Awesome and strange

The Man Who Knew Infinity: A Life of the Genius Ramanujan. Robert Kanigel. Charles Scribner's, New York; and Rupa, Calcutta, 1991. 438 pp. US $22.36, Indian edition Rs 195.

The story of Ramanujan's life is probably well known to most readers of *Current Science*, so I shall not summarize it in this review. Robert Kanigel has written the first serious general biography of Ramanujan. This is important enough, so that my review is not just comments on his book, but also includes comments on Ramanujan's life and mathematics which come from my more than 15 years of study and research related to part of Ramanujan's work, discussions with Indian mathematicians and scientists about Ramanujan, and my reading of what little published material exists on Ramanujan.

It is impossible for me to know what many Indians think when Ramanujan's name is mentioned, but I suspect there is a mixture of awe and a feeling of strangeness. The awe usually comes from Ramanujan's honours, such as being the first Indian fellow of Trinity College and the second (but usually thought to be the first) Indian elected a fellow of the Royal Society. His accomplishments that led to these honours are thought of as something beyond knowing. The strangeness is often thought to be related to mysticism, for many of the people who have written about Ramanujan emphasize this aspect of his life.

My feelings are also a mixture of awe and strangeness. The awe comes from the realization of what he did, most of it with no aid from any other mathematician. The strangeness is my realization that, still, more than 70 years after Ramanujan's death, no one begins to understand how he discovered most of the gems he found.

In the late thirties, G. H. Hardy wrote a mathematical biography of Ramanujan[1]. He told little of Ramanujan's life, since he knew little beyond what he had written in an obituary more than fifteen years earlier and what he and others had written ten years earlier when Ramanujan's collected papers were published. Hardy wrote that his book was not a systematic account of Ramanujan's work, but a series of essays suggested by it. It is now fifty years later, and still too early to write a definitive mathematical biography, since too many of Ramanujan's results are still not understood. As a substitute, the proceedings of the Ramanujan meeting held in Urbana, Illinois, in May 1987 (ref. 2) can be consulted. The proceedings of various meetings in India held in 1987 and 1988 and special issues of various Indian journals should also be consulted.

There have been a few general books on Ramanujan. The best is probably by S. R. Ranganathan[3]. Interesting stories are included, some of which I believe and others I strongly doubt. As the author wrote, 'Some of the information collected for this biography and the earlier one by me is of a trans-rational nature.' Here is an example. Ranganathan and K. S. Krishnaswami Ayyangar invoked Ramanujan with the help of a Ouija board. Among other questions they asked what Ramanujan had been working on in the third notebook, which Ranganathan had access to in the Madras University library. While turning pages, Ramanujan supposedly said, 'I now remember, I was working on mock-theta function.' However, now that the third notebook has been published, we can look at it and see what Ramanujan was doing. There is nothing dealing with mock-theta functions on the few pages in this notebook that contain mathematics. Unlike what Ranganathan wrote, there is more here than just tables of numbers. Most of the tables of numbers are labelled. The one that is not labelled is a list of all integers less than or equal to 12,005 whose factors are 2, 3, 5 and 7 and no other primes.

In addition to the stories Ranganathan gave, there is a collection of letters and comments about Ramanujan compiled by P. K. Srinivasan[4], family notes on Ramanujan, and a chronology of Ramanujan's life written by S. Narayana Iyer which was in the Archives of the Madras Port Trust and has been transferred to the Indian National Archives in New Delhi.

All of these and some other sources were used by Robert Kanigel to help flesh out his treatment of Ramanujan's life in India. To understand the area where Ramanujan grew up, Kanigel took a trip to South India. There he talked with Ramanujan's widow, Janaki Ammal, and many others who could help fill in as much of the background as possible. Since my first trip to the banks of the Cauvery River was in 1987, when I was over 50, I am not able to say how well he has captured the subtle details of India, but the overall picture comes through nicely. Ramanujan had no special push toward mathematics from anyone other than himself, but he did acquire intellectual curiosity from his mother and the schools nurtured this, at least before he went to college.

