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Computer aids in production engineering

N. Chandrasekaran

Solutions to several industrial production problems call for a systems approach involving integration of computer-based techniques for performing material flow analysis with press tool design. Software packages have been developed that perform the complex mathematical analyses required for optimizing design and system parameters. Computational tools are thus key to competitive production engineering.

Modern production engineering environment requires that a technological system is primarily controlled by means of extensive scientific inputs. The inputs are provided by mathematical formulations, which tend to be rather complex when in the field of solid mechanics. Solutions to the mechanics problems often entail making suitable approximations that will simplify the mathematics. The advent of powerful modern digital computers has dispensed with the need to accept inexact solutions. Most accurate solutions can be obtained using numerical treatments. Another feature is that these techniques allow the development of software packages in modular form. Still, the metal forming industry has been frustrated by the fact that the commercially available software packages are mostly irrelevant to their needs, because the formulations, though sound in fundamentals, often tend to ignore certain critical engineering aspects of the problem that are peculiar to the practitioner.

Whether it is forging, extrusion, rolling, deep drawing, flow turning or any other metal forming operation, the flow of metal within the body is responsible for bringing about permanent shape changes. Metal flow is associated with displacements, strains and stresses. Each of them is related to the other by material constitutive equations, flow rule, associated flow rule, displacement–strain relationships, etc.[1–4] Apart from the deformation behaviour of the work material, the machine tools (press, rolling mill, etc.) and the tooling (die, punch, etc.) involved in the production process significantly affect the overall system response. Suffice it to state at this juncture that numerous parameters representing the machine tool, production tooling and work material play equally important roles.

This leads us to the crux of the problem. Traditionally, academics and textbooks have treated metal flow and tooling problems in isolation, as evidenced by the published literature. The important corollary is that one can perform the metal flow (stress-distribution) analysis, but cannot use the information to design the tooling, or vice versa; in other words, that technology lacks scientific input. One reason for this trend may be the non-availability until recently of systems with enormous computing power at the desk-top level. Advancements in the field of computers have further brought powerful graphics engines with the systems, which facilitate problem definition.

It is generally accepted that no single software can solve a wide range of practical problems. In this article I describe the salient features of selected fundamental software packages that were found to be useful in the consultancy work performed by me for the American Dow Chemical Co., the Canadian Cosma International, Fabricated Steel Products Division of Indal Ltd, etc. Since the methodology essentially uses a short-term approach that derives benefit from long-term development work, I first briefly discuss the motivating factors.

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A parallel and a lesson

It may come as a great surprise to note that the realization that the material manufacturer (the steel companies), the car maker and the user (the subcontracting stamping companies) must work in unison dawned on the North American automotive industry not until well into the eighties. It is also a fact, even at the cost of sounding uncharitable, that this was triggered more by the inroads the Japanese were making into the traditional American car markets. The only way the industry would have survived the onslaught was by cutting cost and production time. This, in turn, could be accomplished only by reducing the number of costly trials, which calls for a scientific understanding of the problem. This is a classic example of a crisis fuelling a development that should have anyway taken place. One should bear in mind that the typical time taken to freeze a tool-design and fabrication process is anywhere between a half to two years and the costs involved often run to over a quarter million dollars.

Also consider the other facts. The automotive industry is deemed to be a high engineering, low-technology set-up, mainly because the toolings for the components that are in production are designed by tool makers with little or no scientific input. The mathematical complexities of plastic deformation-process analysis are so great that communication links between universities, which are the primary source of such mathematical models, and industries snap no sooner than they are established. Universities realized that the solutions to production problems are often either not publishable or not tractable. Industries, in turn, could find little use for the abundance of theoretical models appearing in the literature. But once control became the main issue, the emergence of consortiums, funded by industries, became inevitable. In the process, the university-trained scientists gained access to the production environment and modified the theoretical models to suit the production requirement.

The phase India is going through today is not very different. It is also understandable that we cannot afford the luxury of high software-development costs. Fortunately we can devise ways of avoiding this expenditure. *Time*'s prophecy, when it named the computer 'Man of the Year', that the microchip can allow the developing countries to neutralize certain gains of the West or the Far East by decreasing the number of development steps is relevant here.

Metal flow lines

Figure 1,*a* shows a test specimen of a cylindrical collar, which is one of the shapes used for predicting failure in forging and machinability in machining (see refs. 5, 6 for related work). In the figure, *r* and *z* denote the radial and axial directions respectively. A grid pattern, employed for performing a finite-element analysis (FEA), is shown only in the first quadrant. distorted mesh shape in Figure 1,*b* is obtained afte specimen has been subjected to open-die upset fo operation. The predictions were made using a com program called FARM originally developed at the versity of Hannover[7]. The arrows in Figure 1,*b* depic velocity directions at the grid intersection points known as nodal points). The directions repr tangents drawn to the metal flow lines at the respe points. The magnitude of the material flow veloci proportional to the length of the arrow at a location. Alternatively one could have devised ex ments to obtain the same pattern. I have rec developed a program called VISIO, which perfor complete strain and stress analysis once the dist mesh shape is thus obtained[8].

The flow of metal during extrusion (see Figure not dissimilar to the flow of toothpaste emerging o a collapsible tube when it is gently squeezed. Ob that the diameter of the container (tube) is greater the diameter at the exit. The area reduction is bro about by providing a gentle conical taper. The ta region where the plastic deformation occurs is kr as the die region. When the extrusion pressu applied at one end, the material has a tendency to

