Concept of discreteness, continuity and the Cantor continuum theory as related to the life-time and masses of elementary particles

Raja Ramanna

National Institute of Advanced Studies, Indian Institute of Science Campus, Bangalore 560 012, India

It is shown that with the use of the Cantor continuum theory it is possible to derive the systematics of the lifetime and masses of elementary particles.

The theory gives a new meaning to the foundations of quantum theory as Cantor's theory deals with discreteness and continuity, which is the essence of quantum mechanics.

LET $A_1, A_2, \ldots, A_n$ be the various possible physical *events* and $a_{11}, a_{12}, a_{13}, \ldots$, etc. be the various *conditions* which generate the *events*, e.g. A_1 takes place when conditions a_{11}, $a_{12}, a_{13}, \ldots, a_{1n}$ exist. We write all possibilities in a matrix form as follows:

Events	Sequence of conditions						
A_1	a_{11}	a_{12}	a_{13}	$\ldots$	a_{1n}	$\ldots$	
A_2	a_{21}	a_{22}	a_{23}	$\ldots$	a_{2n}	$\ldots$	
—	—	—	—	—	—	—	(I)
—	—	—	—	—	—	—	
A_n	a_{n1}	a_{n2}	a_{n3}	$\ldots$	a_{nn}	$\ldots$, etc.	

The values of the a's can be 1 or 0 depending on whether the condition is relevant or not to that particular *event*.

In order to convert the above matrix to represent actual physical quantities and be able to interpret them as physical laws, we invoke the Cantor diagonal construction which Cantor used in showing the difference between the various types of transfinite numbers, particularly between countable and non-countable infinities. The theory in itself is a mathematical differentiation between continuous and discrete quantities and can also be made the starting point for the foundations of quantum mechanics, since it deals with physical quantities, which under certain conditions are discrete and in others continuous.

It will be recalled that Cantor proved that if we measure all the points on a continuous line, there are not enough integers to count them all. The Cantor diagonal method consists of writing all the points on a line lying say between 0 and 1 (or in any continuous interval) in decimal notation[1]. Let $S_0, S_1, \ldots, S_n \ldots$ be points on a straight line. We can presume that we can continuously count all the points by taking the decimals to an infinite number.

$$
\begin{array}{lllllllll}
S_0 = & 0.X_{00} & X_{01} & X_{02} & X_{03} & \ldots & X_{0n} & \ldots \\
S_1 = & 0.X_{10} & X_{11} & X_{12} & X_{13} & \ldots & X_{1n} & \ldots \\
S_2 = & 0.X_{20} & X_{21} & X_{22} & X_{23} & \ldots & X_{2n} & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
S_n = & 0.X_{n0} & X_{n1} & X_{n2} & X_{n3} & \ldots & X_{nn} & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots
\end{array}
$$

It will, however, be noticed that a number, say S, will not be found in the enumeration if S is created such that

$$S \neq 0.X_{00} \cdot X_{11} \cdot X_{22} \quad X_{33} \quad \ldots \quad X_{nn} \quad \ldots$$

and each diagonal element is replaced.

This is because the particular diagonal element will always be different by definition. This demonstrates that if one tries to enumerate all the points continuously, there are an infinite number of missing entries showing that a countable ∞ is different from a non-countable ∞.

This method can be used to determine the missing *event* from matrix (I) for each diagonal. If, for some reason, the events are all ordered, there will be only one such sequence.

After having shown the existence of two kinds of infinities, Cantor went on to show that an infinite number of infinities can be formed by what is known as the Cantor continuum. The method consists of showing that from a set of positive integers, it is possible to construct another set of integers with a different cardinal number[2]. If a set A consists of three integers (1, 2, 3,), a set B can be constructed {1, 2, 3,}, {1, 2,}, {1, 3,}, {2, 3,}, {1}, {2}, {3}, and 0, i.e. with 8 different elements. As shown later, this continuum property can be used to get the levels of atoms if a proper correspondence is made. Cantor has in general shown that a set with n elements, can generate another set with 2^n elements of a different cardinal number.

From these very general considerations, we proceed to consider the special case of atomic spectra and elementary particles.

In classical and relativistic theories, we can say that an event either happens or does not happen. In quantum mechanics, we have a very different situation where *events* are described by abstract wave functions whose interpretation depends on probability considerations.

