sphere. The measured Pancharatnam phase equals half the solid angle subtended by the slice PCQBP at the centre of the sphere and equals $2\theta$.

polarization transformation (3 parameters) is represented by a rotation about an axis joining the centre of the sphere to some point on the sphere (2 parameters) by a certain angle (3rd parameter). Let me also note that this representation is isomorphic to a similar representation for the 'ray space' of a quantum mechanical spin-1/2 particle. Pancharatnam made two important contributions: (i) He proposed a very reasonable, physically motivated criterion for comparing the phases of two different states of polarization of light, i.e., waves in two different states are in phase if their interference given maximum intensity, (ii) Pancharatnam noted that if a wave in state 2 on the PS is in phase with a wave in state 1 according to the above criterion, a wave in state 3 is in phase with 2, then 1 and 3 will not, in general, be in phase with each other. He gave the exact formula for the excess phase, i.e., equal to half the solid angle subtended by the spherical triangle formed by joining 1, 2; 2, 3 and 3, 1 by geodesics. It was noted by my colleagues Ramaseshan and Nityananda[7] that this result is an early example of the 'Berry Phase'.

## SURVEY OF GEOMETRIC PHASE EXPERIMENTS IN OPTICS

Experimental realizations of geometric phases have been reported in several fields, e.g. molecular physics, neutron spin rotation, nuclear magnetic reasonce and optics. A good collection of references can be found in Suter *et al*[8]. Here I wish to give a brief review of the experimental realizations of geometric phases in optics, presented in a logical, rather than chronological order, and then go on to describe an interesting framework proposed by me recently for describing the evolution of light beams along arbitrary space curves while they undergo mirror reflections and arbitrary polarization changes on the way. This framework emerged as an attempt to understand all the reported geometric phase experiments in optics in a single framework.

The basic challenge in any experimental observation of a geometric phase is the difficulty of separating this small effect from the dynamical phase which is always dominant. One strategy is to measure a related quantity in which the dynamical phase by definition cancels. This is the strategy used in the first of these experiments. The basic idea of this experiment proposed by Chiao and Wu[9] and reported by Tomita and Chiao[10] is very simple and can be explained with the help of Berry's original example of a quantum mechanical particle of spin $\sigma$ undergoing rotations in space. A circularly polarized photon corresponds to $\sigma = \pm 1$. A circularly polarized beam of laser light travelling along a mono-mode optical fibre twisted in the shape of an arbitrary space curve with the two ends pointing in the same direction acquires a geometric phase $\delta$ which is equal in magnitude to the solid angle $\Omega(C)$ subtended by the circuit traced out by the direction of the beam on the sphere of directions, the sign being opposite for the two senses of circular polarization. The two circular polarizations, however, see exactly the same amount of dynamical phase. If, therefore, we could measure the difference of the geometric phases seen by RHC and LHC, the dynamical phase cancels exactly and one measures $2\delta$. The angle of rotation of the direction of polarization of a linearly polarized beam after the beam has gone through a cycle of directions is precisely half this quantity which is measured in[10] by use of helically wound optical fibres of varying pitch, hence as a function of the solid angle in the space of directions. The solid angle dependence of the rotation angle is seen to be linear as predicted, with the expected slope. I could demonstrate this with the help of a very simple gadget consisting of a glass rod bent in a shape consisting of four straight sections connected by 3 bends such that light propagating along the rod would start out in the $x$-direction, then propagate in the $z$-direction, then in the $y$-direction and finally in the $x$-direction again. The other part of the gadget is a small plate which represents the wavefront of the propagating wave with a hole in the centre to let the rod through, and a red line representing the electric vector. Sliding this plate along the rod represents propagation. On the sphere of directions, light propagating along this 'fibre' describes a closed circuit consisting of three great-circle arcs, each arc being a quarter circle. This spherical triangle encloses an area (1/8th of the sphere) which subtends $\pi / 2$ sterradian solid angle. To duplicate light-propagation along a bend, I would slide the plate around the bend in the most natural possible way, namely without any rotation about the instantaneous direction of propagation. Transporting a vector (the red line) this way, which amounts to keeping the component normal to the plane of the bend invariant and rotating the component in the plane of the bend by the angle of the bend has been known in literature as Fermi-Walker Transport. It has, however, been a common practice to call it 'parallel-transport'. I shall use this term to describe such a transport from now on. As we shall see later, this transport plays an important role in the proposed framework for light propagation. It is easily seen that transporting the plate by this rule along the three bends (closed circuit on the sphere of directions) results in a net rotation of the electric vector by a right angle, i.e., $\pi / 2$ radians. Bending the glass rod in different shapes which amount to different solid angles on the sphere of directions will give a different angle of rotation of the polarization.

