To obtain more physically realistic models, one must proceed to establish dynamic models in which the slip is one of the unknowns derived from the state of stress and the strength of the material at the source region. The general problem of dynamic models is based on the idea of crack formation and propagation in a prestressed medium. A discussion of the dynamic models of earthquake source mechanism is beyond the scope of this paper (see e.g. refs 15 and 16).

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Uncertainties with strong motion earthquake parameters

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A large measure of uncertainty is associated with the earthquake ground motion. The characteristics of the earthquake ground motion are examined to identify the sources of uncertainties. Seismic risk analysis is presented to establish the distribution of peak ground motion parameters. Random process models of the earthquake ground motion are discussed.

Aseismic design of a structure involves prediction of the nature of ground motion at the site during the service life of the structure. This in turn requires: (i) identification of the potential sources of strong motion earthquakes; (ii) geometry of each source; (iii) magnitude, epicentral location and focal depth, and time history of occurrence of past earthquakes for each source; and (iv) attenuation laws.

A large measure of uncertainty is associated with each of the above factors. The cumulative effect of these uncertainties makes the earthquake-induced ground motion at a point, a time-dependent random process vector. In earthquake engineering, it is convenient to resolve this vector into three random processes—two along perpendicular directions in the horizontal plane and one vertical. The uncertainty in the earthquake-induced ground motion may, therefore, be represented to varying degree of completeness by: (i) ensemble of sample functions; (ii) hierarchy of joint probability distribution, or characteristic functions; (iii) envelope functions, and intensity moments[1]; (iv) spectral density functions and spectral moments[2], Fourier and response spectra; and (v) gross properties of ground motion components in terms of random variables, such as, peak values of ground acceleration, velocity and displacements, r.m.s. value, duration, spectral intensity, etc.

For long structures with a dimension significantly large as compared to the characteristic wavelengths in earthquake ground motion, the spatial randomness must also be considered and the ground motion must be modelled as a multi-parametered random process vector. This introduces additional uncertainties.

In this paper we consider the cumulative effect of uncertainties associated with various factors to establish the distribution, and other characteristics of ground motion at a point in terms of: (i) peak ground acceleration, velocity and displacement treated as random variables; and (ii) ground motion time-histories, treated as random processes. First, we consider the characteristics of the earthquake induced ground motion.

Characteristics of earthquake ground motion

Most earthquakes of engineering significance are of tectonic origin and are caused by slip along geological faults. While specific source mechanisms leading to a slip vary in different regions of the earth, four basic types of faulting can be identified with strong-motion earthquakes[3]. In most earthquakes, the actual slip mechanism is a combination of two or more types of faulting. Often slip occurs on an irregular surface and on more than one fault. The characteristics of ground motion during an earthquake in the vicinity of the causative fault (near-field) are strongly dependent on the type of faulting and the time-history motion of fault displacement. As we move away from the fault (far-field), the nature of ground motion is primarily determined by the travel-path geology. The nature of ground motion at a point on the earth's surface is also influenced by local site conditions, that is, soil properties and topography. Characteristics of source mechanisms, travel-path geology and local site conditions, therefore, determine the nature of ground motion due to an earthquake. The basic characteristics of resulting seismic waves depend primarily on: stress drop during the slip; total fault displacement; size of the slipped area; roughness of the slipping process; fault shape; and proximity of the slipped area to the ground surface. As the waves radiate from the fault, they undergo geometric spreading and attenuation due to loss of energy (internal friction) in the rocks. Since the interior of the earth consists of heterogeneous formations, seismic waves suffer multiple reflections, refractions, dispersion and attenuation as they travel. Seismic waves arriving at a site on the surface of the earth are thus a result of complex superposition giving rise to an irregular motion which may be modelled as a random vector varying randomly in space and time.

Earthquake ground motion at a point on the surface of the earth consists of both translation and rotation components. For most problems the rotational component can be disregarded and ground motion treated as a random vector with three orthogonal translational components. Each component can be expressed either by an acceleration, velocity or displacement function of time. Although the three forms contain equivalent information and can be derived from each other by differentiation or integration, it is generally convenient to represent and record earthquake ground motion as acceleration from which velocity and displacement are derived through integration, if required. The ground acceleration, velocity and displacement time-history of a typical ground motion (NS.EL–Centro, 18 May 1940) are shown in Figure 1.

The dynamic behaviour of structures during an earthquake is determined primarily by the amplitude, frequency content and duration of ground motion. The amplitude of strong-motion earthquake acceleration records generally exhibit (i) a rapid build-up at the beginning of the motion; (ii) a nearly constant value during the strong-motion shaking; and (iii) an exponentially decaying tail. Their frequency characteristics are reflected by the Fourier amplitude spectra; power spectral density (psd); response spectra; or response envelope spectra[4]. Most strong motion earthquakes possess broad-band characteristics, and their acceleration time-history can be adequately modelled as a uniformly modulated stationary random process. The high frequency components of ground motion attenuate with distance faster than the low frequency components, and this strongly influences the spectral characteristics of ground motion with distance. The important parameters defining the gross characteristics of earthquake motion at a site are the peak values of ground acceleration (A_g), velocity (V_g), displacement (D_g), the r.m.s. value of ground acceleration (σ_g); the response spectra (SA, SV, SD), the spectrum intensity (SI), a site intensity such as (I), and the duration (T'). These may be treated as random variables. In the next section we derive the distribution functions of the peak ground motion parameters through seismic risk analysis. A similar approach may be used for estimating other parameters.