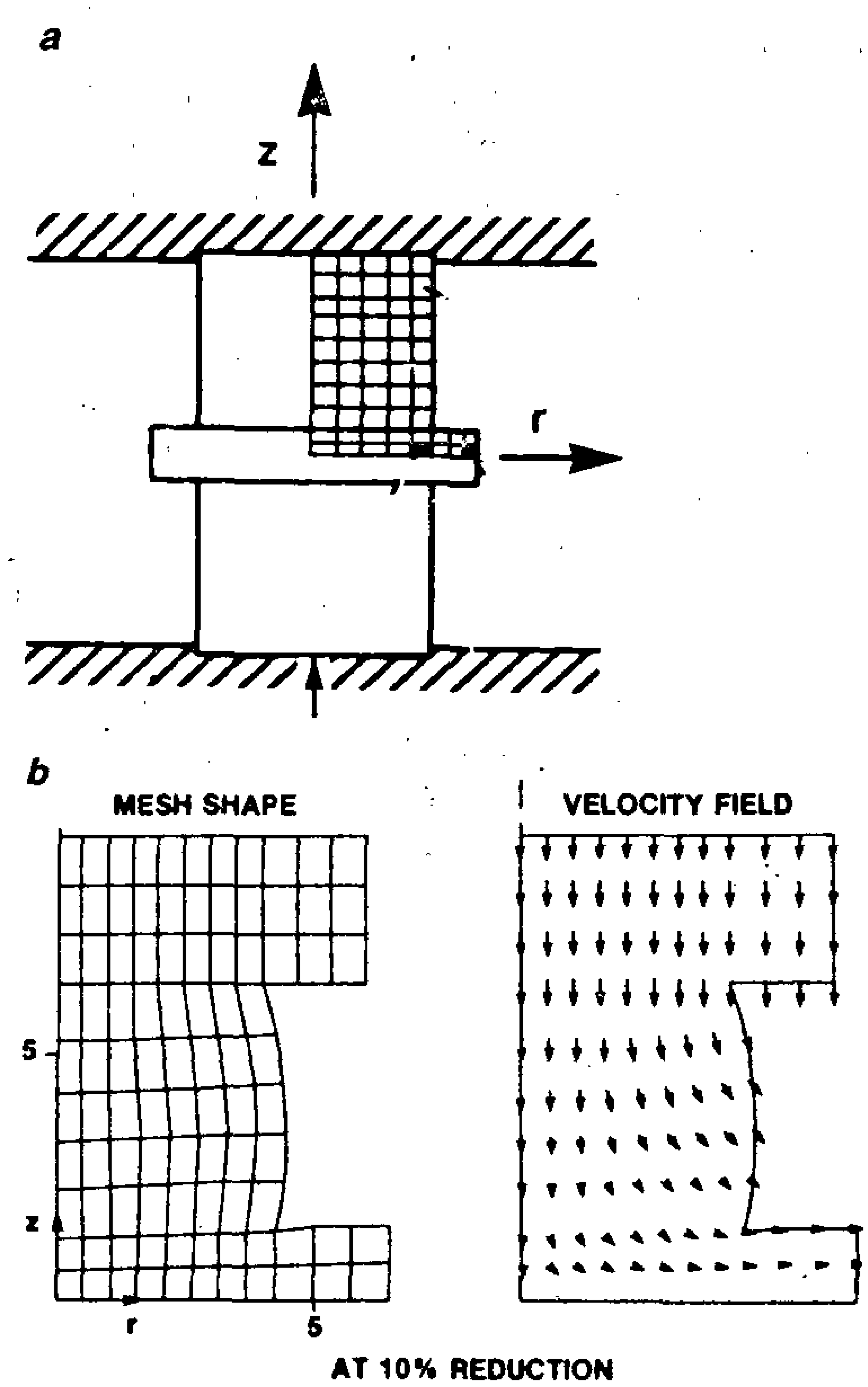

Figure 1. *a*, Cylindrical collar test specimen. *b*, Predicted ve field and mesh shape at 10% reduction.

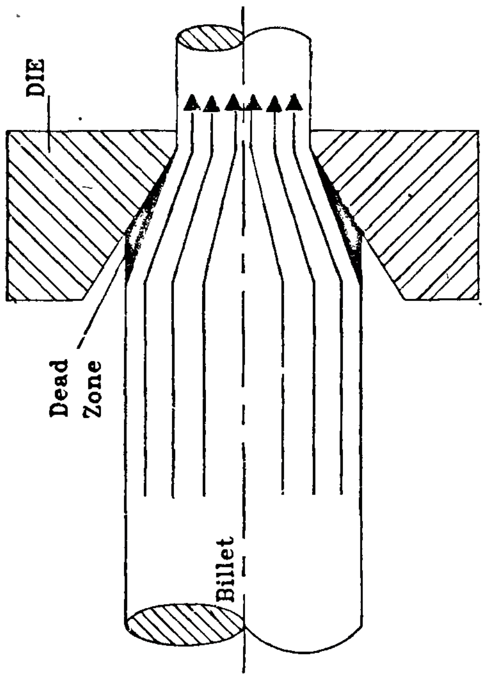

Figure 2. Schematic illustration to depict the development of 'dead zone' during extrusion.

naturally along certain streamlined paths. A tool engineer who designs the shape of the die must, in order to constrain material flow, ensure harmony between the preferred flow path of the metal and the tooling profile. Often a designer has no clue to how exactly the metal flows if appropriate analytical tools are not readily available. Even simplistically assuming that the flow takes place along straight-line paths in the deformation zone, it is no easy task to calculate the slope of the path at the workpiece–die interface. If the die inclination angle is made steeper than this slope, a dead metal zone will form (see Figure 2).

Following Nagpal and Altan[10], Chandrasekaran[9] designed a streamlined die profile (see Figure 3). The non-reentrant shape shown is a fourth-degree polynomial, whose coefficients have been optimized by employing the classical plasticity theory-based approach[9]. The program STRM-DIE generates the optimized contours and directs its output to a CAD software. This avoids data spills and the contour can be directly machined using an interface to a numerically controlled machine tool.

It is pertinent to point out that most advanced materials, like metal matrix composites (MMC) for instance, acquire strength mainly through plastic deformation processing. Forging, extrusion, rolling, etc. are a few of the preferred forming techniques employed to impart strength and other desirable properties. One may not be able to process materials like MMC at all unless tools are made employing advanced design concepts that lower the forming severity considerably.

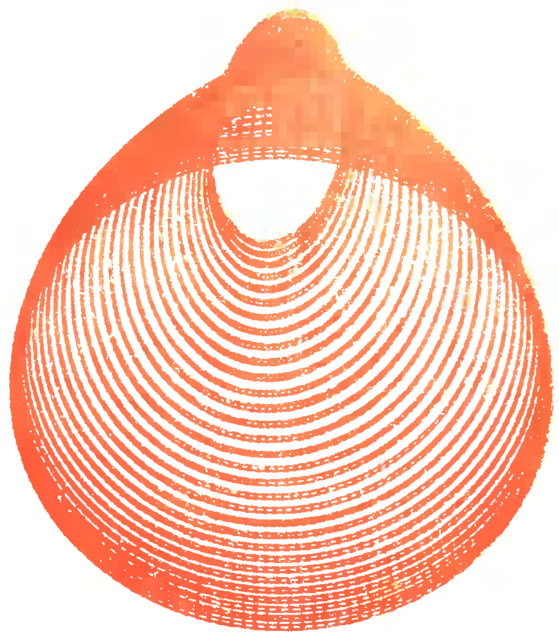

Figure 3. Internal contour of a streamlined extrusion die profile.

Sheet metal forming

In sheet metal forming, often the starting shape is a flat sheet-metal blank. The two main components of a press tooling in a metal stamping operation are the die cavity and the matching punch (see Figure 4). One common feature of all toolings is that, when the punch is made to come down slowly by the top platen of the press (the machine tool) until it halts close to the bottom of the die cavity, the gap formed between the cavity and the punch determines (may not exactly represent) the shape of the component. To predict the exact shape, a geometry (graphics)-based 'punch contact' program must be used. Even while accepting the fact that, under the applied (punch) load, the component can be forced to take a particular shape, one can argue that the shape changes once the punch

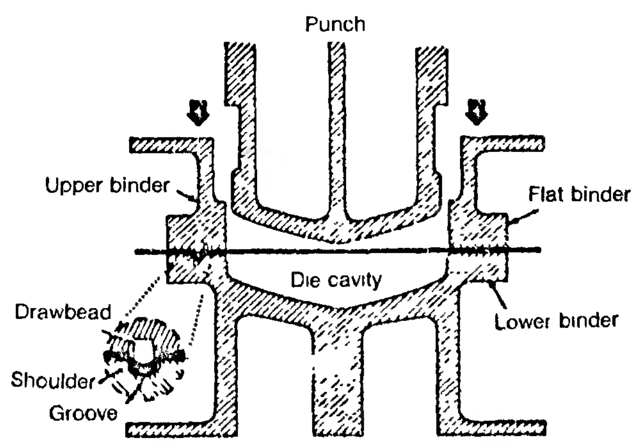

Figure 4. Schematic sketch of a typical stamping set-up. Insert shows a draw bead.