Soon after the experiments of Tomita and Chiao, it was proposed by Kitano *et al.*[11] that geometric phases could be observed in experiments in which the light beam was made to trace a space-curvewith the help of metal mirror reflections. These experiments have a new feature, namely, the lack of adiabaticity in the evolution of the beam associated with reflections. The law of parallel transport described above is not valid under these conditions[12]. Under ideal metal-mirror reflections, for example, a right-circularly polarized beam becomes left-circular and vice-versa. Kitano *et al.*[11] suggest that under ideal metal mirror reflections the evolution be looked upon as being one in a modified $\vec{k}$-space, with the sign of $\vec{k}$ flipped for every alternate mirror reflection. An actual experimental realization of this idea was reported by Chiao *et al.*[13] who use a helical interferometer consisting of two symmetric helical arms of opposite helicity with the shape of each arm, involving four mirror reflections, being adjustable to give varying solid angle on the sphere of directions. By measuring a fringe shift, Chiao *et al.* measure a quantity which is equal to four times the geometric phase seen by a circularly polarized photon in traversing either arm. They interpret this result in terms circuits on the 'space of spin-directions' of the photon. This is a 2 to 1 mapping in which the states $|\vec{K}\rangle|RHC\rangle$ and $|-\vec{K}\rangle|LHC\rangle$ are mapped onto the same point on the sphere.

Let me come next, to the experiments that measure the other kind of geometric phase in optics, namely the Pancharatnam phase due to circuits on the PS described earlier. This involves a cycle of polarization transformations. These experiments also fall in two categories:

(i) those that do not involve mirror reflections in an essential way, and

(ii) those that do.

Two experiments of the type (i)[14,15] were reported from the Raman Research Institute. In the first of these, my colleague Joseph Samuel collaborated with me. The principle of both these experiments is the same and is very simple. The linearly polarized beam from a stabilized He-Ne laser is split into two beams by means of beam splitter (figure 2). In the experiment of ref. 15, one of these, say