Distribution of peak ground motion parameters

The occurrence of earthquakes, both temporal and spatial, involves a large measure of uncertainty. The multiple reflections, refractions and dispersions of seismic waves at irregular boundaries in the course of their travel to a site compound the level of uncertainty even further. This results in a high level of variability in the estimates

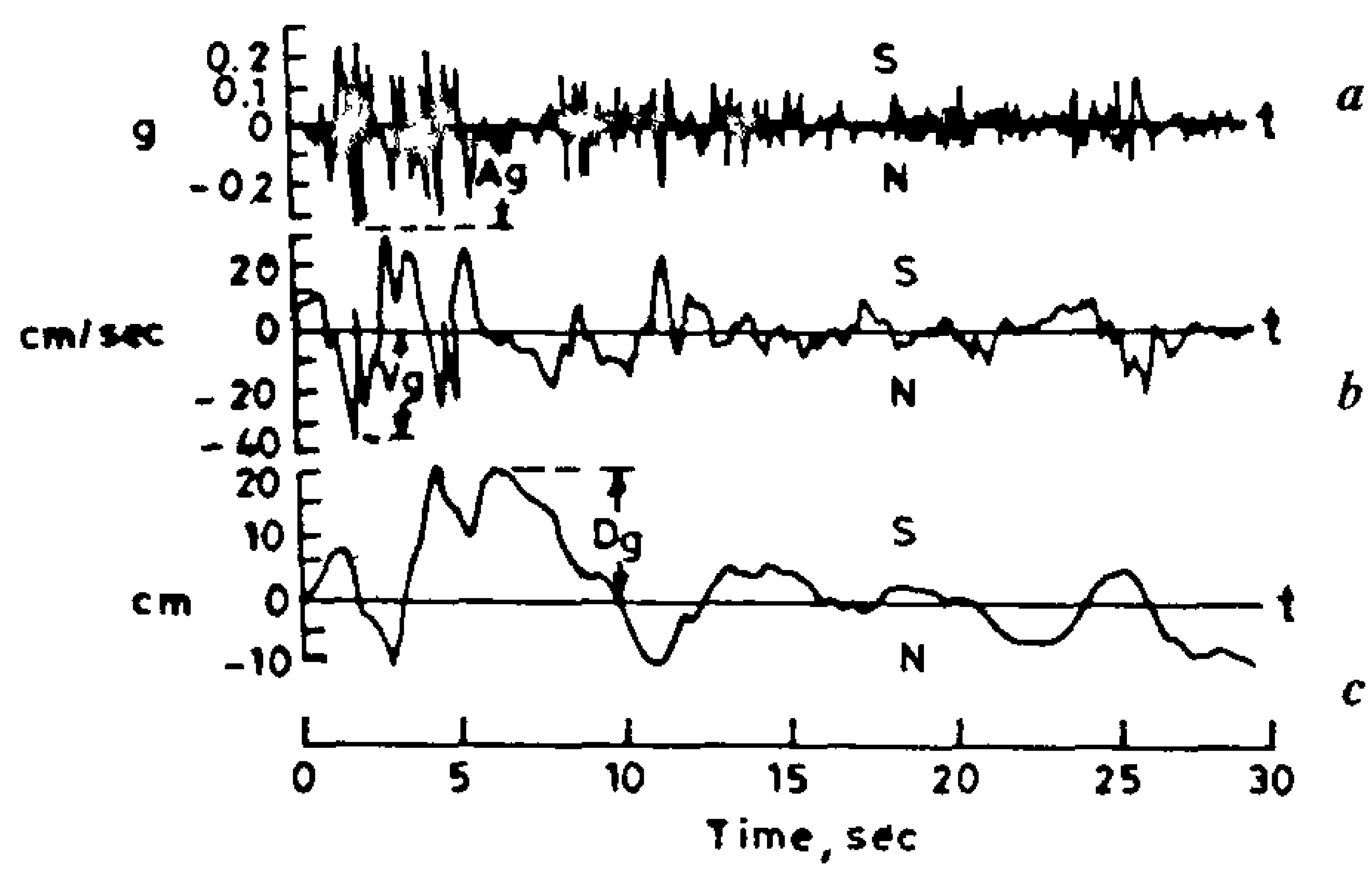

Figure 1. Ground motion time histories of El-Centro, May 18, 1940, NS-component earthquake record. (*a*) acceleration, (*b*) velocity, and (*c*) displacement.

of ground motion intensity at a site during the service life of a structure, which is the major contributor to the total seismic risk. This is determined by embedding the random vibration analysis into seismic risk analysis.