The Uncertainty Principle further adds to complexity of defining the *conditions* for an *event*. Even this is not established until a measurement is made and digested by the *observer* with a consciousness.

In this present matrix formulation, because of the existence of quantized parameters, wave particle duality and of virtual states, it is necessary to assume that the matrix elements could have values continuously between 0 and 1. This could come about due to averaging effect over time or as a result of the influence of other parameters.

In general, the differential equation provides causal relations in classical and quantum mechanics. In quantum mechanics, the solution of the Schroedinger equation depends on the type of boundary conditions one imposes on it. Some quantities get quantized while others remain in a continuum. For example, for a particle in a box, energy is quantized while position remains in a continuum. Thus their cardinality is different. We have from Cantor's theory of the continuum that n entries from a lower cardinality will have for the next higher cardinality 2^n entries.

Cardinality is used essentially to differentiate between the discrete and continuous. We consider an actual example from atomic spectroscopy to apply the Continuum theory to quantum mechanics.

It is known that there is a certain natural width ΔE_0 corresponding to an intensity distribution of a spectral line emitted by an atom. In classical theory this is due to the reaction force of the emitted radiation on the emitting source. In quantum mechanics, following the perturbation theoretic treatment due to Weisskopf and Wigner (see ref. 3), one gets a formula for ΔE_0 which is consistent with the time - energy uncertainty relation where ΔE_0 is interpreted as an estimate of the accuracy with which an energy level is known, as Δt is the lifetime of the excited state of the atom. The continuum of the line width thus coexists with the discrete structure of the levels. At higher energies the levels will begin to overlap and a *continuum* of nonenumerable infinite number of states will come into existence.

Let us assume that the ratio of the energy width in the continuum state which is riding on the discrete structure and the energy of the level, be proportionate to the ratio of the number of discrete states (n) and the next cardinal number given by 2^n.

i.e. $$\rho\,(n/2^n) = \Delta E_0/E\,, \tag{1}$$

where ρ is some constant of conversion from cardinal states to energy states and $\Delta E_0 = (h/\Delta t)$ is the spread of the level of energy E and is related to the time an atom can retain excess energy before re-emitting it in the form of photons.

We now apply the above formula to the case of a free electron falling to the various levels of the hydrogen atom. Table 1 gives the *cardinal levels*, the simplest series of the levels of the hydrogen atom and the energy widths.

Table 1. Cardinality and the width of spectral lines of hydrogen for constant ρ

Card No.	Quantum No.	2^n	$n/2^n$	E (ergs) Energy level	ΔE_0 width (ergs)
1	8	2	0.500	3.37×10^{13}	1.7×10^{19}
2	7	4	0.500	4.39	2.2
3	6	8	0 375	5.99	2.2
4	5	16	0.250	8.61	2.2
5	4	32	0.156	13.45	2.1
6	3	64	0.094	24.00	2.25
7	2	128	0.055	53.90	3
8	1	256	0.031	214.00	6

The ratio of the values of $n/2^n$ and $\Delta E_0/E$ is nearly the same provided we allow for the variation of the value of ΔE_0. ρ is some constant which has to have the value of 10^{1-6}. It will be noticed that the three levels which are nearer the ground state could belong to a separate family having a different cardinality. These first three levels could be of cardinality 3 which in turn can give rise to 8 levels of higher cardinality.

From equation (1) it was shown that the natural width of the spectral lines of the hydrogen spectrum could be obtained using the Continuum Theory of Cantor. Here we show that the level spacing given by the theory is consistent with the quantum theory of the hydrogen atom.

Let D_1 be the level spacing of the hydrogen atom as given by the Bohr formula.

$$D_1 = \text{Const}\ (1/p^2 - 1/q^2), \tag{2}$$

where p and q are the quantum numbers and the constant is the Rydberg constant. From equation (1) we have D_2, i.e.

$$D_2 = \rho\left(\frac{n}{2^n} - \frac{m}{2^m}\right) = \Delta E_0\left(\frac{E_n - E_m}{E_m E_n}\right) \tag{3}$$

Table 2 gives the values of D_1 and D_2. Here the cardinal number 8, which is also the number of discrete levels in hydrogen, corresponds to the quantum number $q = 1$, and the ordering of the cardinal numbers is in the reverse order to the quantum numbers. The last column gives $\rho/\Delta E_0$.