Kanigel follows Ramanujan around South India, looking in vain for someone who could really appreciate his mathematics and provide the minimal financial needs that would allow him to work full time on mathematics. He found someone—R. Ramachandra Rao, collector of Nellore. There are a number of written accounts of this meeting, and they differ significantly. In his obituary notice, P. V. Seshu Aiyar says he suggested that Ramanujan approach R. Ramachandra Rao with a letter of recommendation, which he wrote. Ramachandra Rao said it was his nephew who approached him with Ramanujan and there is no mention of a letter from Seshu Aiyer or anyone else. He said he could not understand Ramanujan's mathematics at the first meeting, but that at the second meeting Ramanujan showed him easier results, and these convinced him of Ramanujan's ability. He provided financial aid for a while, and a year later he introduced Ramanujan to Sir Francis Spring, who gave him a sinecure post in his office. R. Radhakrishna Ayyar claimed to have written to his father-in-law in Nellore to try to arrange a meeting with Ramachandra Rao. Finally, C. V. Rajagopalachari, a classmate of Ramanujan at the Town High School in Kumbakonam, said that he stopped Ramanujan from returning to Kumbakonam from Madras, and arranged for R. Krishna Rao, nephew of Ramachandra Rao, to take Ramanujan and Rajagopalachari to meet Ramachandra Rao. He said there

were three meetings, all fruitless, but that after the last meeting Ramanujan mentioned he had a letter from Professor Saldhana of Bombay, and they returned, and this letter convinced Ramachandra Rao that Ramanujan's work should be studied carefully and that he, Ramachandra Rao, should help Ramanujan. To complete the story, Narayana Iyer claimed he hired Ramanujan at the Port Trust, and that only a few months later was Sir Francis Spring introduced to Ramanujan and told of his mathematical ability.

These details are mentioned so you can see some of the problems Kanigel had in writing this book. At times there is little or no information about what was happening to Ramanujan, while at other times there are many conflicting accounts. Kanigel has taken the parts of the story that he sees as making a coherent account of Ramanujan's life, but often mentions other accounts and supplies adequate references so the reader can read alternative accounts. I don't always agree with his choices, but he provides a consistent account.

One topic that often leads to controversy is Ramanujan's religious views. Most of Ramanujan's friends and acquaintances who wrote about his view of religion wrote that he was an orthodox brahmin who regularly visited Sree Sarangapani Temple in Kumbakonam. There is little doubt that as a child growing up Ramanujan regularly visited this temple. A number of people have also written that Ramanujan told them that the Goddess Namagiri gave him formulas in his sleep. To counter this there is an article by N. Subbanarayanan, son of S. Narayana Iyer. He wrote about Ramanujan staying in their house in Triplicane, and how the joint work Ramanujan and his father did in the evening on the chalkboard was a great nuisance to the others who were trying to sleep. Several nights he saw Ramanujan get up at 2 in the morning and write down something on the slate. His father asked Ramanujan what he was writing, and Ramanujan said he worked out mathematics in his dreams and was jotting them down to remember them. Who does one believe, or are both stories true? My feeling is to trust the more specific account, the second one, as the one Ramanujan really believed. He may have told the other to some people who did not understand mathematics and how it is discovered.