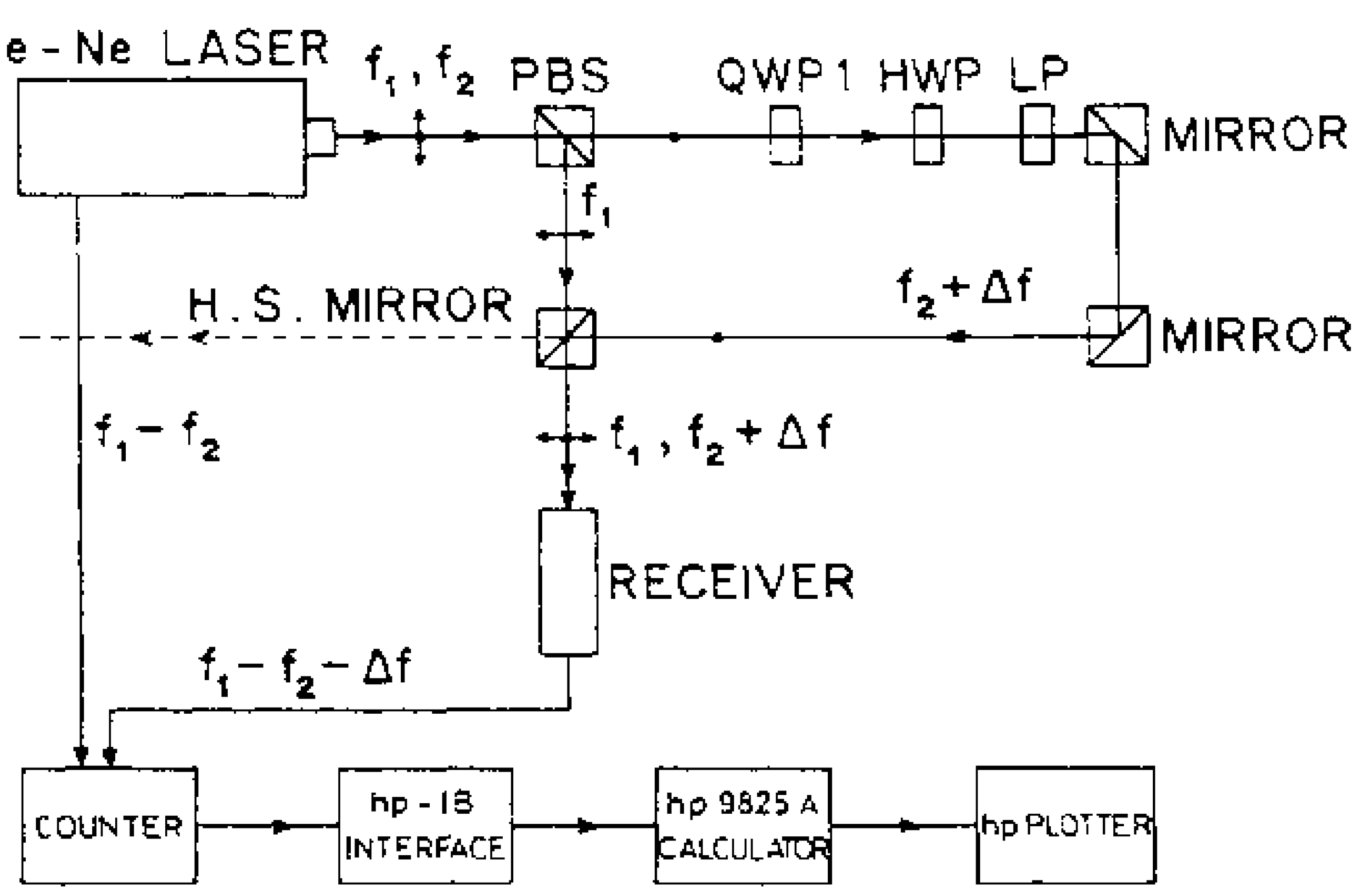


**Figure 2.** Schematic diagram of the experimental set-up of ref. 15. A stabilized, single-mode He-Ne Zeeman laser (6328 Å) which outputs two frequencies $f_1$ and $f_2$ ($f_1 - f_2 \approx 2$ MHz) in the two orthogonal linear polarizations is used. PBS is a polarizing beam splitter, H. S. Mirror is a half-silvered mirror. The phase change produced is measured by the counter and can be looked upon as the time-integral of an 'instantaneous frequency shift' $\Delta f$ of the measurement beam.

the measurement beam is taken along a cycle of polarization transformations represented by the circuit APBQA on the PS (figure 1) by means of (i) a quarter-wave plate (QWP1) oriented with its principal axes at 45° to the electric vector in the beam, that corresponds to the part *AP* of the circuit, (ii) a half-wave plate with its axes oriented at an angle $90° + \alpha/2$ to those of QWP1 that correspond to PBQ and (iii) a linear polarizer LP, that corresponds to QA. In the process, the beam acquires a geometric phase equal in magnitude to half the solid angle subtended by the area APBQA at the centre of the sphere.

The absolute value of this phase is difficult to determine, because it would, in general, be buried in a much larger magnitude of dynamical phase arising out of unequal path lengths in the measurement and the reference beams. The quantity that *can* be measured with adequate sensitivity, however, is the *change* in the geometric phase as the circuit APBQA is changed to APCQA by rotating the half-wave plate (HWP) about the beam axis through an angle $\theta$.

This would show up as a relative phase change between the two beams as recorded by the receiving system of the Hewlett-Packard laser interferometer system used in the experiment, schematically shown in figure 2. This instrument is capable of recording on-line, phase changes of $\lambda/40$ and upwards alongwith their sign, where $\lambda$ is the wavelength of He-Ne laser light.

The sign of the phase change observed in the experiment depends upon the sense of rotation of the HWP which determines the sense of rotation of the circuit on the PS. By the solid angle formula, keeping track of all the relevant factors of 2, one expects to observe a phase change of $2\theta$ radians for a rotation $\theta$ radians of the HWP. Figure 3 shows an actual observed record of phase change as a function of the angle of rotation of the HWP. In the experiment, the HWP was rotated through two full rotations in one sense and then through two full rotations in the opposite sense, while the phase change was being continuously recorded by the instrument. The good agreement with the expected results is evident. For the circuits shown, the change in the dynamical phase on rotation of the HWP will be zero if a good optical quality HWP is used and the axis of rotation coincides