It is known that the hydrogen lines are grouped in various series such as Lyman, Balmer, Paschen, etc. It, therefore, suggests that ρ and ΔE_0 need not necessarily be constant but can have quantized values. In Figure 1, $n/2En$ is plotted against $-\text{Log}\ E$. The degeneracy of the hydrogen lines makes it difficult to bring out the changes in the values of ρ and ΔE_0. But the changes in the slopes are clearly visible as we move from series to series.

Using the equations $\rho/\Delta E_0 = 0.06 \times 10^{14}$ (Table 2); $\rho\,(8/2^8) = \Delta E_0/E$; $\Delta E_0/E = \Delta\lambda_0/\lambda$, where $\Delta\lambda_0$ is the natural line width, $\Delta\lambda_0 = 1.17 \times 10^{4}$ ÅU, and $E\lambda = hc$, ρ takes the value of 1×10^{6}.

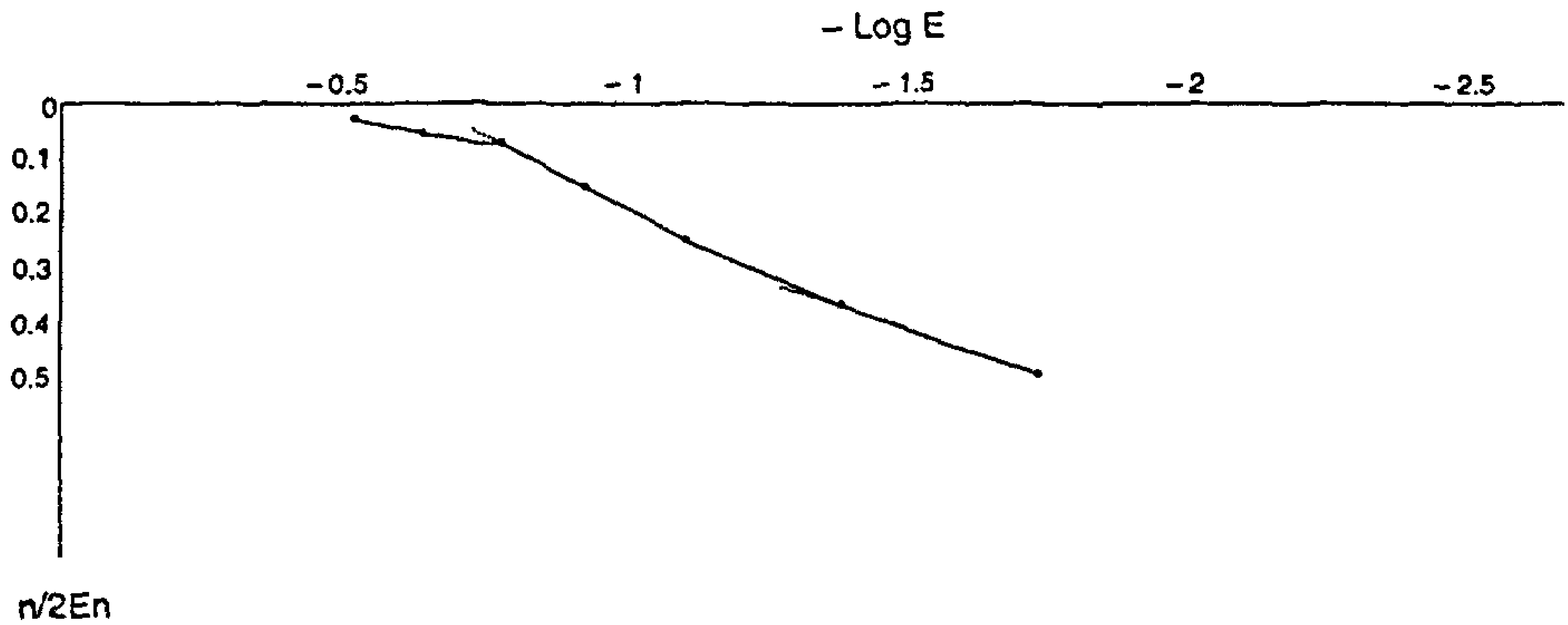

Figure 1. – Log *E* vs. *n* /2*En* showing the change in the values of ρ due to the formation of Lyman, Balmer and Paschen series. The changes in the slopes are clearly visible.