The location of a site in relation to an earthquake may be specified by the *focal distance* (R); or by the *epicentral distance* (X) and the *focal depth* (H). Clearly,

$$R = (X^2 + H^2)^{1/2}, \tag{1}$$

Peak ground acceleration, peak ground velocity, and peak ground displacement, are respectively the absolute peak values of the ordinates in the ground acceleration, velocity and displacement records at a site. An empirical relationship between the peak ground motion parameters (A_g, V_g or D_g) and the magnitude and focal distance, called attenuation relations, can be expressed in the following general form[5,6],

$$Y = b_1 \exp(b_2 M) R^{-b_3}, \tag{2}$$

where Y may denote A_g, V_g or D_g; and b_1, b_2 and b_3 are constants which depend on the quantity represented by Y. Esteva and Rosenbleuth[5] suggesting that, for southern California, the average values of constants (b_1, b_2, b_3) may be chosen as (2000, 0.8, 2), (16, 1.0, 1.7) and (7, 1.2, 1.6), if Y denotes A_g, V_g or D_g in units of centimetres and seconds, and R is measured in km. It may be remarked that available data show a large scatter around the mean curve represented by 2[7]. We shall discuss this aspect later.

Seismic risk analysis

Seismic risk at a site is usually expressed in terms of the probability of site intensity exceeding a certain value in a given period of time. The term 'intensity' is used here in a generic sense to denote any one, or a function of several, ground motion parameters[8]. To determine the seismic risk at a site, it is necessary to construct stochastic models of source parameters, both temporal and spatial, and the travel path. A seismic source is characterized by its geometry, temporal characteristics and size of the seismic events. The geometric shape of the source on the surface of the earth can be idealized as a point, a line, or an area based on the knowledge of the spatial distribution of past earthquakes and known geotectonic features. The spatial distribution of earthquakes can generally be assumed to be homogeneous in a source, and, if necessary, a source may be subdivided into homogeneous sources. The focal depth of earthquakes is usually assumed to be constant. However, if focal depth data are available, a distribution can be fitted[9,10]. The occurrence of earthquakes, in time, at a source is generally assumed to follow a Poisson distribution, so that

$$P_N(n, t) = \frac{(\nu t)^n \exp(-\nu t)}{n!}, \tag{3}$$

where $P_N(n, t)$ denotes the probability of occurrence of n earthquakes of magnitude greater than say m_0 during the time interval t, and $\nu = \nu(m_0)$ is the expected rate of occurrence of such earthquakes. A Poisson model assumes stationarity and independence of successive events. Both these assumptions remain to be fully substantiated by data and evolutionary nature of earthquake occurrence, specially if fore- and after-shocks are included. However, the model is considered adequate for the service life of most structures[11,12].

The probability density function of the magnitude of earthquakes may be expressed as

$$p_M(m) = \beta \exp[-\beta(m - m_0)], \quad m \ge m_0, \tag{4}$$

where m_0 is the threshold magnitude below which the events are not of engineering interest and $\beta = (1.5 - 2.3)$ is a constant for the source.

It is clear that the variables M and R in equation (2) are random variables, and therefore, the site intensity Y is also a random variable. The distribution of M is given by equation (4), and the distribution of R can be derived for a given source-geometry, assumed spatial distribution of epicentres and focal depth. For example, for a line-source of length L, uniform distribution of epicentres along the line and constant focal depth, it can be shown[13] that

$$p_R^{(r)} = \frac{2r}{L(r^2 - d^2)^{1/2}}, \quad d \le r \le r_0, \tag{5}$$

where d and r_0 are indicated in Figure 2. Assuming that R and M are independent random variables, it can be shown from the theory of functions of random variables that[13]

$$P_Y = P[Y \ge y] = 1 - F_Y(y) = \frac{C}{L} G y^{-\beta/b_2}, \quad y \ge y', \tag{6}$$

where

$$C = \exp(\beta m_0)\,(b_1)^{\beta/b_2}, \tag{7}$$

$$G = \frac{2}{d^{\nu}} \int_0^{\sec^{-1}(r_0/d)} (\cos u)^{\nu-1}\, du, \tag{8}$$