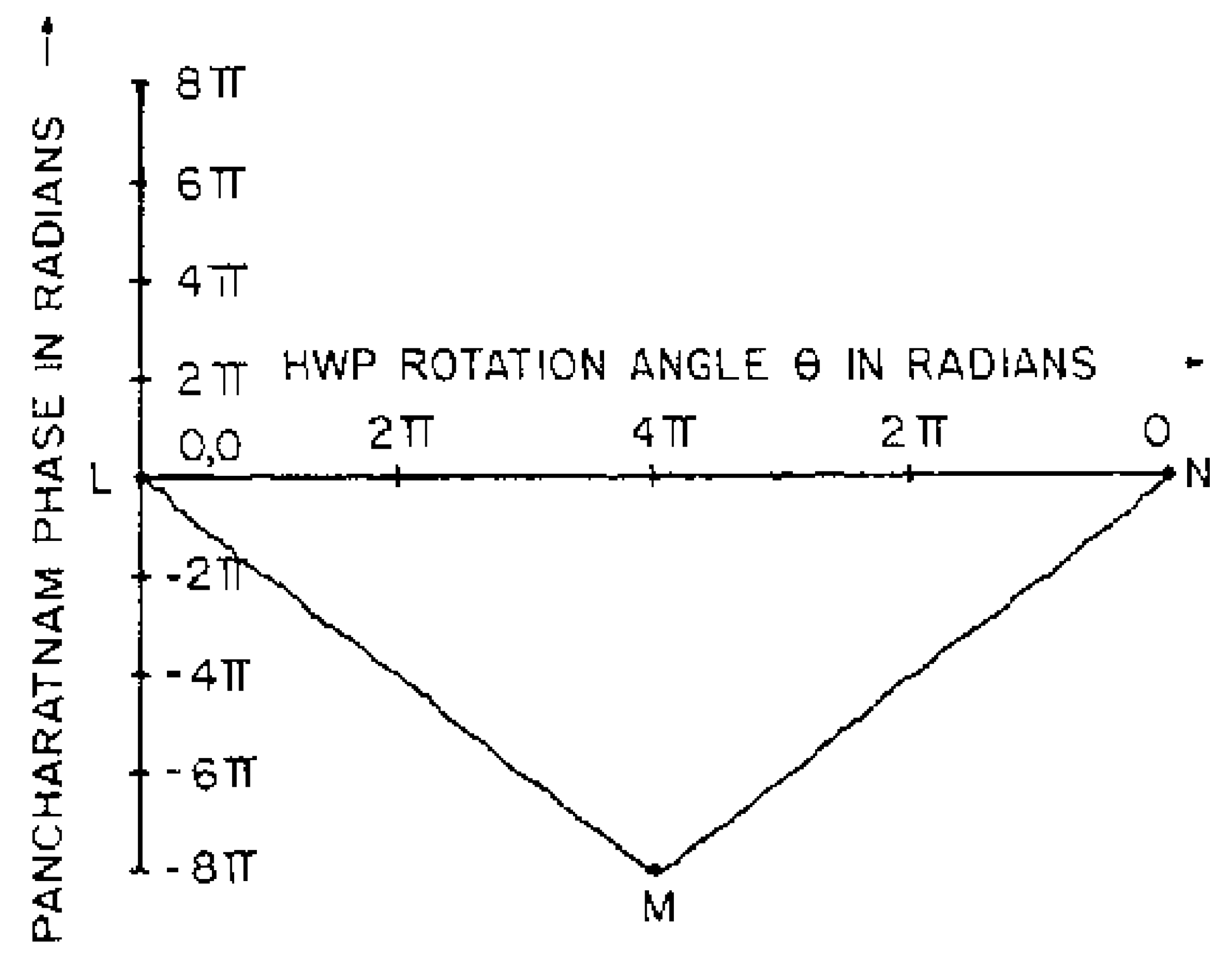


**Figure 3.** Observed variation of the phase of the measurement beam as a function of the angle of rotation of the HWP in a single, unbroken phase measurement in the experiment of ref. 15. Theoretically predicted curves are the straight lines LM and MN. The curve is uniformly sampled in $\theta$, with a sampling interval of 10 degrees.