Table 2. Cardinality and the Bohr theory of the hydrogen atom

1	2	3	4	5	6		7	8
n	$n-1$	$-\left(\frac{n}{2^n}-\frac{n-1}{2^{n-1}}\right)2^8 \times$ $(= D_2/\rho)$	E_n $E_{n-1} \times 10^{26}$	(3)× (4) $\times 10^{26} \times 2^8$	p	q	$D_1 = -R_{H}hc$ $hc\left(\frac{1}{p^2}-\frac{1}{q^2}\right) \times 10^{12}$	$(\rho/\Delta E_0) \times 10^{-14}$ erg^{-1}
8	7	6	11615	69690	2	1	16.5	0.06
7	6	10	1293	12930	3	2	3.06	0.06
6	5	16	324	5184	4	3	1.07	0.052
5	4	24	116	2784	5	4	0.495	0.045
4	3	32	51.57	1650	6	5	0.269	0.041
3	2	32	26.29	841	7	6	0.162	0 049
2	1	0	14 93	0	8	7	0.105	—

For $\rho = 1 \times 10^{-6}$ and cardinality 8, i.e. quantum number 1, Δλ the spectral width comes out to be 1 17×10^{-4} ÅU, which is the value given by quantum mechanics.

This value establishes a consistency between Table 1 and 2 and also establishes a consistency between the Bohr theory of the hydrogen atom and the present work.

We now apply the Cantor continuum principle to another branch of physics which deals with elementary particles. Earlier the energy levels of the hydrogen atom were considered because of the elementary nature of the particles involved, and the fact that the problem had been solved by quantum mechanics and could be used to compare the results of the two theories. The theory of elementary particles concerns problems in the forefront of physics and the solution is far from complete. An attempt is made to derive some order in the distribution of masses of elementary particles as it is essentially a problem of energy levels. The Cantor continuum principle is so general that it can be used in principle to the study of all physical problems involving discreteness and continuity. It is not possible to apply the present theory to problems where there is close interaction between particles as in the case of nuclear physics, but it could stand a trial with isolated elementary particles.

If T_0 is the life-time of an elementary particle, its energy width is given by $\hbar/T_0$. If M is the mass of the particle in ergs and C is some constant, we have from equation (1)

$$\Delta E_0/E = \hbar/T_0 M = C \,.\, n/2^n.$$

Since we know from experimental data the lifetimes of many of these particles and their masses, we can determine the values of $C.n/2^n$. By considering different values for the constant C and by selecting the values of the cardinal number for various values of C, it can be shown that for values of $C = 10E-1$, $C = 10E-8$ and $C = 10E-15$, the values of log $\hbar/TM$ when plotted against log $n/2^n$ give

