Quantum correlations in nonlinear optical signals

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It is shown that the intensity correlation function is central to the understanding of various aspects of nonclassicality of the electromagnetic field, e.g. quantum interference, antibunching, nonclassical statistics, squeezing and nonlocality. Particular emphasis is given on correlations of twin signals generated in nonlinear mixing interactions.

ONE of the most challenging aspects of research in the field of quantum optics has been to understand the nature and implications of *quantum correlations* generated in some optical processes. Quantum correlations between two systems are characterized by either the correlations between operators corresponding to observables associated with individual systems, or the correlated state describing the two systems. These correlations lead to a number of spectacular *nonclassical effects*, such as generation of number states (Fock states)[1], squeezing[2-6], quantum interference[7-9], and quantum mechanical nonlocality[10-16]. In many *nonlinear optical problems*, for example in down-conversion and four-wave mixing, the photons are generated in pairs. The strong correlation between the photons in a pair may lead to such nonclassicalities. In this review we discuss these nonclassical effects and their experimental demonstrations in nonlinear optical processes.

We begin by describing the nonlinear optical processes of interest, namely three-wave mixing interaction of frequency down-conversion and four-wave mixing. All the known nonclassical features of the electromagnetic field, such as quantum interference, antibunching, nonclassical statistics, squeezing and nonlocality, are taken up one by one. The properties of the corresponding nonclassical state are reviewed in terms of the intensity–intensity correlation function, which is proportional to the joint probability of two-photon detection. (Here intensity is expressed in units of photon number density.) The methods of generation of the nonclassical states are also indicated.

Optical nonlinearities

When the electrons in matter are strongly excited by a high intensity light beam, 'overtones' of light are created. For a nonlinear material subject to an intense electric field E, the susceptibility χ is field-dependent and can be written as a power expansion in E. The induced electric polarization **P** of the medium is then given by

$$\mathbf{P}_i = \varepsilon_0 [\chi^{(1)}_{ij} E_j(\omega) + \chi^{(2)}_{ijk} E_j E_k + \chi^{(3)}_{ijkl} E_j E_k E_l + \ldots], \qquad (1)$$

where ε_0 is the dielectric permittivity of vacuum and $\chi^{(n)}$ is the nth-order susceptibility tensor (of rank n+1) characteristic of the medium. The power series expansion (1) of the polarization is essentially a perturbative approach to nonlinear optics. It is valid in the regime of weak nonlinearities only, when the electronic structure of the medium is not much perturbed by the applied field. This approach cannot be used in the regime of strong nonlinearity, which arises when field frequencies are very close to a narrow resonance and saturation of optical transitions occurs in the medium. In this regime, the electronic structure of the medium is perturbed strongly by the applied field.

For very weak fields, it is often sufficient to retain only the first term on the right-hand side of equation (1), which denotes the usual linear response of the medium to the applied electric field. Nonlinear effects of self-focusing, self-trapping and self-induced transparency are observed when a single beam of light propagates in a nonlinear medium. Most other nonlinear phenomena require the simultaneous presence of a number of coherent beams in the medium. The number of beams taking part in a nonlinear mixing interaction in a medium is the number of beams entering the medium plus the number of beams generated by the medium itself. The $\chi^{(2)}$ processes are *three-wave mixing* interactions of the type

$$\omega_1 + \omega_2 \rightleftharpoons \omega_3, \qquad (2)$$

where ω's are the frequencies. For the degenerate case of $\omega_1 = \omega_2 = \omega$ (say), this leads to the second-harmonic generation, $\omega + \omega \rightarrow 2\omega$. The reverse of second-harmonic generation is called the degenerate parametric *down-conversion*, in which one beam of frequency 2ω splits into two beams of frequency ω each. The second- and all even-order susceptibilities are identically zero in the bulk of a centrosymmetric medium. The terms involving the third-order susceptibility $\chi^{(3)}$ correspond to *four-wave mixing* (FWM) interactions of the type

$$\omega_3 + \omega_4 \rightleftharpoons \omega_1 + \omega_2. \quad (3)$$

For a particular order interaction to take place with appreciable probability, the phase-matching conditions (energy and momentum conservation laws) are to be satisfied. For three-wave mixing this can be achieved in a uniaxial non-centrosymmetric crystal exhibiting birefringence. For a negative birefringent crystal with the index of refraction for the ordinary wave (n_0) larger than that for the extraordinary ray (n_e) at a particular frequency, phase-matching for degenerate down-conversion may be achieved by choosing a direction of propagation Θ of the pump with the optic axis of the crystal such that n_e (Θ) (at 2ω) = n_0 (at ω).

It is now understood how different types of nonclassical states can be generated using a variety of nonlinear interactions both in resonant and in nonresonant systems. Under the assumption of negligible depletion of the input pump fields, so that pump beams can be treated classically, all nonlinear interactions generating two output beams (such as down-conversion and four-wave mixing) can be written in the parametric approximation as an effective bilinear interaction of the following second-quantized form

$$H \approx G a_1^+ a_2^+ + H.c, \quad (4)$$

where the coupling constant G is related to the nonlinear susceptibility for the process under consideration, and a_1^+, a_2^+ (a_1, a_2) are the creation (annihilation) operators corresponding to the two output modes. The Hamiltonian H creates and destroys photons in pairs. The output fields are taken to be initially in the vacuum state $|0, 0\rangle$, and the state at time t will then be a *two-photon state* $|\mathrm{TP}\rangle$ given by