$$\nu = \beta(b_3/b_2) - 1, \tag{9}$$

and

$$y' = b_1 \exp(b_2 m_0)\, d^{-b_3}. \tag{10}$$

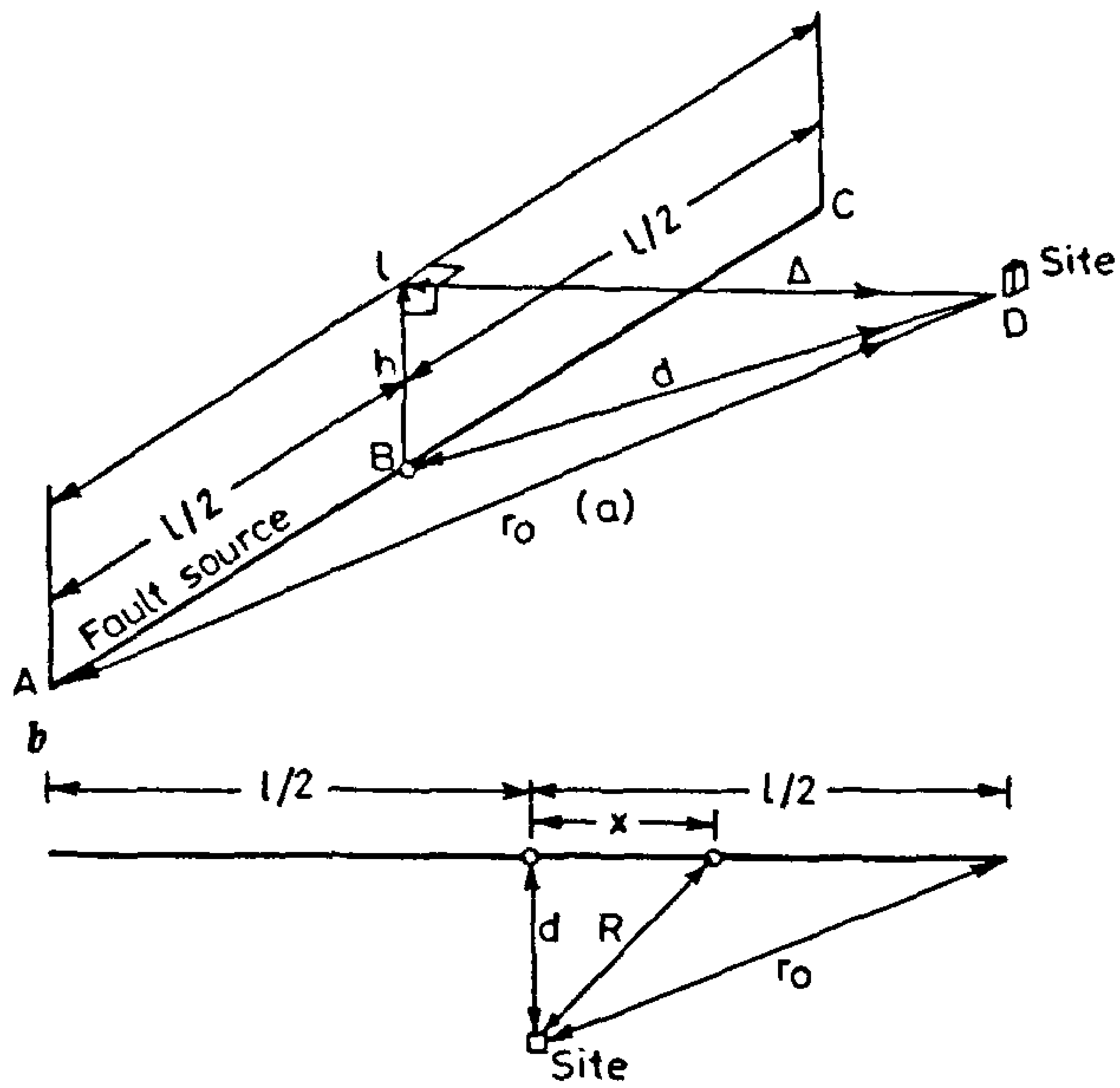

Figure 2. Line source: (*a*) perspective, and (*b*) plan (Cornell, 1967).

Note that parameter G depends on the geometry of the source and can be similarly derived for the area source. For a point source

$$G = (r)^{-(1+\nu)}.$$

Equation (6) gives the probability that the site intensity Y will exceed the value y given that an event of magnitude $M \geq m_0$ has occurred at the source. Since the occurrence of such events at a source is assumed to be Poisson with arrival rate ν, it is clear that events with site-intensity: $Y \geq y$, are 'special events' with arrival rate $P_y\nu$, and

$$P^*_N(n, t) = \frac{(P_y\nu t)^n}{n!}\exp(-P_y\nu t),$$

$$n = 0, 1, 2, 3, \ldots, \quad (11)$$

where $N^*(t)$ is the counting process of special events $Y \geq y$ at the site.

Let $Y_m(t)$ denote the maximum value of intensity over a t-year period, and let

$$Y_m = Y_m(t) \mid t = 1,$$

be the annual maximum intensity. It can be shown that

$$F_{Y_m}(y) = \exp[-\hat{\nu}CGy^{(-\beta/b_2)}], \quad y \geq y',$$

$$\simeq 1 - \hat{\nu}CGy^{(-\beta/b_2)}, \quad y \geq y', \quad (12)$$

if $\hat{\nu}CGy^{(-\beta/b_2)} \ll 1$, for probabilities of interest in design.

The annual return period T_y is given by

$$T_y = \frac{1}{1-(F_y/y)} = \frac{1}{\hat{\nu}cG}\, y^{(-\beta/b_2)}, \quad (13)$$

where $\hat{\nu} = \nu/L$. The T-year intensity y_T is obtained from equation (13)

$$y_T = (\hat{\nu}CGT_y)^{(-\beta/b_2)}. \quad (14)$$

It may be noted that equation (12) represents type–II asymptotic extreme value distribution of largest values.

The above results have been derived for a single source. If a site experiences shaking by more than one source, the above results can be easily extended, assuming the various sources to be independent. Consider m such sources. It is clear that $P[Y_m(t) \leq y]$ is the probability that the maximum value from each source is less than or equal to y, that is

$$F^{(t)}_{Y_m}(y, t) = \prod_{j=1}^{m} F^{(t)}_{Y_m}(y, t),$$

$$= \exp\left(-\sum_{j=1}^{m}\hat{\nu}_j C_j G_j y^{-\beta/2b_{2j}} t\right), \quad y \geq y', \quad (15)$$

where $F^{(t)}_{Y_m}(y, t)$ is the distribution of the maximum, in time t, for jth source, and y' is the largest y'_j. If constants β, $b_i = 1, 2, 3$ are the same for each source, equation (15) reduces to

$$F^{(t)}_{Y_m}(y, t) = \exp(-C\hat{\nu}Gy^{-\beta/b_2}t), \quad (16)$$

where

$$\hat{\nu}G = \sum_{j=1}\hat{\nu}_j G_j. \quad (17)$$

Equations (16) and (17) indicate that each source contributes approximately in an additive way to the risk. The design intensity for a specified probability ($y_{t,p}$), the return period for a specified intensity, and T-year intensity can be obtained from equation (15) or equation (16) for a multiple source condition.