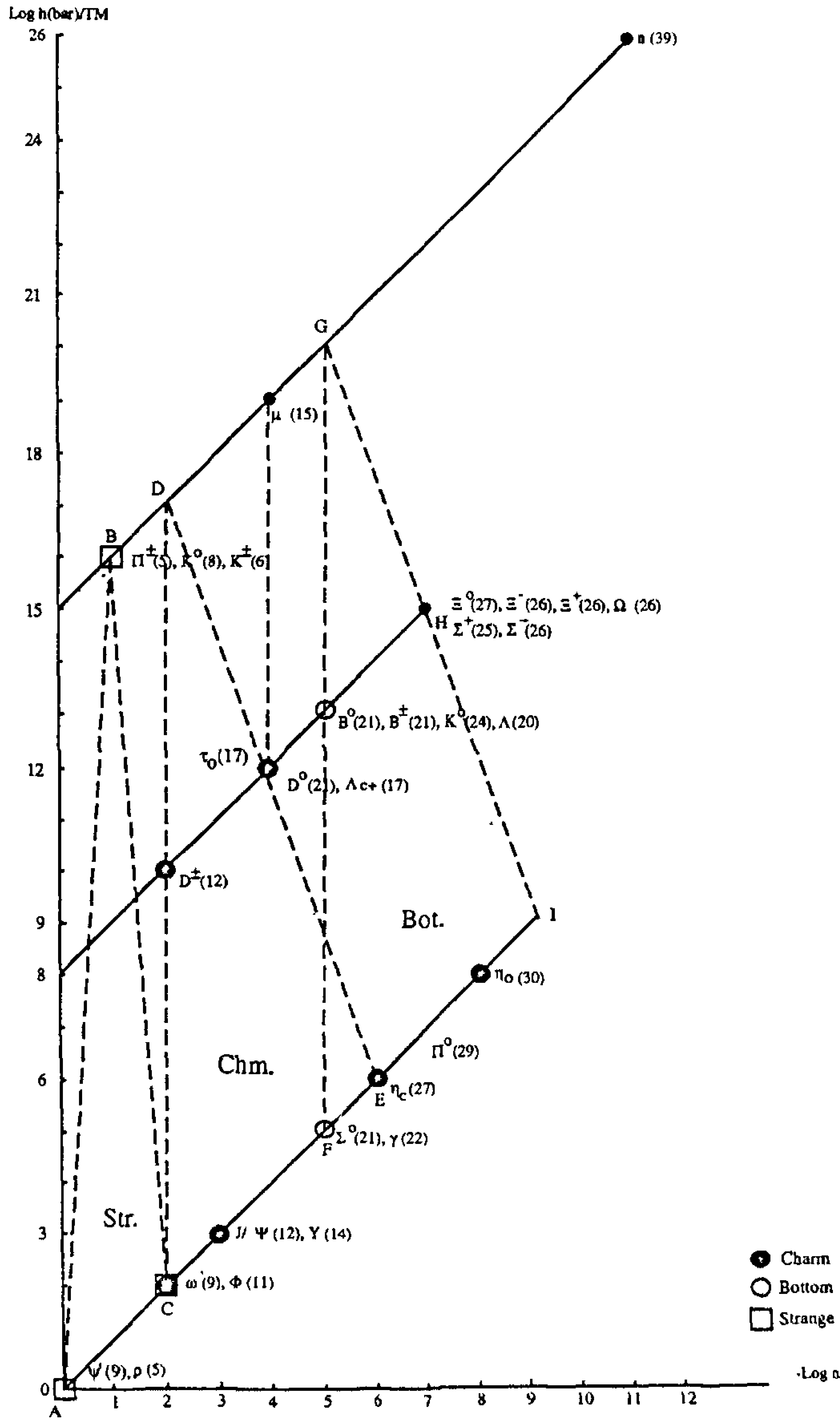

Figure 2. Families of elementary particle and cardinal number determining discreteness and continuity. (Figures in brackets denote cardinal numbers.) For representing the information from Table 3 in this figure the nearest integers have been used for the Log. values.

straight lines. From Table 3 and Figure 2 we can infer the following:

(a) It is possible to see some ordering among the masses and life-times of elementary particles through the choosing of an appropriate cardinal number.

(b) Though the choice of the cardinal number may seem arbitrary, the intercepts of the three lines grouping the particles divide them according to their life-times, as follows:

(1) Those whose life-times are between 1000 sec and 10^{-8} sec.

(2) Those whose life-times are between 10^{-10} sec and 10^{-13} sec, and

(3) Those whose life-times are beyond 10^{-17} sec and 10^{-24} sec.

(c) It is seen from Figure 2 and Table 3 that particles described as strange (Str), charm (Chm) and bottom (Bot) are found in the sides of triangles ABC, CDE and FGI respectively. It suggests that Ψ' should be a strange particle. All particles found at H are hyperons. One could expect new particles at points marked D, G and I.

(d) The cardinal number by itself represents the overlap of the continuous elements with the discrete elements as given by equation (1). For example, the neutron has a high cardinal number of 39. This means that its discreteness and thus its identity/reality is well preserved. In the case of those with lower cardinal numbers further investigation is required to see whether it is possible to connect their interactive properties, implying that a high component of continuous elements implies a greater capability of interaction with other particles leading to a loss of its identity.

Real and virtual events

We now associate all observable *events* with the possible sequence of conditions given by the matrix and associate all *virtual events*, which as we know are predicted by quantum theory, with an infinite number of the missing sequences, i.e. those different from the diagonal elements of the matrix.