(and with it the load) is withdrawn. This is because of material elasticity. Prediction of such 'springback' is a major challenge.

The plastic deformation process is initiated as soon as the descending punch makes the first contact with the sheet metal. The starting blank is normally placed on top of the die. The deformation continues as the metal is pushed through the die cavity by the punch. A suitable clamping pressure, indicated by the thick arrows in Figure 4, is applied to the work metal by means of a binder (also called a blank holder). During deformation, the blank slides between the die and binder and flows around the die and punch surfaces and corners. Hence lubrication conditions, the applied blank-holding pressure, the corner and profile radii of curvatures, the ability of the metal to undergo plastic deformation (also called formability), etc. influence the process. Figure 5 gives a clearer picture. It depicts a 3D cut-away view of a cylindrical cup-drawing operation. The binder has purposely not been shown, in order to bring out other important features.

Need for applying appropriate back tension arises when forming complex shapes, because the friction constraint offered by the binder pressure may not be adequate. This is achieved by providing draw beads, shown as an insert in Figure 4. If the part must be formed not by (metal) drawing, but by stretching, it can be accomplished by incorporating lock beads instead of draw beads.

Metal flow analysis

Imagine etching a grid pattern on the sheet surface, a pattern that is compatible with deformation. For example, an axisymmetric deformation can be more efficiently monitored using a pattern formed of concentric circles intercepted by radial lines. Also, assume that material particles are located at the nodal points. (These bear no relationship to the second-phase or other microscopic particles.) If the movements of the nodal points are monitored, they then represent the particle trajectories or the metal flow lines.

The inner closed contour ABCE of Figure 6 (only one quadrant is shown) represents the plan view of a die cavity used in the forming of an irregularly shaped component. Assume that a sheet metal blank placed on the die surface is transparent in regions above unsupported regions. (Otherwise, one cannot see the die cavity in the plan view). In other words, what one sees is only the flange portion of the metal blank, and what we study is flange deformation. An experimental grid pattern for this set-up, for establishing metal flow lines should, obviously, consist of square grids in the regions surrounding the straight walls and axisymmetric patterns around the corners. The boundaries of the flange are unknown, i.e. the blank shape is not yet developed. With a small increment of downward displacement to the punch, the descending punch will make contact, obviously, only with the transparent region. The contact and motion would trigger the onset of plastic deformation in the component. The material is fed from the flange. The provision of a grid pattern allows experimental establishment of the metal flow lines. In that case, one observes that, in the corners, the angles subtended by the radial lines decrease, indicating circumferential compression and the segments of radial lines between adjacent circles lengthen, indicating radial tension. What it means is that the material is fed into the die cavity by means of circumferential compression and radial tension. This is also called shrink flanging; obviously a reference to the physical condition of the flange. Another type of metal forming operation that induces circumferential tension to feed the material is termed stretch flanging.

In the theoretical simulation, to achieve the objective of predicting the movement of the nodal points or the displacements (Figure 6), a method of characteristics based slip-line field (SLF) solution approach must be

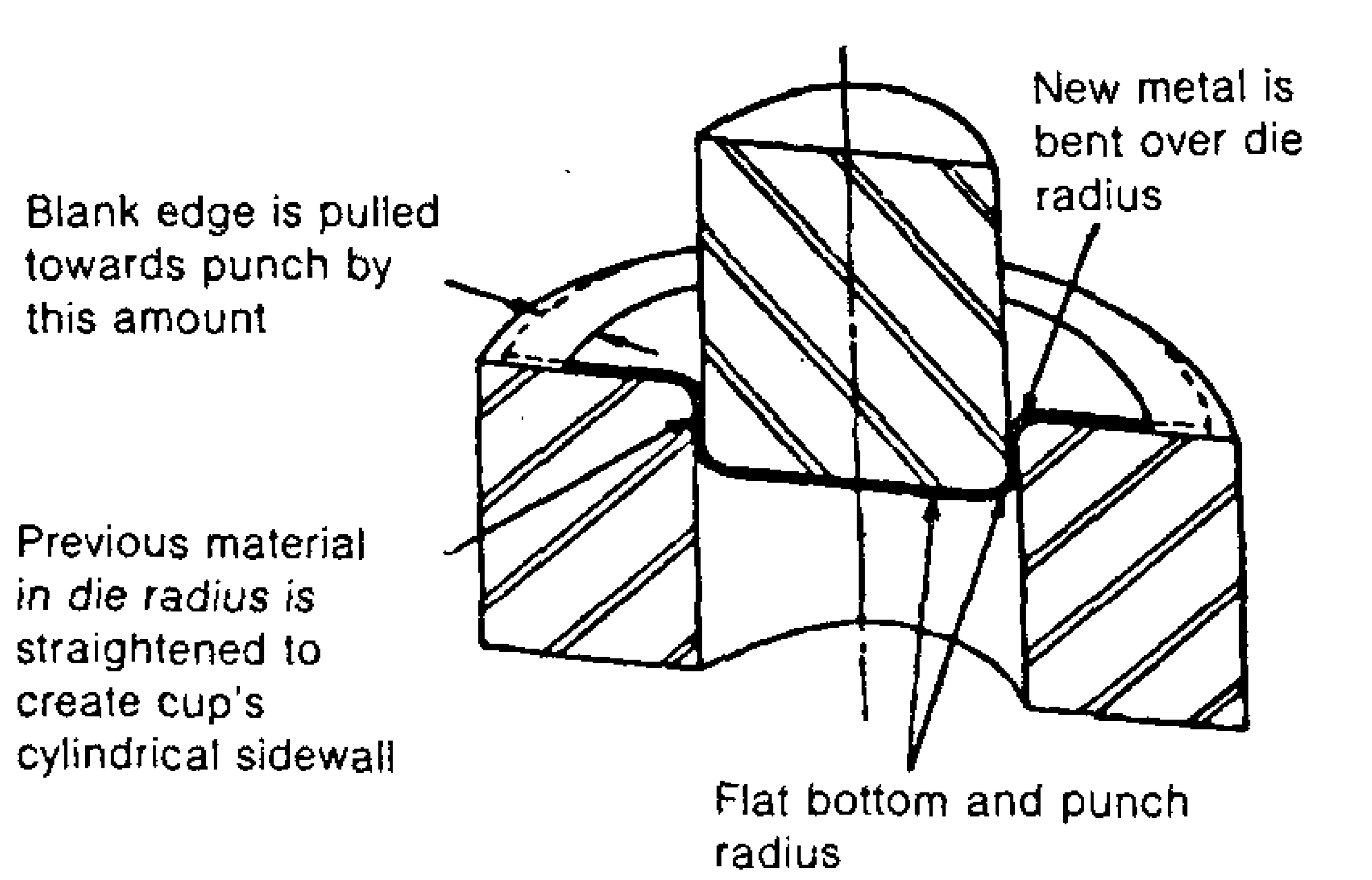

Figure 5. Three-dimensional cut-away view of a cup drawing operation.