The next part of the book gives Hardy's background. This is less familiar, but also very interesting. Hardy grew up near the public (what we call private in the US) school Cranleigh, where his father taught geography and later was bursar, and his mother ran the preparatory school. He was privately tutored in mathematics by Eustace Thomas Clarke, who had been a top mathematics student at St John's College, Cambridge. He won a scholarship to Winchester, one of the top public schools, hated his time there, and went to Trinity College, Cambridge, rather than the usual Oxford choice of most Winchester students. In 1913 he was fellow of Trinity College, fellow of the Royal Society, but most important, he and his younger colleague J. E. Littlewood were the best mathematicians in Great Britain, the two most likely to appreciate a letter from an unknown clerk that contained mathematical claims of a very unusual nature. Hardy was the fourth mathematician written to by or about Ramanujan, but the first one to respond positively. After Hardy's positive response to Ramanujan's first letter, Ramanujan's prospects improved. Within three months Ramanujan was awarded a scholarship by the University of Madras, and he was asked to go to Cambridge. He refused, partly for religious reasons, partly because his mother and other family members did not want him to go, and probably partly because of his unsatisfactory experiences in colleges. A year later he agreed to go to Cambridge. One reason for this change was his meeting E. H. Neville, a fellow of Trinity College, who reassured him that he could be a vegetarian in Cambridge, that his English was good enough, and that he would not have to take any exams. His mother began to realize that Ramanujan's prospects would be better if he went to England, and she withdrew her objections after dreaming of Ramanujan surrounded by Europeans and the Goddess Namagiri telling her not to stand in the way of her son's fulfilling his purpose in life. There was also a trip to Namakkal with Narayana Iyer, which was written about by Narayana Iyer's son, who also made most of the journey. An explanation of this journey has passed down orally in the family and seems very plausible. Kanigel reports these, and admits, as he should, that he does not know exactly what happened but that, in the end, Ramanujan was eager to go to Cambridge.

Cambridge was a place full of both problems and delights for Ramanujan. The major problem was food, which Ramanujan solved after a fashion by doing all his own cooking. Since he regularly worked round the clock, and would not take breaks to prepare food, health problems were in store for him. However, the benefits of Cambridge included G. H. Hardy, whom Ramanujan saw regularly, a marvellous library which he used at least periodically, and courses on mathematics. As an example of the last point, the first spring Ramanujan was in Cambridge he attended Arthur Berry's lectures on elliptic integrals. After writing some formulas Berry looked at Ramanujan and saw a face glowing with excitement. Berry asked if Ramanujan was following the lecture and after an answer of yes he asked if Ramanujan had anything to add. Ramanujan went to the blackboard 'and wrote results Berry had not yet proved, and which, Berry concluded later, he could not have known before'. This quotation is from Kanigel. Mahalanobis, from whom he got this story, wrote quoting Berry, using the same words up to the word 'proved', but then Mahalanobis continued: 'I remember Mr Berry was greatly impressed. He said that Ramanujan must have reached these results by pure intuition as Professor Hardy had advised Ramanujan to attend the lectures on elliptic integrals because Ramanujan had not studied that subject before.' Kanigel's summary of this is not completely accurate, but very reasonable for a general audience. However, the full quotation is interesting since it shows how Ramanujan's knowledge was not appreciated. In India he had acquired a deep knowledge of many aspects of elliptic integrals. His excitement was just what one would expect of someone who was seeing work being described that he had previously done all alone, with no one to appreciate it, or talk to about it. At last Ramanujan was in a place where there were people to talk to about his real love, mathematics. In the library he found an 1894 paper by L. J. Rogers which had a proof of an identity he had been trying to prove for a number of

years. After seeing Rogers' proof, Ramanujan was able to find a new proof, which in one sense was more direct than the first proof of Rogers. Hardy wrote that he talked to Ramanujan almost daily, and that he regretted not having asked Ramanujan what books he had read or where he found a known theorem, but that he was a mathematician and so had more interesting things to talk to Ramanujan about, these being the new results Ramanujan was finding everyday.

Two very good Indian mathematicians have remarked that it was a shame Ramanujan went to England to work with Hardy rather than to Germany to work with E. Hecke. Their argument is that Ramanujan's work on modular forms was not really understood or appreciated in England as it would have been by Hecke, and so the type of mathematics Ramanujan did in England was not as deep as it could have been. Kanigel does not address this issue, nor should he since it is a very technical one, but he raises one point in which he thinks Hardy was not an ideal mentor and friend to Ramanujan. He says that Hardy was good for Ramanujan as a mathematician, but that Ramanujan was a man as well as a mathematician, and Hardy did not really understand the nonmathematical side of Ramanujan, and in encouraging only the mathematical side helped distort Ramanujan's life. Both of these are serious questions, and, while I do not have a definitive answer to either, I shall attempt to answer them.