with the laser beam. Any dynamical phase change resulting from rotation of the plate would show up as a ripple on the straight line, but *must* repeat with a maximum period of $2\pi$ radians. The fact that the linear phase change continues beyond a full rotation of the HWP and returns to the original value after an equivalent amount of reverse rotation makes it almost impossible to attribute it to anything but a geometric phase. The experiment thus provides a striking demonstration of the anholonomy and the unboundedness associated with geometric phases. Particular care was taken to verify that the absolute sign of the observed phase change is in the direction predicted by theory. To do this one needs to establish the absolute sense of rotation of the electric vector in the circularly polarized beam coming out of QWP1. This in turn needs absolute identification of the fast and slow of QWP1. I conducted independent path length experiments to identify the fast and slow axes of all the wave plates and discovered in the process that the specifications provided by the suppliers of some of our QWP'S (Ealing 34–5835) in this regard were wrong. This was later confirmed by the manufacturers (Special Optics, U.S.A.). In my first attempt I did get the wrong sign for the Pancharatnam phase for this reason! I hold a somewhat cynical view towards sign determinations (including my own). Observing the correct sign usually means that the number of mistakes made is even. In this experiment, therefore, I tried to make the sign determination as convention-free as possible. Let me also point out that the curve of figure 3 implies that Pancharatnam phase is odd under reversal of the sense of traversal of the circuit on the PS. Another interesting point: If the HWP were rotated with a uniform angular velocity, which is more or less what is done in our experiments, the horizontal axis in figure 3 could be regarded as the time axis and the slope of the observed straight line as a frequency-shift of the laser beam.

This aspect was highlighted by Simon *et al.*[16] in one of the three reported Pancharatnam phase experiments that involve mirror reflections[16–18]. They suggest its use in the fine-tuning of lasers. Tompkin *et al.*[18] study the effect on the observed geometric phase of replacing the ordinary metal mirror with a phase-conjugate mirror. In these experiments, one half of the circuit on the PS is traversed during the first pass of the beam through the polarization-transforming elements and the other half is traversed during the reverse pass after a 180° mirror reflection (see figures 4a and 4b). The relative phase change of $\pi$ between the two components of the electric field accompanying a perfect metal-mirror reflection introduces a new feature in the problem which is similar in nature to that encountered in the case of the momentum-space experiments with mirrors[11,13]. This new feature led me to propose that in geometric phase experiments, the relevant state-space is a direct-product of the PS and the momentum-space ($\vec{k}$-space) of light. For problems in which $|\vec{k}|$ is constant, the surface of the $\vec{k}$-sphere or the 'space of directions' replaces the $\vec{k}$-space. This yields a unified description of all these experiments in terms of a single entity of which the different experiments are special cases. In the first exercise[19], I constructed a model space which is a direct product of a one-dimensional $\vec{k}$-space and the PS. The 1-d $\vec{k}$-space is tagged on to the radial coordinate of the PS. This