Some extensions and comments

The discussion in the preceding sections covered the basic approach to engineering seismic risk analysis, which can be integrated with random vibration analysis to compute the total risk. In this section, we shall discuss some extensions, and comment on possible improvements.

Limited magnitude distribution

In the preceding treatment, the earthquake magnitude could take any value above the threshold value m_0. On physical grounds it can be argued that there should be a limit on the size of earthquakes[14]. Such a limit can be established on the basis of historical or geophysical data for a region or a source. Let m_1 be the upper limit on the magnitude of the earthquake. The probability density function for M can be expressed as

$$p_M(m) = k_{m_1} \beta \exp[-\beta(m-m_0)], \quad m_0 \le m \le m_1, \tag{18}$$

where

$$k_{m_1} = \{1-\exp[-\beta(m_1-m_0)]\}^{-1}, \tag{19}$$

is the normalizing factor. The restriction on magnitude to an upper limit m_1, defines a focal distance r_y, beyond which no earthquake will cause ground motion in excess of y. This distance is

$$r_y = (y/b_1)^{-1/b_3} \exp(b_2 m_1/b_3), \tag{20}$$

and the distribution for $Y_m(t)$ can be expressed as[15],

$$F_{Y_m^{(t)}}(y,t) = \exp[-\hat{\nu}_y t(1-k_m)] \times \exp[-k_{m_1} C \hat{\nu}_y G_y y^{-\beta/b_2}], \quad y \ge y'; \tag{21}$$

where $\hat{\nu}_y G_y$ is given by equation (17). The subscript y on $\hat{\nu}$ and G implies that sources, or part thereof, are considered within the radius r_y. Computations of seismic risk show that the major contribution to risk comes from more frequent smaller earthquakes close to the site, and the risk is not significantly decreased by the upper-limit on magnitude[15].

Scatter in attenuation relations

The attenuation relations, equation (2), provide only a crude correlation with the large scatter of observed data[7]. To incorporate the effect of scatter on seismic risk estimates, the attenuation relations may be defined as

$$Y = (b_1 e^{b_2 M} R^{-b_3})\,\varepsilon \tag{22}$$

where ε is an assumed random error, defined as the ratio between observed and measured ground intensities. Esteva[7] showed that ε is approximately normally distributed with zero mean and standard deviation σ, lying in the range 0.20 to 1.10. Integrating over the error term in the seismic risk analysis, it can be shown that for fixed R, that is, for a point source

$$P_Y = P(Y>y) = (1-k_{m1})\,\phi^*(Z/\sigma) + k_{m1}\,\phi^*\left(\frac{Z}{\sigma}-\frac{\beta\sigma}{b_2}\right)\exp(\beta^2\sigma^2/2b_2^2)\exp(\beta m_0)\times R^{-\beta b_3/b_2}\left(\frac{y}{b_1}\right)^{-\beta/b_2}, \tag{23}$$

where $\phi^*(\,.\,)$ is the complimentary cumulative distribution function of the standardized Gaussian distribution, and

$$Z = \ln(y) - \ln[b_1 \exp(b_2 m_1) R^{-b_3}]. \tag{24}$$

The above result for the point source can be used to compute the excedance probability for arbitrary sources numerically. The effect of including the attenuation law uncertainty can result in almost doubling the computed risk.

Cornell and Vanmarcke[15] examined in detail the sensitivity of seismic risk estimates to various parameters and found that risk may be sensitive to the focal depth H, to the attenuation constant b_3, and to the ratio β/b_2. Esteva[16] suggested replacement of focal-depth H, by 'effective focal-depth' $(H^2+2O^2)^{1/2}$, to improve the fit in the near-source zone. Basu[9] has fitted uniform and truncated lognormal distributions to the focal depth data for the Indian subcontinent and incorporated these in seismic risk analysis.

Random process models of earthquake ground motions

Housner[17] was the first to suggest that acceleration records of earthquake exhibit characteristics of randomness. If a large number of ground motion records were available for a particular site, the parameters of a random process model could be determined directly by statistical analysis. However, this approach is not possible at present in any part of the world, due to availability of only a few strong-motion records. It, therefore, becomes necessary to use considerable judgement in constructing and validating stochastic models of ground motion on the basis of a few available records at a site, or at comparable locations coupled with seismological and geological data, and local site conditions.

Let $Z_i(t)$, i = 1, 2, 3, represent the three components of ground displacement at a point due to an earthquake. Consistent with general characteristics of strong-motion earthquakes, each component of the ground acceleration can be expressed as

$$\ddot{Z}_i(t) = A_i(t)\,\ddot{Y}_i(t);\quad 0 \le t \le T',$$
$$= 0;\quad \text{otherwise},\quad t > T', \qquad (25)$$

where $i = 1, 2, 3$; $A_i(t)$ are slowly varying random functions of time called the envelope; $\ddot{Y}_i$ are a segment of stationary random processes; and T' is the duration of the motion. Thus, each component of ground acceleration is a realization of a nonstationary random process. The stochastic model represented by equation (25) may be described to varying degrees of completeness by the joint distribution functions, the joint moment or spectral functions and other properties such as level crossings, peaks, etc.