First, I feel that Ramanujan was very fortunate in writing to Hardy. Hardy's initial response is a model of how to write to an unknown great mathematician or scientist. He first expressed great interest in the results, but then said he must see proofs before making a definitive judgement; some of the results were rediscoveries, and Hardy said so; others Hardy thought were too special, of interest only because of their difficulty rather than their intrinsic interest; others were of great interest, but only if proofs could be given. In other words, Hardy treated Ramanujan seriously, as a colleague and coworker, not as a student or someone doing something he did not understand or appreciate. Ramanujan was interested in having his work published. Hardy either rewrote or helped rewrite all the early papers Ramanujan published after he arrived in Cambridge. In the greatest of the early papers, 'Modular equations and approximations to π', this involved introducing Ramanujan to some earlier work, as given by Weber, which Ramanujan had rediscovered. Then Ramanujan could add references to earlier work, and as a side benefit the reader had another place to go in order to start to understand what Ramanujan had done. This can be very useful, for Ramanujan's style is often cryptic at best. I wish Hardy had added many more details, but his editing and rewriting was as minimal as possible to help preserve Ramanujan's style and thought. All the early English papers go back to ideas Ramanujan had in India, and that is also true of some of the later papers. In particular, the joint work with Hardy on asymptotics of the partition function, which Littlewood singled out as the one piece of work of Ramanujan that was worthy of his genius, was foreshadowed by a claim in his first letter to Hardy, and modified in the second. (See Selberg[5] for comments on this.) One type of result that was started in England was the work on congruences for the partition function. This came from numerical data Major MacMahon provided to Hardy and Ramanujan, and if this data had been available earlier Ramanujan would clearly have made the same observations, and would have been able to provide some of the same proofs he later found. In looking for influences of Hardy in Ramanujan's work, I fail to find any major ones. Ramanujan had his own views on mathematics and what he wanted to do, and it seems likely that it would have been very hard to dislodge these views. I have already mentioned his appreciation and use of the 1894 paper by Rogers. So Ramanujan learned from others when the ideas were ones he was comfortable with and interested in. However, there is a second case where he does not seem to have taken up a method that was used to prove one of his conjectures. This appears in a 1917 paper of Mordell, where modular-function arguments are used to prove a conjectured factorization of Ramanujan. There is no indication that Ramanujan used these arguments on other problems he had. It is likely that Ramanujan's final illness, which started in 1917, was responsible for this, but it may also be that he was too busy doing other work to take time to learn arguments completely different from those he had used. Given enough time, Ramanujan would have been very interested in Mordell's arguments, but time was something Ramanujan did not have.

What would have been the situation in Gemany? Food would have been at least as scarce as in England, the language would have been much harder than the English Ramanujan had finally mastered, and the Indian friends he had in England would have largely been absent. He would have had Hecke to talk to, but probably few others. In the twenties Hecke rediscovered Mordell's argument and made a general method out of it, now called Hecke operators. It would have been very useful for Hecke to be exposed to Ramanujan, with his deep feeling and understanding of formulas, but the gap in mathematical sophistication between Ramanujan and Hecke, who had the full German number-theoretic background at his disposal, was so great that it would have taken much longer for Ramanujan to learn this mathematics than it took him to learn what Hardy tried to teach him. Hardy was not sure Ramanujan ever learned Cauchy's theorem, so Ramanujan did not really learn much from Hardy. The real problem was not that Ramanujan could not learn new mathematics, but that this took time away from his real love, discovering new mathematics.