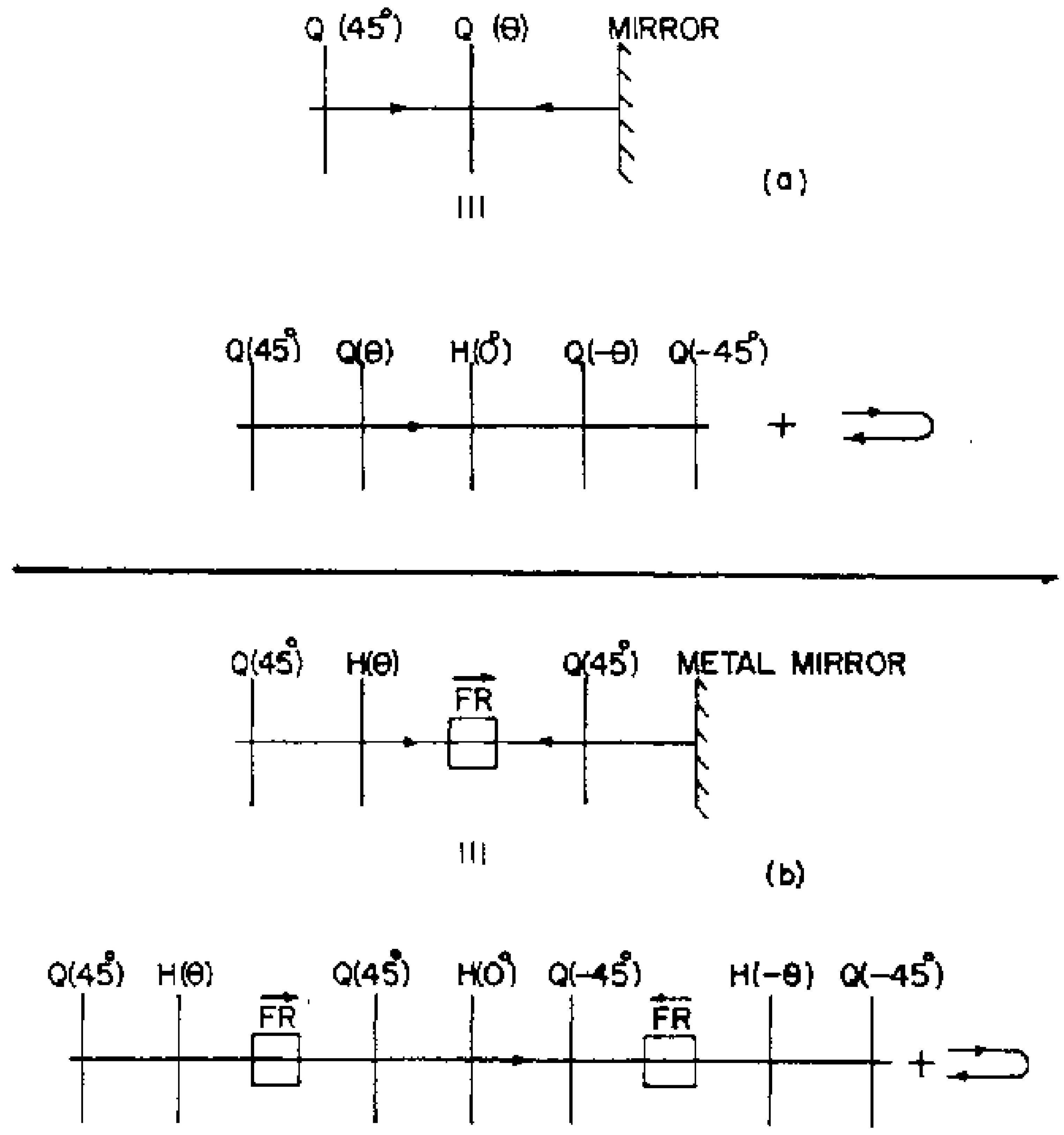


**Figure 4.** Illustration of the use of the decomposition scheme in case of one-dimensional Pancharatnam phase experiments : (a) the experiment of Simon *et al.*[16] and Chyba *et al.*[17], (b) the metal mirror part of the experiment of Tompkin *et al.*[18]. Arrow on FR represents the direction of the magnetic field, H stands for a half-wave plate and Q stands for a quarter-wave plate.

product-space suffices for a description of all the 1-d Pancharatnam phase experiments and can be easily visualized. The observed Pancharatnam phase is explained in this model in terms of half the solid angle subtended by the 'projected circuit' on the PS at the centre. These projected circuits are different in shape compared to the circuits drawn in refs. [16–18] and do not require a generalized PS[18]. As a matter of cynical caution perhaps, I also repeated the Pancharatnam phase experiment with mirror (Michelson interferometer configuration) on my interferometer set-up and verified that for a given sense of circular polarization coming out of QWP1, a given sense of rotation of QWP2 (or HWP) give the same sign of phase change in both, the single-pass and the double-pass experiments. This is in agreement with the prediction from the new circuits on the PS.

In the second part of this exercise[20], I proposed (i) a decomposition scheme for an arbitrary evolution of a light beam which involves arbitrary changes of polarization, changes of direction and mirror reflections and (ii) a representation of such an evolution in terms of paths in a sub-space of the projective Hilbert Space of a massive spin-1 particle which enables the problem to be cast in the framework of Aharanov and Anandan[3].

## THE DECOMPOSITION SCHEME

The decomposition scheme allows the separation of the general problem into two problems, the first involving evolution on the PS alone and the second involving adiabatic propagation of a given polarization along a space curve. Both these problems being separately well-understood, one thus has a complete solution of the problem. The main ingredients that go into this scheme are (a) a new way of describing a mirror reflection in terms of a rotation of the beam, followed by a wave-plate (a half-wave plate for ideal metal-mirrors) and (b) a law of transport along the light path for polarization-transforming elements having preferred axes. This is just the law of parallel transport I described earlier.

Using these ingredients, for any general train of elements, one can construct an equivalent train which consists, in the first part, only of polarization-transforming elements and no change in the direction of the beam and, in the second part, propagation along a space-curve and no polarization-changing elements. Two examples of such a decomposition are shown in figures 4a and 4b. The corresponding circuits on the PS are shown in figure 5. These agree with the circuits obtained in the 1-d $\vec{k}$-space model[19].