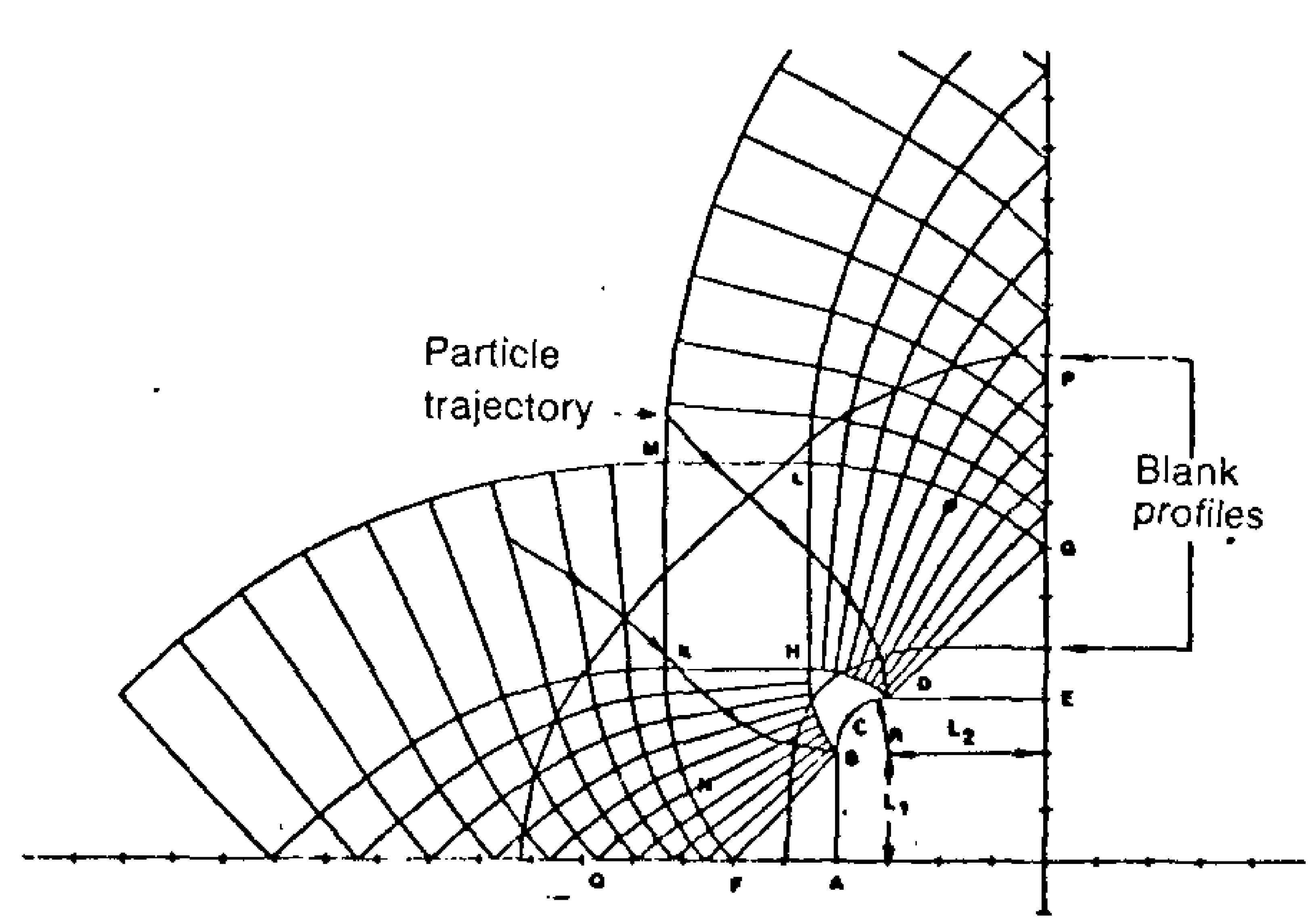

Figure 6. Slip-line field mesh and the optimized blank contours for one quarter of a rectangular pan.

followed[11,12]. This allows the construction of an orthogonal network of slip lines, which represent the directions of maximum shear stresses. The program OPTI-BLANK uses this methodology to construct an SLF and, as a follow-up, to evaluate the blank shape (see Figure 6).

Consider the following scenario. A component manufacturer gets costly tooling from (say) Japan, where it works perfectly. But the component suffers a high rejection rate in his country. Should the manufacturer then blame local material supplier and import the material too? The shape of the starting blank is considered to be technological know-how and information on this is hard to obtain. The material supplier is at a loss to convince the stamper that the failure was caused more owing to nonoptimization of starting blank geometry. This is one of the primary reasons why the giant steel mills of North America were the first to moot the idea of forming a consortium.

Process and tooling features

Metal flow analysis provides other valuable information. Process and tooling features, summarized here, can in fact directly be deduced from predicting or observing such flow patterns.

(i) The corner and profile radii of the die affect material flow, which in turn affects forming severity. Hence these parameters must be optimized. The punch design should also take similar considerations into account. See, for example, Figure 7, which illustrates how the