If all the observable physical *events* are described by the elements of the matrix and if all virtual *events* are associ-

Table 3. Cardinality and ordering of elementary particles

1 Particle		2 Mass (M) (in ergs)	3 Decay time (T) (secs)	4 h (bar) TM	5 n	6 $n/2^n$	7 C
Mesons and baryons							
1. $\Pi^{\pm}$	Pion	0.2243*E*-3	2.6*E*-08	1.8*E*-16	5	1.5*E*-1	*E*-15
2. Π^0	Pion	0 216*E*-3	8.7*E*-17	0.56*E*-7	29	0 54*E*-7	*E*-0
3. ρ	Meson	1.232*E*-3	4.3*E*-24	0.198*E*-0	5	0.156*E*-0	*E*-0
4. ω	Meson	1.25*E*-3	0.67*E*-22	1.25*E*-2	9	1 82*E*-2	*E*-0
5. μ	Meson	0.169*E*-3	2.197*E*-6	2.98*E*-19	15	3.1*E*-4	*E*-15
6. τ	Meson	2.854*E*-3	0.3*E*-12	1.2*E*-12	17	1.3*E*-4	*E*-8
Strange particles							
7. $K^{\pm}$	Kaon	0.789*E*-3	1.24*E*-8	1.07*E*-16	6	0.94*E*-1	*E*-15
8. ϕ	Meson	1.019*E*-3	1.5*E*-22	6 869*E*-3	11	5.0*E*-3	*E*-0
9. K^0	Kaon	0.789*E*-3	5.2*E*-8	0.25*E*-16	8	0 31*E*-1	*E*-15
			or 8.9*E*-11	0.15*E*-13	24	1.43*E*-6	*E*-8
Charm particles							
10. D^0	Meson	2.97*E*-3	4.4*E*-13	0.8*E*-12	21	1.29*E*-4	*E*-8
11. $D^{\pm}$	Meson	2.99*E*-13	9.2*E*-13	0.38*E*-10	12	0 29*E*-2	*E*-8
12. η_c	Meson	0.876*E*-3	6 0*E*-18	0.2*E*-6	27	0.19*E*-6	*E*-0
13. J/Ψ	Meson	3.097*E*-3	1 0*E*-20	3.4*E*-3	12	2.9*E*-3	*E*-0
14. Ψ'	Meson	3.77*E*-3	0.26*E*-22	0.0107*E*-0	9	0.018*E*-0	*E*-0
Bottom							
15. $B^{\pm}$	Meson	8.43*E*-3	14.2*E*-13	0.88*E*-13	21	1.0*E*-5	*E*-8
16. B^0	Meson	8 44*E*-3	14.2*E*-13	0.38*E*-13	21	1.0*E*-5	*E*-8
17. γ	Meson	15.13*E*-3	1.5*E*-20	0.46*E*-5	22	0.52*E*-5	*E*-0
18. η_0	Meson	5.48*E*-3	6 0*E*-18	3.19*E*-8	30	2.79*E*-8	*E*-0
19. n	Neutron	1.504*E*-3	898	7.8*E*-26	39	7 0*E*-11	*E*-15
Hyperons							
20. Λ		1.784*E*-3	2.6*E*-10	2 2*E*-13	20	1.91*E*-5	*E*-8
21. Σ^+		1.902*E*-3	0.8*E*-10	6.9*E*-15	25	7.45*E*-7	*E*-8
22. Σ^0		1 907*E*-3	5.8*E*-20	0.9*E*-5	21	1.00*E*-5	*E*-0
23. Σ^-		1.915E-3	1.48*E*-10	3.7*E*-15	26	3.8*E*-7	*E*-8
24. Ξ^0		2.103*E*-3	2.9*E*-10	1.7*E*-15	27	2.0*E*-7	*E*-8
25. Ξ^-		2.113*E*-3	1.642*E*-10	3.0*E*-15	26	3 8*E*-7	*E*-8
26. Ω		2.675*E*-3	0.822*E*-10	4.7*E*-15	26	3.8*E*-7	*E*-8
Ξ^+							
27. Λ_{c+}		3 649*E*-3	2.3*E*-13	1.2*E*-12	17	1.29*E*-4	*E*-8

ated with the missing sequences, which now form a set by themselves, it provides for a 'duality'. This arises because for every *real event A*, there is a *virtual event A'*, opposite in character, since a one-to-one correlation exists between A and A'. Every sequence in the missing set arises from an element denied in the diagonal set.