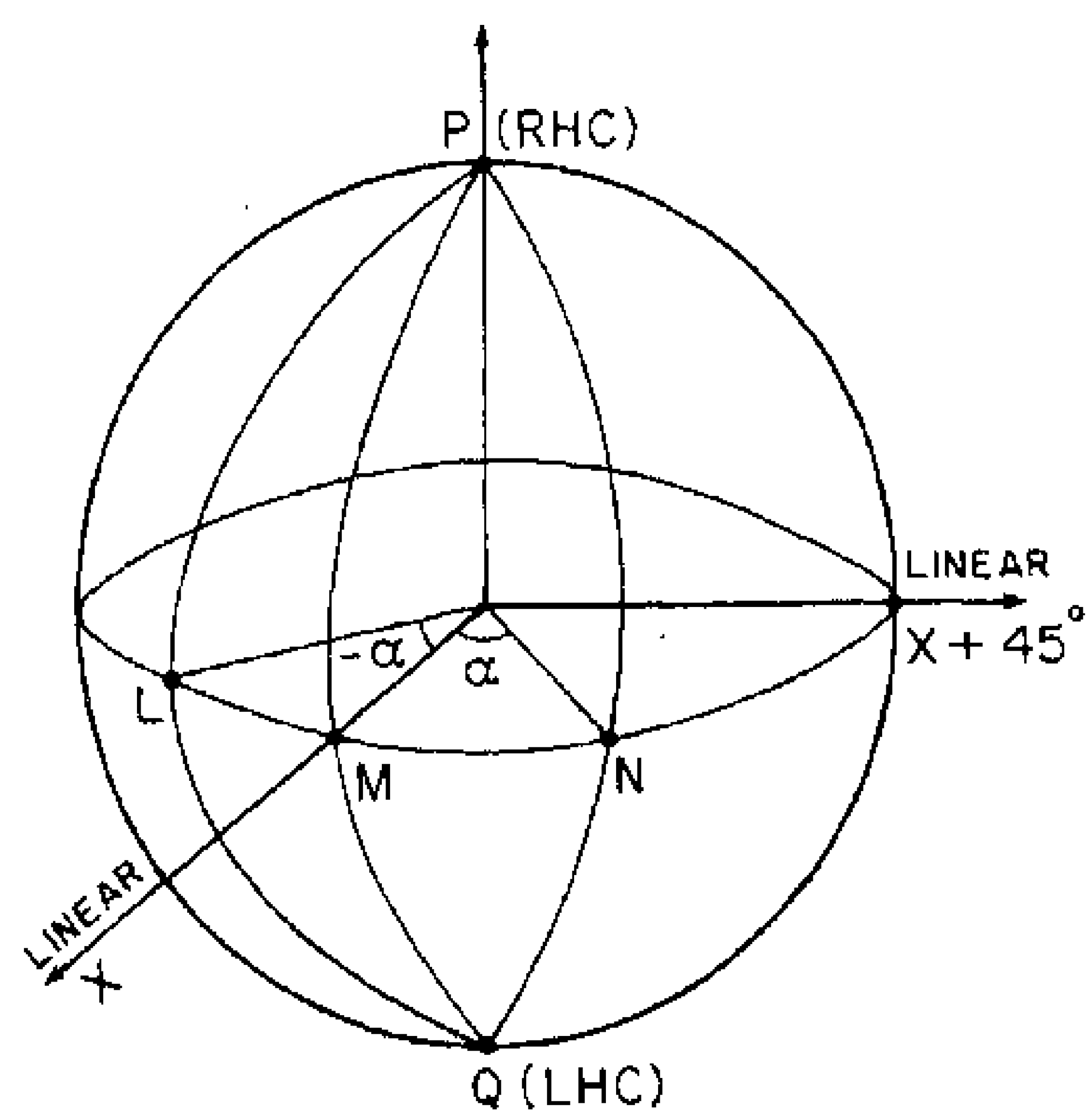


**Figure 5.** Circuits on the PS corresponding to the one-dimensional Pancharatnam phase experiments: **(a)** In the experiments of Simon *et al.*[16] and Chyba *et al.*[17], the relevant circuit is MQNLQM, **(b)** In the experiment of Tompkin *et al.*[18], the relevant circuit is MPNQMQLPM.

The full power of such a decomposition becomes obvious, however, only in three-dimensional propagation of light beams. To illustrate this, let us analyse the experiment of Chiao *et al.*[13]. Each of the two arms of their helical interferometer involves four nearly ideal metal mirror reflections which, in this scheme, are replaced by four half-wave plates. These, when properly transported to one end of the light path using the law of transport, separate into two pairs, each consisting of two half-wave plates with their axes oriented at right angles to each other. Such a pair is equivalent to a plane glass plate, hence to 'nothing'. The four reflections therefore add to 'nothing' and the evolution of the beam in each arm, therefore, is equivalent to a simple propagation along a space curve.