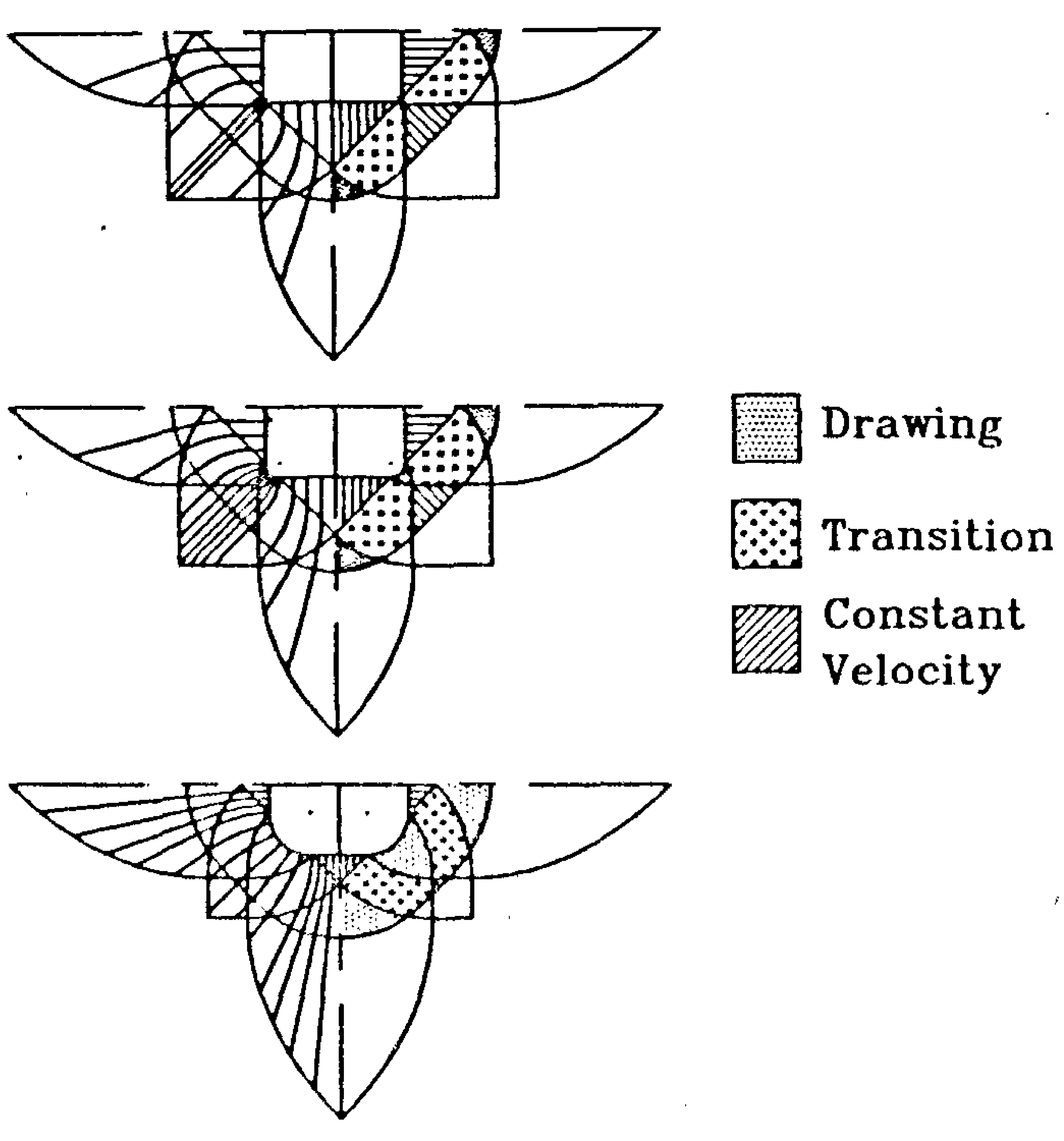

Figure 7. Metal flow analysis performed to illustrate the effect of change in die corner radii.

deformation pattern is altered with changing die corner radii. The results were obtained from metal flow analysis performed using SLF theory[13]. Observe that increase in die corner radius has the beneficial effect of increasing the size of the drawing zones. The figure shows the metal flow trajectories only in the third quadrant. In the areas identified as rigid zones, material simply moves as a rigid body. Plastic deformations occur primarily in the drawing zones.

(ii) During deformation the thickness of the sheet metal changes, and the effect varies with location. For example, a highly localized deformation caused by (say) material locking due to high friction can induce drastic thinning and eventual failure of the material.

(iii) Non-uniform thickening of the sheet metal flowing between the die and the binder can even lift the binder, with or without the beads. At the least, the thickening gradient will disallow the uniform binder contact that exists at the beginning of the deformation. Hence binders must be developed.

The programs DRAW-AYS and DRAW-BEAD and the elastic-plastic finite-element program STRTCH-AYS deal with aspects (i) to (iii) (see Box).

(iv) Normally binder pressure is applied by means of a double-acting cushion pressure. Any increase in this pressure restricts proper flow of material into the die cavity; the starved regions of the stamping then fail. On the other hand, a decrease in blank holding pressure feeds too much material, and the component wrinkles. Tool engineers or press operators might believe that by trial and error this can be set right. But the dynamics of the deformation process may not make the optimization that simple.

(v) Mechanical engineering drawings of components are extremely complicated. One of the most advanced commercially available computer design packages, ANVIL, is being used by the Big-3 (General Motors, Ford and Chrysler) to transmit their engineering drawings across to stampers. The package allows one to see how a component actually looks. This statement might sound simplistic, but the fact is that to enable me to 'read' a drawing (that of both component and trial tooling), Dow Chemical Co. must depute a group of their engineers. One often has no clue to what the shape of the starting blank ought to be. The mode of deformation, whether shrink flanging, stretch flanging, pure bending, bending under tension, rigid body movement, plane strain or pure shear, varies from zone to zone. But a broad-based blank development can be performed using OPTI-BLANK.

(vi) A plastic deformation analysis, performed using, for instance, DRAW-AYS, DRAW-BEAD, STRTCH-AYS, etc., provides complete strain and stress distribution solutions. To estimate the forming severity or, in other words, to determine whether the part can be successfully formed or not, a forming limit diagram

(FLD) for the given material, depicted in strain space, must be predicted or experimentally determined. Usually, the strain history during forming is non-uniform, and hence FLD predictions should be based on both linear as well as piece-wise linear or bi-linear strain paths. The program FLD-PDCT can accomplish this task.

Grid strain measurement

American industries use a very simple, straightforward technique to troubleshoot one class of stamping problems[14]. Circular or square grids are chemically etched or marked on the surface of the starting blank. The distortions of the grids are monitored during deformation and strains are computed by comparing the original grid shape with the deformed shape (see Figure 8).

The program STRAIN takes as input the nodal (x, y, z) coordinates of the formed component and computes the three principal components of the strain. Two of the principal strains, ε_1 and ε_2 (also called the major and minor strains), are along the plane of the sheet surface, while the third, ε_3, is through the thickness. To maintain volume constancy during forming, the three logarithmic strains add to zero. The results are normally marked in a strain space (ε_1 vs ε_2).

Straight lines passing through the origin in the strain space represent linear strain paths during deformation. Referring to Figure 9, a homogeneous (compression) deformation has a slope of $-1/2$. Actually it is $-r/(r+1)$, where r is the anisotropy parameter (r-value). Materials undergoing uniaxial and plane strain tension follow paths with slopes of $-(r+1)/r$ and infinity respectively. Pure shear and biaxial stretch have slopes of -1 and $+1$ respectively. The formability limits of materials depend entirely on the strain path. For instance, the failure strain is lowest for a plane strain deformation. These limits can be evaluated by means of experiments or obtained on the basis of theoretical predictions made using the program FLD-PDCT (refs. 15–17). The points representing the limits can be joined by a curve to construct an FLD.

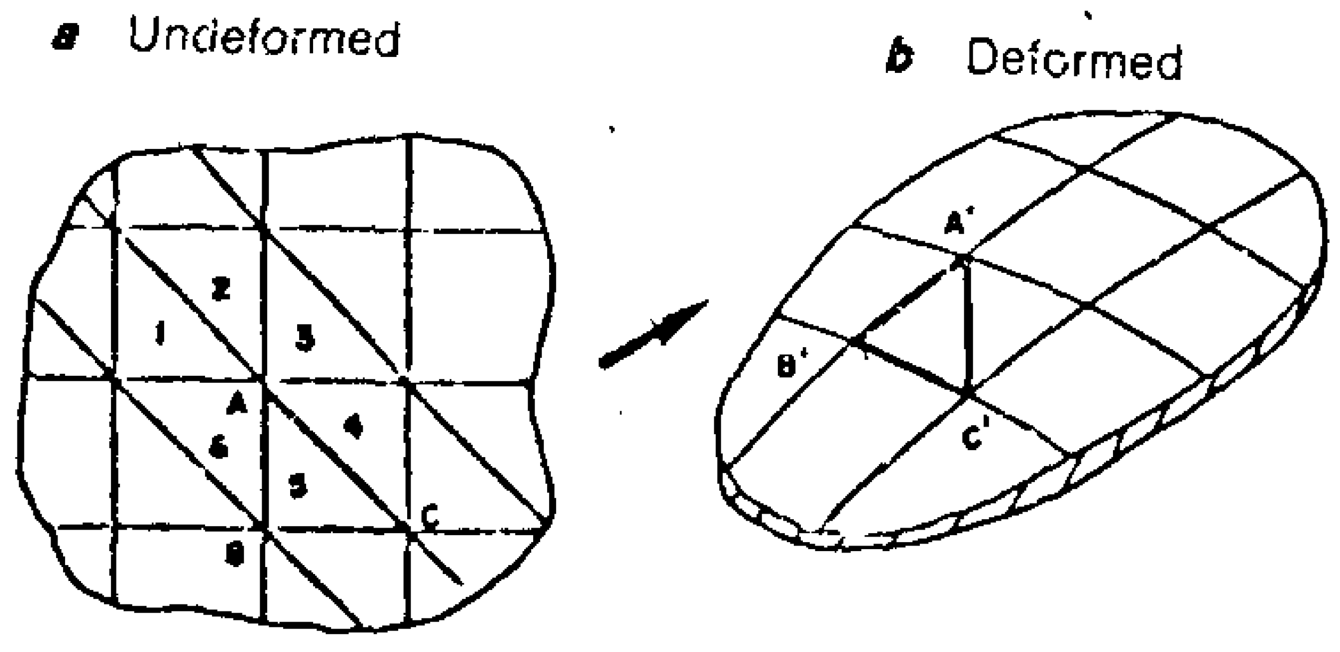

Figure 8. Undeformed and deformed configurations of triangular elements of a sheet stamping.