Whether the wave and particle nature of matter which are both observable, are found in A or A', depends on whether *events* belong to either (A)'s or (A ')'s respectively. It also depends on the validity of the Complementarity Principle of Bohr. If particle and wave behaviour are described as both belonging to A only, then the wave and particle property can be observed simultaneously which is contrary to that predicted by Bohr (see ref. 4). If one is from *event* set A and the other from set A', their detection will be mutually exclusive.

Conclusion

This work is based on Buddhist Logic of Conditional Reality, Kshana and Nothingness[5]. In the matrix I, if the causes of events are dependent on each other it leads to what is known as 'conditioned reality'. While in classical physics, space and time are separate, in reality this is not so and it leads to new physics. In quantum mechanics, the wave nature is merged with particle nature and leads to many paradoxes but this is all a part of 'conditioned reality'. 'Kshana' is the theory of time averages and 'nothingness' is the existence of virtual states. These are described in a forthcoming paper entitled 'Causality, cardinality and conditioned reality' in the project of history of Indian science, philosophy and culture, Calcutta.

1. Ramanna, R., *Sci Cult*, 969, 35, 649 - 660.
2. Courant, R. and Robbins, H., *What is Mathematics: An Elementary Approach to Ideas and Methods*, Oxford University Press, 1978, pp. 84.
3. Heitler, W., *The Quantum Theory of Radiation*, 2nd edn., Oxford University Press, 1944
4. Ghose, P., Home, D. and Agarwal, G. S., *Phy. Lett. A*, 1992, 168, 95-99.
5. Stcherbatsky, F. Th., *Buddhist Logic*, Oriental Books, New Delhi, 1984; Originally printed in *Bibliotheca Buddhica Series*, 1930, vol. 26, Parts 1 and 2.

ACKNOWLEDGEMENTS. My thanks are due to Prof. B. V. Sreekantan for many useful discussions without which the paper would not have been possible. I thank Prof. C. V. Sundaram for his continued support and encouragement. My thanks are also due to Dr Dipankar Home for carefully going through the manuscript and making very valuable suggestions.

Received 16 August 1993; revised accepted 17 September 1993

Indian strains of hepatitis-C-virus: Prevalence and detection

M. R. Das, Naushad Ali, B. Aruna*, Rama Durga*, A. Nambiar, Z. Rehana, M. S. Shekhar, A. Siddiqui** and C. M. Habibullah*

Centre for Cellular and Molecular Biology, Hyderabad 500 007, India
*Department of Gastroenterology, Osmania General Hospital, Hyderabad 500 012, India
**Department of Microbiology, University of Colorado Health Sciences Center, Denver, Colorado 80262, USA

A simple polymerase chain reaction (PCR) method for detecting Indian strains of hepatitis-C virus (HCV) directly using serum samples is described. Owing to known genetic variability in HCV strains, commonly used primers for detecting US and Japanese strains were found unsuitable for Indian strains. We report the successful use of primers designed from minimum variable regions of the HCV genome for detecting Indian strains of the virus by PCR. The PCR products have been authenticated and one of them sequenced. We also show that the method we have developed can detect the presence of HCV in ELISA-negative (using commercially available kits) patients who had received blood transfusion.

THE existence of a parenteral non-A, non-B (NANB) hepatitis agent was first reported[1-3] in 1989. A number of reports based on enzyme immunoassays indicate that this virus, termed hepatitis-C (HCV), is a predominant cause of post-transfusion NANB hepatitis around the world[2,4-8]. Anti-HCV is associated with most community-acquired NANB hepatitis cases in the United States[2,9] and Western Europe[5,10,11] and is a major cause of cryptogenic chronic liver disease in Italy[12]. In addition, anti-HCV is associated with most cases of hepatocellular carcinoma (HCC) in Japan[13], Italy[11] and Spain[15]. On the other hand, practically no information is available on either the prevalence or the nature of HCV in India.

More than 50% of the patients who acquire acute HCV infection develop chronic hepatitis[16]. Clinical manifestations of HCV infection include acute hepatitis which may be resolving or fulminant, chronic hepatitis, cirrhosis and hepatocellular carcinoma. Some of the