We shall first discuss the properties and models of individual components. Three types of basic models, and their minor variations reflecting increasing level of complexity, have been used to model earthquake ground acceleration: (i) white noise[17–19], (ii) stationary process[20, 21] and (iii) nonstationary process[22–26].

If the short duration of initial rise and exponentially decaying tail are disregarded, the central strong-motion portion of acceleration records on firm ground can be treated as a segment of stationary random process. Byrcroft[18] showed that properly scaled segment of white noise have response spectra similar to the response spectra of real earthquakes and can, therefore, be used to model the strong-motion portion of ground acceleration.

The stationary process noise model is an improvement over the white noise as it can be shaped to represent the frequency characteristics of the actual ground motions. The model makes it possible to incorporate the effect of local soil conditions explicitly. A segment of stationary process is, therefore, a more realistic model of the strong-motion portion of ground acceleration records. The following analytical expression due to Tajimi[20] based on the work of Kanai[27], called Kanai–Tajimi spectrum, has been extensively used to represent the power spectral density (psd) of ground acceleration:

$$\phi(\omega) = \phi_0 \frac{1 + 4\,\zeta_g^2\,(\omega/\omega_g)^2}{[1 - (\omega/\omega_g)^2\,]^2 + 4\,\zeta_g^2\,(\omega/\omega_g)^2}\,. \qquad (26)$$

A filtered white noise with psd given by equation (26) can be generated by the following second order filter

$$\ddot{U} + 2\,\zeta_g\,\omega_g\,\dot{U} + \omega_g^2\,U = W(t), \qquad (27)$$

where $W(t)$ is white noise with psd ϕ_0 and $\ddot{Z}(t) = \ddot{U}(t) + W(t)$. From an analysis of actual records, it is found that $\zeta_g = 0.6$ and $\omega_g = 5\pi$ correspond closely to the spectral properties for firm ground. For a specific site, the parameters ζ_g and ω_g should be chosen suitably to represent local site conditions.

The variance and other spectral parameters for the Kanai–Tajimi spectrum are given by

$$\sigma_g^2 = \frac{\pi\phi_0\,\omega_g}{2\,\zeta_g}\,(1 + 4\,\zeta_g^2)\,,$$
$$\Omega_g = (\lambda_2/\lambda_0)^{1/2} \simeq 2.1\,\omega_g\,,$$
$$\delta_g = \left(1 + \frac{\lambda_1^2}{\lambda_0\,\lambda_2}\right)^{1/2} = 0.67\,. \qquad (28)$$

In equation (28), cut-off frequency $\omega_0 = 4\,\omega_g$ and $\zeta_g = 0.6$ (ref. 28). Bolotin[22] proposed the following expression to represent a comparable auto-correlation function for stationary ground acceleration:

$$R(\tau) = \sigma_2^2\,\exp(-\alpha)\,|\tau|\,\cos\beta\tau\,. \qquad (29)$$

The nonstationarity in the ground acceleration records arises primarily through envelope functions $A_i(t)$ in equation (25). These are slowly varying random functions of time. A simple and adequate nonstationary model of ground acceleration can be constructed by assuming $A_i(t) = A(t)$ to be a deterministic function and $\ddot{Y}_i(t)$ a set of stationary random processes with specified psd. The model may be constructed either by multiplying a segment of filtered white noise by a modulating function (MFWN) or alternatively by multiplying white noise by a modulating function first and then filtering the product process (FMWN). If the filter characteristics and the modulating functions are identical for the two models, the difference in their characteristics depends on the smoothness of $A(t)$. For earthquakes with long quasi-stationary motion, both models yield similar characteristics.

Several functions have been used to model the envelope of the ground acceleration records. The following expressions are commonly used:

Iyengar and Iyengar[29]

$$A(t) = (A_0 + A_1 t)\,\exp(-at)\,H(t);$$
$$n = 1 \text{ or } 2,\quad a > 0\,. \qquad (30)$$

Shinozuka and Sato[24]

$$A(t) = A_0\,[\exp(-at) - \exp(-bt)\,]\,H(t);$$
$$b > a > 0\,. \qquad (31)$$

Jennings et al.[30]

$$A(t) = A_0\,(t/t_1)^a\,H(t),\qquad 0 \le t \le t_1\,,$$
$$= A_0\,H(t - t_1)\,,\qquad t_1 \le t \le t_2\,, \qquad (32)$$

$$= A_0 \exp[-b(t-t_2)]\ H(t-t_2), \qquad t_2 \le t \le t_3,$$

$$= A_0[.05c + d(T'-t)^2]\ H(t-t_3), \qquad t \ge t_3,$$

where $H(t)$ is heaviside unit step function.

The envelope functions represented by equation (32) cover significant features of a complete range of recorded earthquake ground accelerations. Jennings *et al.*[30] have suggested suitable values for the parameters of the envelope function to model four different types of earthquake motions (A, B, C and D) which are considered to be significant for engineering structures.

