Anyon superconductivity

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I enumerate the arguments which suggest that the anyon gas can exhibit superconductivity. The vortices of the superconducting anyon gas are charged and I point out some of their properties. One of the unique signatures of anyon superconductors is the violation of parity and time reversal invariance. I discuss some of the experiments which are trying to look for parity and time-reversal violation in the copper oxide superconductors so as to decide if these are anyon superconductors or not.

In the last few years the subject of anyon superconductivity[1] has become a very active area of research. This is primarily because of the hope that anyons may provide the mechanism for copper oxide superconductors[2]. The purpose of this article is to give a short account of this exciting field of research. The plan of the paper is the following. First I give a quick introduction to anyons[3]. Those who are interested in a more detailed exposition should consult my article on this subject which has appeared in one of the recent issues of this journal[4]. After that I point out the arguments which led people to believe that anyon gas can exhibit superconductivity. Then I discuss a field theory model for charged anyons which was given by us[5] much before this field became active. I also discuss some of the properties of anyon superconductors. In particular it turns out that one of the most striking predictions is the violation of parity (P) and time-reversal (T) invariance in anyon superconductors. Finally I discuss the P and T violation phenomenology in the context of copper oxide superconductors so as to decide if these are anyon superconductors or not. It turns out that the experimental situation is still unsettled but the general consensus is that anyons can provide a mechanism for superconductivity but it is unlikely that the copper oxide superconductors are anyon superconductors.

What are anyons?

For more than sixty years we know that all particles must either have integral or half-integral spin and hence must obey Bose–Einstein or Fermi–Dirac statistics respectively. While this is certainly true in three and higher space dimensions, in the last 14 years or so it has been realized that this need not be the case in two space dimensions. In particular, it has been shown by Leinaas and Myrheim[3] in 1977 that in two space dimensions the particles can have *any* fractional spin and can satisfy *any* statistics which is interpolating between the Bose–Einstein and Fermi–Dirac statistics; hence the name anyons for such particles. In a nutshell, the reason why anyons are allowed in two and not in higher dimensions is that, whereas the configuration space is multiply connected in two dimensions, it is only doubly connected in three and higher dimensions. In ref. 3 it has been shown that if one takes one anyon slowly around the other in anticlockwise direction then the phase acquired is $e^{i\theta}$ while it is $e^{-i\theta}$ if it is taken around in clockwise direction with $0 \leqslant \theta \leqslant \pi$, θ being a continuous parameter. Note that $\theta = 0$ or π corresponds to boson or fermion respectively. Several conclusions follow from here. Some of them are: (i) The anyons must necessarily violate the discrete P and T symmetries ($\theta \neq 0, \pi$). This is because either P or T transformation changes $(-i\theta)$ to $(+i\theta)$ by changing anticlockwise to clockwise direction and vice versa. We shall see that this P and T violation will turn out to be a very characteristic signal of anyon superconductors. (ii) The anyons are sort of in between bosons and fermions in the sense that the repulsion between two anyons monotonically increases as θ goes from 0 to π with there being no repulsion between two bosons. As a result the trajectories of two anyons cannot cross each other and one can in principle distinguish crossing 'in front' from crossing 'behind'. This has very profound consequences. Unlike bosons or fermions one now finds that the phase due to the exchange of two anyons depends in principle on the position of all other anyons. This fact makes three- and multi-anyon problems highly nontrivial and that is why till today one has not been able to obtain exact solutions to the ideal (noninteracting) gas of anyons. For example, it has been shown[6,7] that, whereas the ground state energy of two anyons in an oscillator potential monotonically increases as one goes from bosons to fermions, in the corresponding n-anyon problem with $n \geqslant 3$ there is always at least one crossover between the ground state energies. It is worth remembering here that in both Bose and Fermi cases we rely on exact solutions to the ideal gas problem both as a guide to intuition and as a starting point for perturbation theory. In particular, in these cases the exact solutions are obtained very simply as linear (symmetrized or antisymmetrized) combinations of one particle states. I believe that new mathematics may be required to solve the simple but challenging problem of the ideal gas of anyons and that unless one can solve this basic problem no serious, reliable calculation can be done involving interactions like in the problem of anyon superconductivity.

Why bother about anyons?

Is the above discussion about anyons merely of academic interest since anyons only live in flatland and not in our three-dimensional real world? The answer is no. The point is that there are many condensed matter systems which are essentially planar in character, i.e. for these systems the states of motion in the transverse direction are quantized and at sufficiently low temperature the energy to excite them is not available. As a result the hope is that, even though at the basic level the objects are bosons or fermions, the quasiparticles which provide the most direct and appropriate discussion of these systems could be anyons. This has in fact already been realized in the case of fractionally quantized Hall effect in the sense that, according to the best available explanation due to Laughlin[8], the quasi particles responsible for this effect are charged anyons. In fact it is this success which has inspired Laughlin to suggest that anyons could also provide the mechanism for the copper oxide superconductors.