While Hardy did not really understand modular functions, his appreciation of them was much greater than many people think. They remember part of Hardy's comment on Ramanujan's function $\tau(n)$, 'We may seem to be straying into one of the backwaters of mathematics', but forget he continued with 'but the genesis of $\tau(n)$ as a coefficient in so fundamental a function compels us to treat it with respect'.

Kanigel's point about Hardy's lack of understanding of the human side of Ramanujan has some merit. Hardy disliked small talk, and would have encouraged Ramanujan to talk mathematics. However, he discussed philosophy with Ramanujan, so I am not sure the case for Hardy pushing Ramanujan too hard in mathematics is proven. It is just as likely that Ramanujan pushed mathematics all the time, for this was his main love and for the first time he had people to talk to at the level of equals. There are

many reasons why it would have been good if Littlewood had been around longer, but he was called for war work shortly after Ramanujan arrived. These reasons include the mathematics he would have discussed with Ramanujan, and his sociability. He was a different person from Hardy, an excellent dancer, and, later in life, a mountain climber. I could see him taking long walks with Ramanujan, while it is unlikely Hardy would have done so.

Finally, in 1917, the tragedy of Ramanujan's health, probably caused to a large extent by poor eating, finally caught up with him. The last two years in England were primarily spent in various sanatoria, and the last year in India was spent doing what was probably his greatest work as he was dying. All of this, and later developments like the influence of Ramanujan's work on two high-school students, Atle Selberg and Freeman Dyson, and the discovery of the sheets that contain the last great work of Ramanujan, first by J. M. Whittaker at Watson's home and then by George Andrews at Trinity College, are, covered by Kanigel. This is a marvellous story, and, as has been said before, no one would believe it if it had not happened. We will never have the definitive biography of Ramanujan, for there are too many open questions, too much we do not know and shall never know, but this is a book that can be read with pleasure by anyone who cares about what human beings can do.

Ramanujan grew up in a society that did not understand what he wanted to do, or what he had done, but one that nourished intellectual activity in at least some children. While his college did not bend its rules to allow Ramanujan to keep his scholarship despite failing some courses, it had a good library that helped to deepen Ramanujan's insights, and eventually led to some marvellous mathematics. We must see that such opportunities continue to be available for the children of our time. That is how we can best pay homage to Ramanujan, recalling that he intended to use a substantial fraction of his fellowship from Trinity to provide scholarships for poor students.

1. Hardy, G. H., *Ramanujan*, Cambridge University Press, Cambridge, 1940.
2. Andrews, G. E. *et al.*, *Ramanujan Revisited*, Academic Press, San Diego, 1988.
3. Ranganathan, S. R., *Ramanujan, the Man and the Mathematician*, Asia Publishing House, Bombay, 1967.
4. Srinivasan, P. K., Ramanujan Memorial Number, vol. 1, Letters and Reminiscences, Mathialpet High School, Madras, 1968.
5. Selberg, A., 'Reflections around the Ramanujan centenary', in *Collected Papers*, Springer, Berlin, 1989, vol. 1, pp. 695–706.

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Errata

Optical and infrared excess in radiation from Be stars

K. M. V. Apparao

(*Curr. Sci.*, 1991, **61**, 756)

A key word in the article was mistakenly altered in the editorial office. The first word in the abstract as well as the first word in the text should be 'Be' instead of 'beryllium'. The meaning of the name 'Be' for these stars was explained in the first paragraph. The name 'Be stars' refers to B-emission stars, and not the element beryllium.

The phrase 'beryllium stars' in the last line of page 705 should also read 'Be Stars'.

The present scope of the field of terrestrial cosmogenic nuclides

D. Lal

(*Curr. Sci.*, 1991, **61**, 744)

Owing to a production error, Figures 1 and 2 on page 749 were interchanged. The lower figure on the page (appearing above the caption for Figure 2) is Figure 1.

Emanation of radon from rock minerals

Rama

(*Curr. Sci.*, 1991, **61**, 751)

The author states that since the work described in the article was done with a colleague, the word 'I' in the abstract should read 'we'.