The envelope functions defined by equations (30) and (31) are analytically simpler and are used extensively in theoretical treatments. The nonstationary nature of ground motions has a significant influence on the tail value of the probability estimates, nonlinear response and soil behaviour. Further, the stationary models are inadequate for small near-field earthquake ground motions.

Multi-component ground motion

We have so far discussed the stochastic models of the individual components of ground acceleration during earthquakes. Since the three components of earthquake ground motion act simultaneously on a structure, it is necessary to construct stochastic models which incorporate their joint properties. Also, since the choice of axes along which the components of an earthquake is recorded is arbitrary, it is necessary to establish relations for the transformation of their properties due to the rotation of the coordinate systems.

Consider the three components of ground acceleration defined by equation (25) along the axes (123). Let $A_i(t) = A(t)$ be a deterministic function, and $\ddot{Y}_i(t)$ a zero-mean, stationary random process. Then the elements of the covariance matrix of $\ddot{Z}_i(t)$ are given by

$$K_{z_i z_j}(t,\tau) = E[\ddot{Z}_i(t)\ddot{Z}_j(t+\tau)],$$

$$= A(t)A(t+\tau)E[\ddot{Y}_i(t)\ \ddot{Y}_j(t+\tau)], \qquad (33)$$

$$= A(t)A(t+\tau)[R_{Y_iY_j}(\tau)];\ i,j = 1,2,3.$$

Since the correlation time of the accelerograms is usually very small, the effect of changing the coordinate directions on the covariance functions can be investigated by considering equation (33) for $\tau = 0$, that is

$$E[\ddot{Z}_i(t)\ddot{Z}_j(t)] = A^2(t)E[\ddot{Y}_i(t)\ddot{Y}_j(t)], \qquad (34)$$

which can be expressed in a matrix form

$$R_{\ddot{Z}\ddot{Z}}(t) = A^2(t)R_{\ddot{Y}\ddot{Y}} \qquad (35)$$

Since $\ddot{Y}(t)$ are stationary, the elements of variance matrix $R_{\ddot{Y}\ddot{Y}}$ are constants. Also $R_{\ddot{Y}\ddot{Y}}$ is a real, symmetric and positive definite matrix. It is, therefore, possible to find a canonical transformation matrix **P**, such that

$$\ddot{Y}_p(t) = P\ddot{Y}(t) \qquad (36)$$

and

$$R_{\ddot{Y}_p\ddot{Y}_p} = P^TR_{\ddot{Y}\ddot{Y}}P = \text{diag.}(R_{11}, R_{22}, R_{33}), \qquad (37)$$

where $R_{11} \ge R_{22} \ge R_{33}$ are the principle variances of the matrix $R_{\ddot{Y}\ddot{Y}}$, and the columns of P define the three orthogonal principle directions[31]. Further, from equation (35)

$$R_{\ddot{Z}_p\ddot{Z}_p} = A^2(t)\,\text{diag.}(R_{11}, R_{22}, R_{33}). \qquad (38)$$

Hence, the principal variances of $\ddot{Z}$ are given by $A^2(t)R_{ii}$ (no sum), $i = 1, 2, 3$. Note that variances are functions of time but the principal directions are time-independent. (If different modulation functions, $A_i(t)$, are used for each component, the principal directions will be time-dependent.) Under the assumptions stated above, it follows that the components of ground motion are correlated, but it is always possible to find at least one set of three principal directions along which the components are uncorrelated. The variances along these three principal directions represent maximum, minimum and intermediate values.

By analysing six different strong-motion records, Penzien and Watabe[31] concluded that:

(i) the ratios of the minor and intermediate principal variances to major principal variance are of the order of 1/2 and 3/4 respectively, (ii) the principal values of the cross-correlation coefficients $[\rho_{ij} = (R_{ii} - R_{jj})/(R_{ii} + R_{jj})]$ ρ_{12}, ρ_{23} and ρ_{13} are approximately 0.14, 0.2 and 0.33 respectively, (iii) the major principal axis is generally in the direction of the epicentre and the minor principal axis is vertical, and (iv) the directions of principal axes are reasonably stable over successive time intervals. It is, therefore, reasonable to assume the same envelope function, $A(t)$, in the three directions.

A significant conclusion of the above discussion is that the three components of ground acceleration can be modelled as mutually uncorrelated random processes, provided they are directed along a set of principal axes with the major principal axis directed towards the expected epicentre, and the minor principal axis directed vertically. Hadjian[32] analysed the correlation properties of the horizontal components of recorded ground motion to obtain the probability density function of the correlation coefficient. Recognizing that the uncertainties in the value of the correlation coefficient arise due to two distinct sources, geometric and seismological, Hadjian synthesized a probability density function shown in Figure 3. He suggested that an 'equivalent' rectangular distribution may be used for design codes and simulation purposes.