Anyons and copper oxide superconductors

One of the most exciting developments in the last few years has been the discovery of superconductivity in copper oxide ceramics when these substances are cooled to temperatures below 125 K (i.e. −148°C). This may seem to be a very low temperature. However, it is much higher than the highest temperature, 23 K, below which the normal superconductors exist. One of the main reasons for the excitement is that, whereas one needs the very expensive liquid helium to attain temperatures of 10–20 K, the copper oxide ceramics remain superconductors even at the exceptionally cheap liquid nitrogen temperature (~77 K). While the initial expectation of utopia has died down still it is widely believed that these high-T_c superconductors will bring in revolutionary technological changes.

The essential property of a superconductor is that electric currents can flow in it and that this flow once started has no easy way of dissipating. In normal superconductors the mechanism was first given by the celebrated BCS theory[9]. According to this theory the superconductivity is caused by the attractive electron–electron interaction mediated by phonons resulting in the well-known Cooper pairing between electrons of opposite spin. This is a very delicate mechanism. It requires a net attractive interaction of some kind. This is rather tricky to arrange because in most circumstances the Coulomb repulsion between the electrons is the dominant force. It turns out that this works only if the crystal is not too noisy and the electrons in a pair are well separated. These two requirements make it difficult for this mechanism to work at any but extremely low temperatures. In recent years there have been some claims though that the mechanism may work even at high temperatures.

Soon after the discovery of the copper oxide superconductors, people realized that BCS theory cannot explain the mechanism for superconductivity in these materials. Around that time Laughlin[1] suggested that anyons could provide a mechanism for these copper oxide superconductors. Historically speaking, Laughlin's idea grew out of a bold suggestion made by Anderson that the superconductivity in the copper oxide materials might be caused by the occurrence of the resonating valence bond state in these materials. Even though not many agree with this suggestion, most people tend to agree with the core of the idea that the Fermi liquid principle fails in these high-T_c superconductors and that these high-T_c materials represent one of the first examples of strongly correlated fermion systems.

The key idea of anyon superconductivity due to Laughlin[1] is the following. It is well known that bosons form a superfluid at low temperatures when there is a macroscopic occupation of a single quantum state. Fermions cannot do so due to the Pauli exclusion principle. However, a pair of fermions (Cooper pair) with charge $2e$ behave as bosons at length scales much greater than the size of the pair and hence can enjoy macroscopic occupation of a single pair state leading to superfluidity (and superconductivity in the presence of electromagnetism). The remarkable realization of Laughlin is that this idea could be extended to anyons, i.e. the composite of n anyons such that the composite is a boson might exhibit superfluidity by macroscopic occupation of a single n-body state. If, further, the anyons are charged, then the fluid would be superconducting. For example, notice that if there is a pairing of semions (anyons with $\theta = \pi/2$) in the real space then the pairs of semions are bosons (rather than fermions!) since the exchange of two such pairs gives a phase $(\exp(i\pi/2))^4 = 1$. Could it be plausible that, even in the absence of real space pairing, semions might form a superfluid by pairing just as fermions form Cooper pairs in normal superconductors? Laughlin suggested that indeed this is what happens. Since Laughlin's suggestion, several calculations have been done[10] using a variety of approximations and the literature is unanimous to date on the point that anyon superconductivity is possible in principle.

The picture advocated by Laughlin is that the ground state of the anyon superconductors is a chiral spin liquid. The elementary excitations of this system are neutral spin-$\frac{1}{2}\hbar$ particles called spinons which have a linear energy momentum relation. On doping, the holes introduced into the system bind to the spinons such that the resulting composites are charged semions (charged fermions + neutral semions). These charged semions then pair to form a superconductor.

What are the predictions of anyon superconductivity? While the question is not well settled, some predicted properties like the Meissner effect (expelling of magnetic field from superconductor) and flux quantization in units of $hc/2e$ are indistinguishable from fermion superconductors. Anyon superconductors are also predicted to have linear phonon mode at small q and a small coherence length. One of the unique predictions of anyon superconductivity is the violation of discrete symmetries P and T.

A field theory model for charged anyons

It may be worthwhile to mention here that the first model for charged anyons in relativistic field theory was given by Samir Paul and Khare[5] in 1986 much before anyon superconductivity became fashionable. We showed that if one adds the topological Chern–Simons term to the planar abelian Higgs model then unlike the neutral vortices of Abrikosov (which have been seen in normal type-II superconductors) one now has charged vortices. We further showed that these charged vortices have in general fractional angular momentum which suggests that they could be charged anyons. This was subsequently proved by Frohlich and Marchenti[11] by using the arguments of axiomatic quantum field theory. It is worth mentioning here that the charged vortices exist[12] even in theories with pure Chern–Simons term (i.e. there is no Maxwell kinetic energy term). This is interesting specially in the context of condensed matter physics since in the long wavelength limit the Chern–Simons term (having one space derivative) dominates over the Maxwell term (which has two space derivatives). *A la* neutral vortices, the charged vortices can also be shown to exhibit flux quantization and Meissner effect, a hallmark of superconductivity. Further, since the Chern–Simons term violates the discrete symmetries P and T, the model naturally predicts the violation of these discrete symmetries.