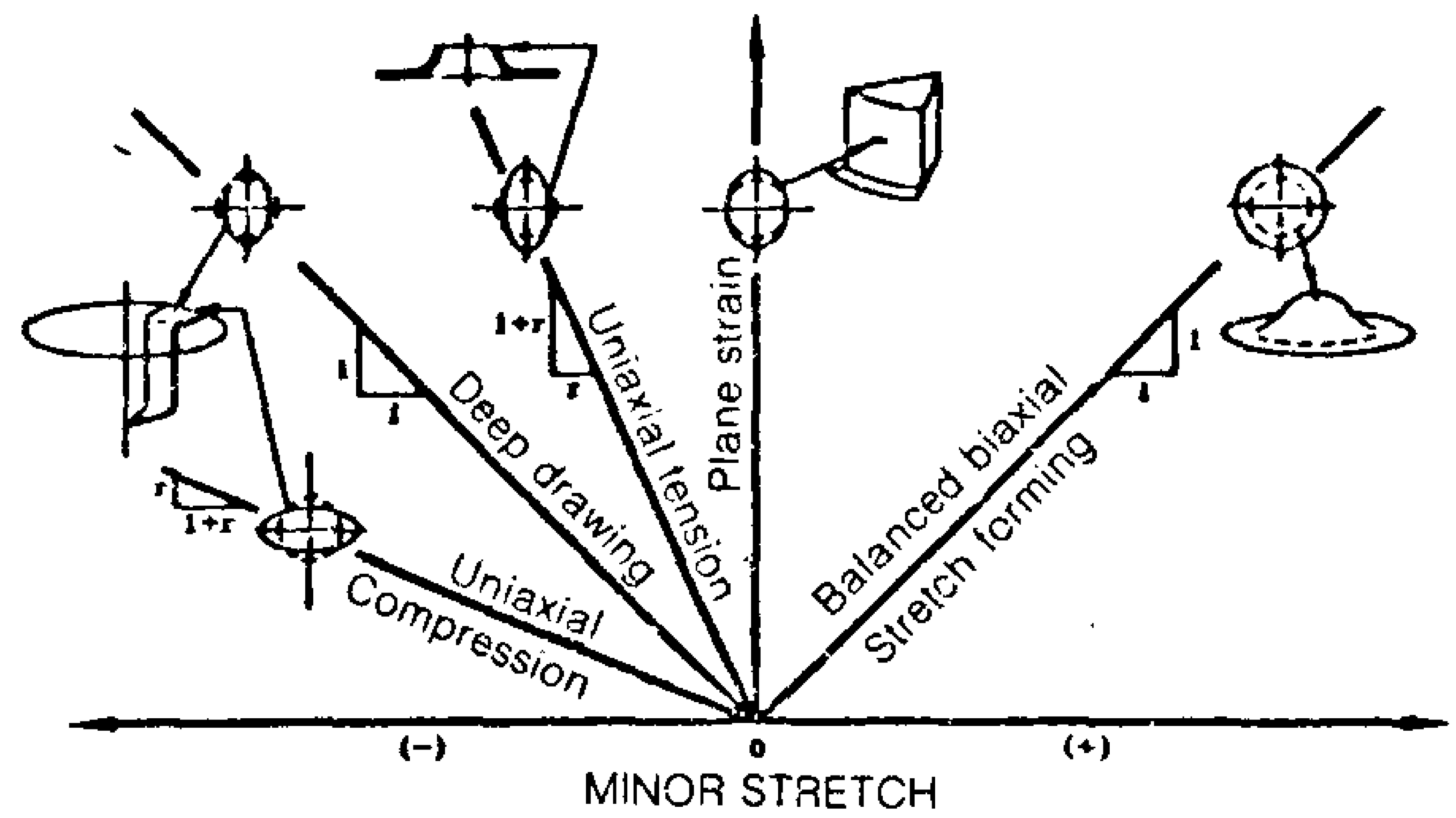

Figure 9. Some typical linear strain paths represented in strain space. During deformation, the shape of grid circles marked on the sheet surface change to ellipses (dashed lines). Stamping configurations that typify these modes are also shown.

The measured strain components can be marked on the FLD. If these points fall close to the limit curve, the forming severity is deemed to be quite high. Also note that it is possible to detect whether the tooling is performing the task according to the design or not. Crowding of points around the first quadrant indicates greater stretch than draw. Without proof, it can further be stated that it is possible to detect misalignment of the punch–die configuration, unintended shifting of blank location, operator error in setting up the binder pressure even between shifts, effect of interfacial lubrication conditions, tool wear, etc.[18,19] For example, consider the forming operation involving production of a truck bumper. The component has symmetricity. The scatter patterns of the strains measured on the left- and right-hand-side regions of the stamping should be the same. Any deviation indicates problems with the blank-locating mechanisms. The Swedish auto company Volvo routinely runs the STRAIN package and carries out the strain measurements even when there are no apparent problems.

Conclusions

Without scientific input, the manufacturing industry's task of building a competitive infrastructure is very difficult to achieve. Nor is it advisable to obtain scientific solutions in isolation. The mathematical modelling of metal deformation process is so complex that I do not foresee the emergence of a single powerful software that can solve a variety of industrial problems. It would still be possible to split a complex problem into several fundamental problems that are tractable. The Indian manufacturing industry must also form a consortium on the lines of those already existing in North America, so that the manufacturers work in unison. India can save a considerable amount of

DRAW-AYS

DRAW-AYS performs a complete analysis of an axisymmetric deep drawing operation (see refs. 20, 21). The press tonnage required for forming the component can also be determined. The program is sensitive to the nature of the binder action, whether applied through a spring-type or double-action hydraulic cushion or any other type of arbitrary variation in pressure versus punch displacement. The program calculates the maximum load that the component can withstand (critical load) and determines whether the operation will be successful by comparing the forming load with the critical load. The post-processor also presents the strain scatter on the FLD.

DRAW-BEAD

If the binder pressure is the only pressure applied to regulate the flow of the sheet metal into the die cavity, the arrangement becomes too restrictive for inducing appropriate back tension. Consider, for instance, the stamping operation shown in Figure 4, involving the forming of an irregularly shaped component. The flange portions of the sheet metal corresponding to the straight wall regions (assume this runs through and into the paper) may have absolutely no back tension at all. Elements in these regions undergo rigid body movement, followed by instantaneous bending and unbending at entry to the die contour (lower binder in Figure 4). The only way back tension can be applied is by means of draw beads embedded on the binder surfaces (depicted as an insert in Figure 4, also see Figure 10).