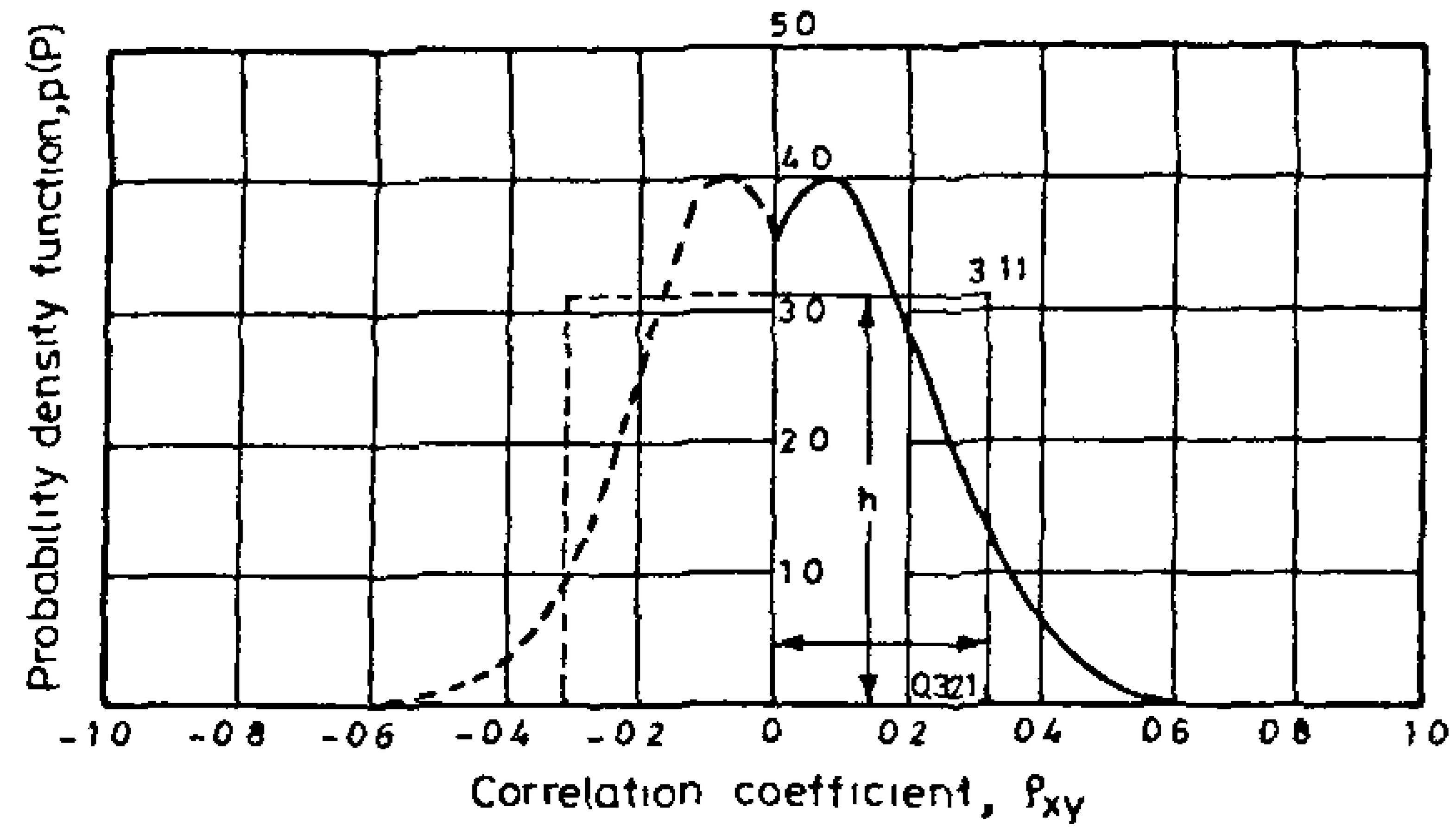

Figure 3. Probability distribution of correlation coefficient, and an 'equivalent' rectangular distribution (Hadjian, 1981)

Estimation of model parameters

Random process models of earthquake ground acceleration are used for two purposes: (i) random vibration analysis of system response to earthquake excitation, and (ii) simulation of an ensemble of ground acceleration records for Monte Carlo type studies. In either case, it is essential to determine the parameters of the model and to validate it. This is done by using both, available seismological information, and past earthquake records at the site, and at comparable locations. The peak values of ground motion parameters, duration of motion, etc. can be estimated from attenuation laws, if potential sources can be identified. Local site conditions can be incorporated in modelling the frequency content. The available ground motion records can be grouped into large, medium and small earthquakes[30] and an ensemble of earthquake records can be compiled for each group, after amplitude and time scaling, if required. The parameters of the model can be estimated on the basis of such information. The limitation of data is a serious problem in earthquake engineering and, therefore, considerable ingenuity and judgement is required for estimation of model parameters.

Random process models of earthquake ground acceleration involve two major components—the modulating function reflecting the nonstationary process reflecting the frequency content. The estimation of parameters and validation of the model is carried out by using a combination of the following characteristics[21,29,33,34]:

(i) peak ground acceleration (A_g); the time (t_m) at which the peak occurs; ratio (t_m/T'),
(ii) time-dependent variance; intensity moments[1]
(iii) rate of zero crossing (N_0), rate of maxima (N_m), ratio (N_0/N_m),
(iv) covariance function; psd; spectral moments[2],
(v) average response spectra, response envelope spectra, time response spectra[35] and
(vi) spectrum intensity.

In time-series models, parameters are estimated on the basis of a comparison between estimated statistics of generated sequences and the statistics of actual records. Digital processing technique may be used to estimate the parameters of FMWN model relying on a single record[36]. Parameters of stochastic models can also be estimated from theoretical considerations[4,26].

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