Are there signals of P and T violation in copper oxide superconductors?

Finally let us discuss if indeed the copper oxide superconductors are anyon superconductors or not. Physics being an experimental science, such questions are decided by experiments and not by things like aesthetic beauty of the theory. As we have seen before, one of the unique predictions of anyon superconductivity is the violation of discrete symmetries P and T. Thus if anyons are to provide a mechanism for copper oxide superconductors then one should be able to see the signals of P and T violation in these materials. In the last two years groups at Bell Labs[13], Stanford[14] and Dortmund[15] have looked for positive optical effects in copper oxide materials. Very careful chiral light scattering experiments at Bell Labs give a nonzero effect that is 100 times smaller and physically very different from that of the Dortmund group. In fact serious doubts have been expressed about the Dortmund group experiment. On the other hand, the Stanford group sees no P and T violating effect in the same crystal even though they are working with a superior set-up. In particular, whereas Bell Labs looked for circular dichroism (i.e. when material absorbs right-handed circularly polarized light more or less strongly than it does the left-handed circularly polarized light) in $YBa_2Cu_3O_{7-x}$, the Stanford group compared the polarization of two light beams of the same handedness transmitted through the (YBaCuO thin film) sample in opposite directions. The arrangement eliminates all effects except those that violate T.

A problem with many of the suggested signatures for P and T violation is that they are appropriate for a single sheet and would thus tend to wash out for many stacked planes of anyons if the different planes choose alternating chirality or any other scheme which averages the chirality to zero. The most promising experimental signatures are then those which may be seen even in a bulk sample of zero (average) chirality. One such possibility is to use surface sensitive probes. Using muon spin rotation studies, experiments have been performed at TRIUMF to detect the spontaneous magnetic moment of anyons but the experiments have failed to detect any such moments[16].

Where do we stand?

In this paper we have provided arguments which suggest that charged anyons can provide a mechanism for superconductivity. A number of calculations have been done and most of them agree that anyon superconductivity is possible in principle. However, the reliability of these calculations may be doubtful since recent exact results[6,7] about n-anyon problems ($n \geq 3$) show that there are crossovers even in the ground state energy of n anyons experiencing harmonic interaction so that perturbative calculations using the bosonic or fermionic basis may not be reliable. I believe that unless one can understand the problem of noninteracting anyon gas, the reliability of interacting anyon problems will always be in doubt.

We have also discussed some of the recent experiments which try to look for positive signals of P and T violation in copper oxide superconductors and even though the situation is not completely clear the general consensus is that on the whole the present experiments seem to rule out the possibility of copper oxide superconductors being anyon superconductors.

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Endocrine control of fish reproduction

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The majority of fishes breed at a particular time of the year and the seasonal reproductive cycle is precisely maintained by endocrine cycle. Environmental stimuli like photoperiod and temperature are presumably received by the brain which releases a decapeptide hormone, gonadotropin releasing hormone (GnRH). GnRH specifically binds to the receptor in the pituitary gonadotroph cells and stimulates the secretion of gonadotropic hormone (GtH). In fish GtH may be of one or two types. Circulatory level of GtH increases during gonadal development and maturation. GtH surge is highest during the breeding season when ovulation or spermiation occurs. GtH regulates ovarian and testicular function by inducing an exceptional steroid hormone which is 17α,20β-dihydroxy-4-pregnen-3-one. However, there appears to be a shift in GtH function; it induces synthesis and secretion of estradiol-17β during previtellogenic phase which in turn induces vitellogenesis or yolk protein synthesis, while during post-vitellogenic phase GtH triggers the synthesis of 17α,20β-dihydroxy-4-pregnen-3-one which is responsible for final maturation leading to ovulation or spermiation. The hormonal cascade of events is perfectly coordinated with the seasonal reproductive cycle of the fish to ensure spawning at a specific time of the year.

THE majority of fishes are seasonal breeders. The seasonal reproductive cycle involves recrudescence of the gonad leading to its final maturation, ovulation or spermiation and spawning. Endocrine activity clearly corresponds to the annual reproductive cycle. In order to spawn at a specific time of the year, fishes must use various environmental cues to initiate gonadal recrudescence so that gametes are matured in time for spawning[1]. Among the various possible environmental factors involved in cuing endocrine activity to perform the reproductive event, duration of daily photophase (photoperiod) and temperature have been assumed to be of prime importance in most fishes[2,3]. It appears that external environmental stimuli are received by exteroreceptors which transfer this message to the brain of a fish. The brain releases a humoral factor, gonadotropin releasing hormone (GnRH). GnRH then acts on the pituitary to release gonadotropin which in turn regulates gonadal function. In most cases gonadotropin action is not direct. It acts through the biosynthesis of gonadal steroid hormones which in turn regulate gonadal growth, maturation and ovulation or spermiation.

Nowadays fish flesh is in high demand in various countries not only for its good taste but also for its better nutritional value. Every country is attempting higher production of food fish and to do this,