The plan view of Figure 11 shows where exactly the draw beads should be placed to obtain the best results. The recommendation is made on the basis of the nature of metal flow lines (see figure) as predicted by slip line field theory. Ideally metal should flow perpendicular to the draw beads.

The shape of these beads should also be optimized. General Motors has so perfected this technique that they provide SHAPE-MATE inserts to the tool shop[22]. This is achieved by performing a draw-bead analysis. The program solves a set of partial differential equations for the mode of deformation in zones extending from 1 to 4 in Figure 10. The sheet metal is assumed to bend instantaneously at point 1 and unbend at 2. The equilibrium equations also take into account friction between points 1 and 2 (between lower binder and sheet metal). After unbending, the material bends again, this time around the bead radius, at point 2, and unbends at 3. Observe that now the friction is between the bead and the metal.

Similar equilibrium conditions prevail between points 3 and 4 and up until the metal straightens out prior to entry into the die cavity. DRAW-BEAD provides a complete tension and strain distribution as output, until the element under consideration moves to the bottom of the punch. On the basis of the solution, an estimate of the forming severity and a determination whether the operation will be successful can be made. If failure becomes imminent, one can resort to altering the input parameters, until fully satisfied. This methodology can drastically cut down the number of costly field trials that would otherwise be required. The data can also be used to estimate the amount of springback likely to occur.

STRTCH-AYS

Chandrasekaran and Karima[23] drew up a state-of-the-art report on the hole flanging operation for a Canadian consortium. During this operation, the lip of the flange is continuously under uniaxial tension. A typical example is the door inner of an automobile (see Figure 12), which, during forming, undergoes a stretch flanging operation, along the opening corresponding to the viewing area of the automobile door. The susceptibility of the lip to fracture, during production of this component used to cause a high failure rate. Proper blank-development procedures have cut the rejection rate quite considerably. The mathematical formulation for this operation has been provided in ref. 23. An elastic-plastic finite-element code[24], STRTCH-AYS, has been successfully used to solve a problem referred to by a chemical company.

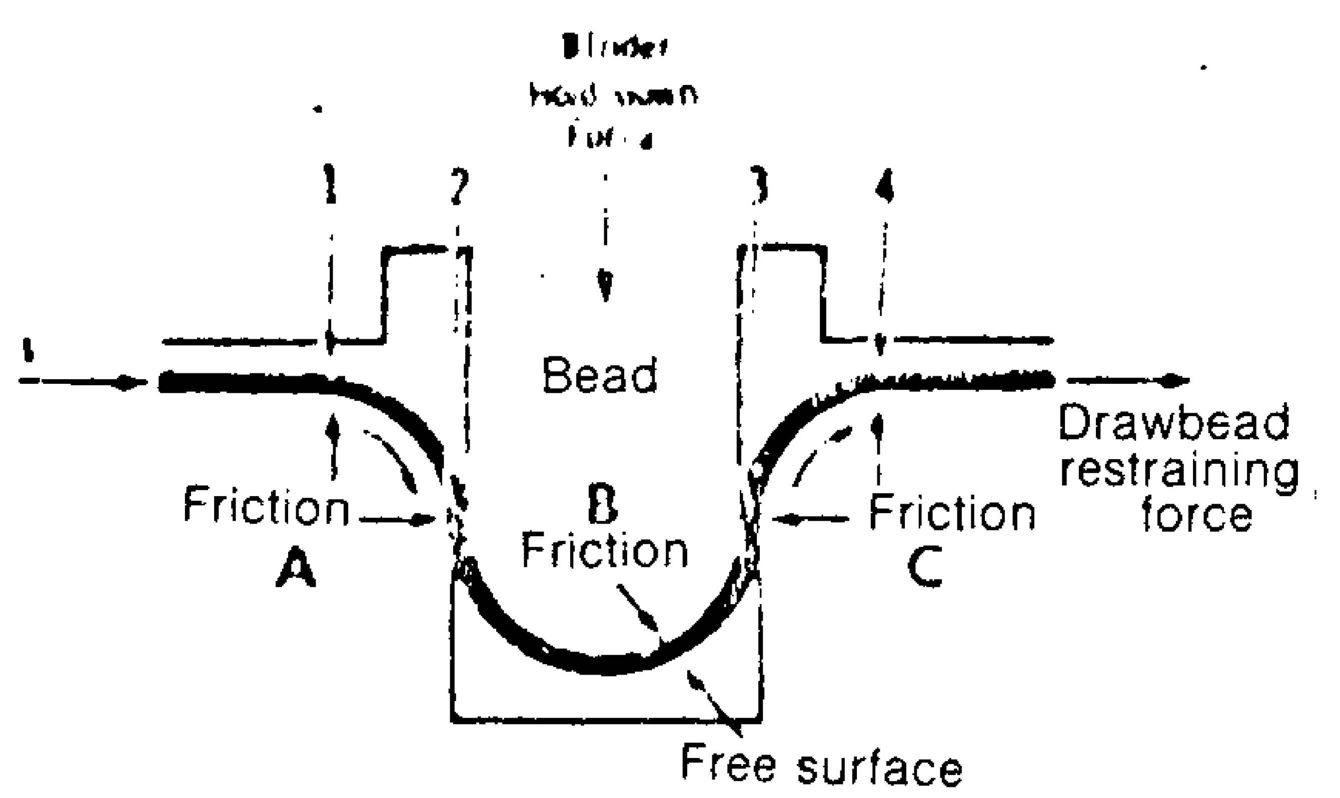

Figure 10. Schematic representation of a typical draw bead.