One therefore expects to observe a phase equal to the solid angle of the light path on the sphere of directions or the $\vec{k}$-sphere. This is what, I believe, they observe. The analysis mentioned above holds for an arbitrary number of mirrors and shows immediately the essential difference between an odd number of mirrors and an even number of mirrors. An odd number of half-wave plates cannot be made to add to 'nothing' (try circular polarization). If, for example, one wished to construct a device that rotates an arbitrary linear polarization by a certain fixed angle $\theta$, this analysis immediately shows that this cannot be done with an odd number of mirrors. On the other hand, with an even number of mirrors it can be done. Each arm of the helical interferometer in the experiment of Chiao *et al.*[13] is in fact such a device.

## THE SPIN-1 REPRESENTATION

In this representation, the projective Hilbert space for the problem is identified with the projective space constructed out of a sub-space of the Hilbert space $\mathcal{H}$ of a massive spin-1 particle. This is constructed in the following way. Take the axis of quantization to be along the initial direction of propagation of light. The sub-space of $\mathcal{H}$ describing this beam is given by the set of three-spinors col.$(c_1, 0, c_2)$, where $c_1$ and $c_2$ are arbitrary complex numbers such that $|c_1|^2 + |c_2|^2 = 1$. In order to describe all states of the beam travelling in all possible directions, one constructs the space of states: $R_z(\phi)\, R_y(\theta)$ col.$(c_1, 0, c_2)$, where $R_z(\phi)$ and $R_y(\theta)$ are spin-1 rotation matrices for rotation about the z-axis by an angle $\phi$ and rotation about the y-axis by an angle $\theta$ respectively. The projective space constructed out of the above space is the relevant state-space for the problem. An arbitrary polarization transformation in the initial section of the beam is represented by a block-diagonal matrix $S$, such that $S_{22}= 1$ and $S_{ij}$ for $i, j=1, 3$ is a $2 \times 2$ matrix which, for intensity-preserving polarization transformations, is an element of the SU(2) group and for transformations involving polarizers which change total intensity, a suitable non-unitary matrix.

If the space-curve defined by the propagating light beam is thought of as being made of straight segments along the directions $\vec{k}_0, \vec{k}_1, .. \vec{k}_n$, connected by 'bends', then the evolution around the bend from $\vec{k}_{i-1}$ to $\vec{k}_i$ is represented by the spin-1 rotation matrix $R_{\hat{n}_i}(\theta_i)$, where $\hat{n}_i$ is the unit vector normal to the plane of the bend and $\theta_i$ is the angle of the bend. A polarization transformation in the i th section of the beam is given by the operator:

$$R_{\hat{n}_i}(\theta_i) \ldots R_{\hat{n}_1}(\theta_1)\, S\, R^{+}_{\hat{n}_1}(\theta_1) \ldots R^{+}_{\hat{n}_i}(\theta_i)$$

where $S$ is the operator corresponding to the same element, transported to the first section of the beam according to the law of parallel transport stated earlier.

This scheme casts the problem in the framework of Aharanov and Anandan[3] and the various geometric phase experiments emerge as special cases of the general problem. In the description of the experiments of Chiao and Wu[9] and Tomita and Chiao[10], only the $R$ operators would be involved. In our experiment[15], only the $S$ operators would be involved. In the other experiments, a combination of the $R$ and the $S$ operators would be involved. In special cases, simpler sub-spaces which are easy to visualise can be constructed, e.g. the space of spin directions of Chiao *et al.*[13], the modified momentum space of Kitano *et al.*[11] and the direct product of a 1-d k-space and the PS[19]. The full implications of the above representation and its relation to previous work, e.g. Bouchiat and Gibbons[21], Samuel and Bhandari[4] and Jordan[22, 23] is at present being studied. I would like to mention at the end that there is yet another class of geometric phase experiments, namely those involving the Lorentz group of transformations[24, 25] which I have not commented upon. Optics thus provides a rich arena for experimental realizations of this fascinating concept of geometric phases. The proposed framework is a by-product of these experiments and it is hoped that the decomposition scheme will be found useful in optical design problems.

I have learnt two lessons from this whole enterprise : (1) Every experiment teaches us something new and is worth doing and (2) Treatment of mirrors requires a lot of reflection!

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