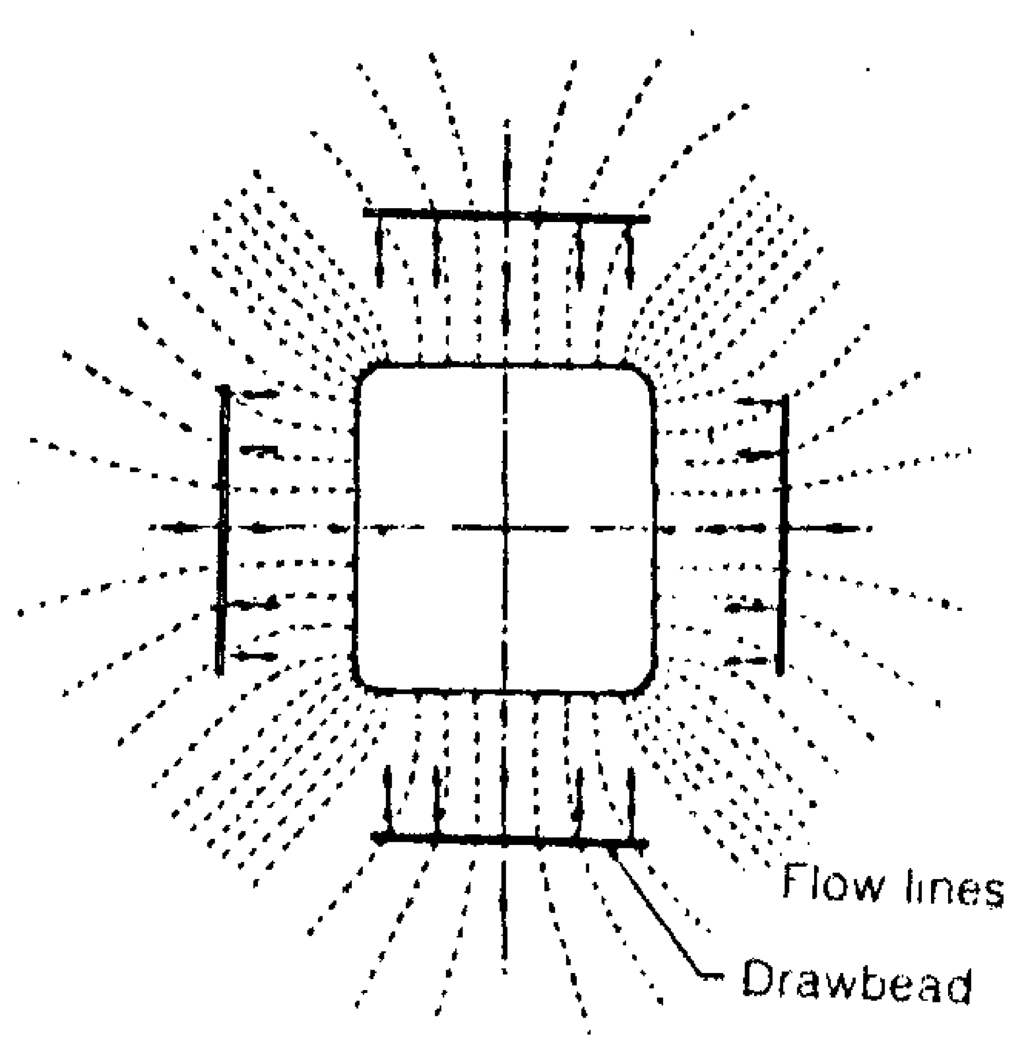

Figure 11. Draw beads and metal flow analysis.

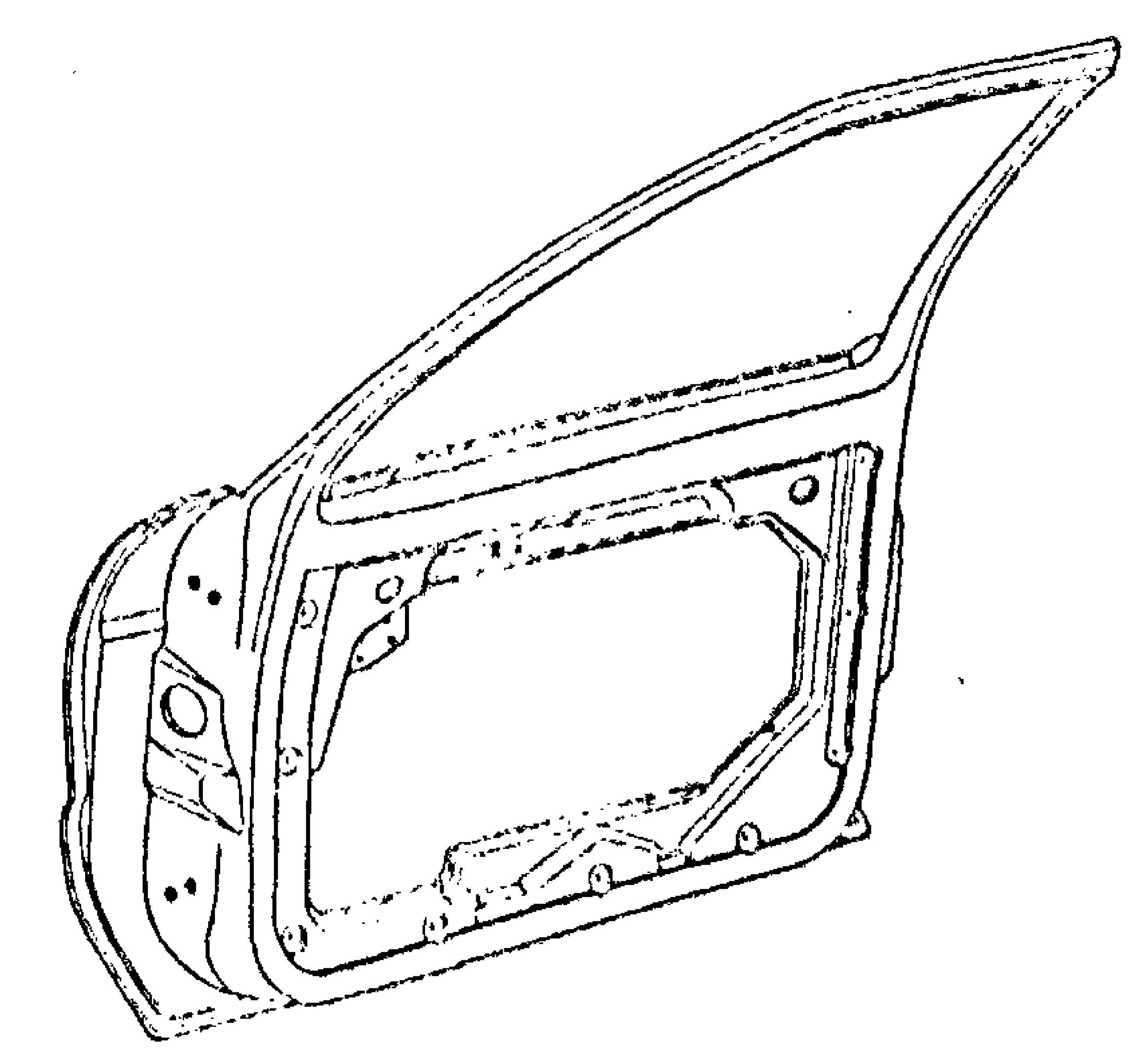

Figure 12. Schematic sketch of an automobile door inner.

development costs and easily place a mechanism that has proven successful elsewhere, in place in a short time.

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REVIEW ARTICLE

Quantum mechanics and statistical mechanics of anyons

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I discuss in some detail the quantum mechanics and statistical mechanics of anyons which are objects in two space dimensions obeying statistics which is interpolating between Fermi–Dirac and Bose–Einstein statistics. In particular I discuss the quantum spectrum of two and three anyons experiencing harmonic oscillator potential. Using these results I discuss the computation of the second virial coefficient of an anyon gas. Approximate results for the third virial coefficient are also given. Some discussion is also given about the possible relevance of anyons in condensed matter physics. Finally it is pointed out that the charged vortices in the abelian Higgs model with the Chern–Simons term provide a concrete model for charged anyons in relativistic field theory.

MANY of us have wondered some time or other if one can have nontrivial science and technology in two space dimensions; but the general feeling is that two-space dimensions do not offer enough scope for it. To my knowledge this whole question was first addressed in 1884 by E. A. Abbot in his satirical novel *Flatland*. The first serious book on this topic appeared in 1907 entitled *An Episode of Flatland*. In this book C. H. Hinton[1] offered the first glimpses of the possible science and technology in the flatland. In 1979 A. K. Dewdney[2] published a 97-page book which contains in detail the laws of physics, chemistry, astronomy and biology in the flatland. However, all these people missed one important case where physical laws are much more