$$|\mathrm{TP}\rangle = \exp(-iHt/\hbar)\,|0, 0\rangle. \quad (5)$$

Quantum interference

The modification of intensity obtained by the superposition of two or more beams of light is what is commonly known as optical interference. Since intensity is second-order in field amplitude, we will refer to the conventional interference as second-order interference. It is normally assumed that the observation of interference effects requires the superposition of *mutually coherent* light beams, i.e. beams which are derived from the division of a single beam so that the phase-difference between them remains constant. However, from a *classical* point of view, the superposition of two waves, whether mutually coherent or not, always leads to interference. For time intervals short compared with the reciprocal frequency spread (coherence time) of the two waves, both waves may be regarded as sinusoidal oscillations, and apart from experimental difficulties, interference fringes should always be observable in principle with two *mutually incoherent* light beams. But in the case of a *quantum* description of the optical field, the state of the field has to be specified, because in quantum mechanics only the expectation value (corresponding to the classical ensemble average) of the light intensity in a state is a meaningful quantity. Thus when the two beams are statistically independent, the interference term disappears as a result of the averaging over a large number of independent trials. Therefore the two beams must have well-defined phases if interference fringes are to be observable in the experiment.

In an interferometer a light beam is split into two components and the two derived beams are recombined after a path-difference is introduced in the distance traveled by them. In his book *The Principles of Quantum Mechanics*, Dirac[17] considers the case of a single photon passing through the interferometer and points out that we should think of this single photon as being associated with both beams simultaneously. If we tried to determine with which beam the photon is associated, during the course of our measurement we would force it into one or the other of the beams and no interference would occur. It is impossible for a photon in one of the beams to interfere with a photon in the other beam, because if it did, 'sometimes these two photons would have to annihilate one another and other times they would have to produce four photons. This would contradict the conservation of energy'. Following this Dirac makes his famous statement: 'Each photon interferes only with itself. Interference between two different photons never occurs.'

Thus the quantum mechanical interpretation of interference is that the photons do not interfere with each other in a conventional interferometer – it is the interference of the probability amplitudes (rather than the probabilities themselves) associated with the two possible indistinguishable paths of each photon that gives rise to fringes. The probability amplitude plays the role of wave amplitude in classical wave theory.

With the development of lasers with long coherence time, it became possible to observe interference effects with light beams from two separate *independent* sources. Many of the early experiments[18] studied the conventional *second-order* interference, which is extremely phase-sensitive, and special techniques (viz. measurement of instantaneous cross-correlation of two photomultiplier outputs) had to be used to reveal the interference pattern, which was not stationary unlike that obtained from coherent beams. *Fourth-order* effects in interference were not noticed until Hanbury Brown and Twiss[19] discovered intensity correlations. In the past few years, there have been several discussions[7-9] focusing on the nonclassical features in interference with independent sources, which show up readily in intensity correlations that are fourth-order in field amplitudes. It has been proved that fourth-order interference is present for both independent and correlated fields, even though second-order interference may not exist. Fourth-order interference is not phase-sensitive and may be easier to observe than second-order. At present, known experimental techniques for producing various quantum states of the field – for example, the resonance fluorescence from a single atom and the parametric down-conversion process – have led to some interesting fourth-order interference experiments with two photons. Fourth-order interference between two *classical* fields with random phases has a maximum relative modulation of 50%, whereas quantum fields may interfere to generate a relative modulation up to 100%.

Let us consider two polarized, approximately plane quasi-monochromatic electromagnetic waves emerging from two points A and B described by complex scalar amplitudes $E_A(t)$ and $E_B(t)$, which are superposed at the receiving plane. Let x_1, x_2 be the positions of the two detectors at the interference plane. If E_A and E_B are random and uncorrelated, the ensemble averages $\langle I(x_1, t)\rangle$ and $\langle I(x_2, t)\rangle$ exhibit no second-order interference. If we evaluate the intensity cross-correlation function $\langle I(x_1, t)\, I(x_2, t)\rangle$ under the same assumption that the two light beams are independent and the phases of E_A, E_B are random, then

$$\langle I(x_1, t)\, I(x_2, t)\rangle = \langle (I_A + I_B)^2\rangle \times [\,1 + \sigma \cos\{2\pi(x_1 - x_2)/L\}], \quad (6)$$

with $L = \lambda D/s = \lambda/\Theta$, where λ is the wavelength, D the distance from the source to the interference plane, s the separation between the sources and Θ the small angle of inclination between the two light paths from A and B. Equation (6) represents a form of interference, involving correlation function of fourth-order, the periodicity ('fringe'-spacing) of the interference pattern being L. The relative modulation amplitude or 'visibility' σ of the interference pattern is given by

$$\sigma = \frac{2\langle I_A\rangle\langle I_B\rangle}{\langle (I_A + I_B)^2\rangle} = \frac{2\langle I_A\rangle\langle I_B\rangle}{(\langle I_A\rangle + \langle I_B\rangle)^2 + \langle (\Delta I_A)^2\rangle + \langle (\Delta I_B)^2\rangle}. \quad (7)$$

Here $\langle (\Delta I)^2\rangle \equiv \langle I^2\rangle - \langle x\rangle^2$ gives the fluctuation in I. From equation (7) we see that the intensity cross-correlation is smallest when $|x_1 - x_2| = (n + ½)L$, $n = 1, 2, \ldots$, but it can never vanish. The visibility σ in the classical case has a maximum possible value of ½, when $\langle I_A\rangle = \langle I_B\rangle$ in the absence of any fluctuations of I_A and I_B, i.e.

$$\sigma \leq ½. \quad (8)$$

Let us now consider a specific example of a quantum mechanical source for two-photon interference, a photon-pair created by the nonlinear process of spontaneous parametric frequency down-conversion. In this process photons in the pump laser beam spontaneously 'split' into pairs of lower-frequency signal and idler photons that emerge from the nonlinear medium within a cone around the pump beam axis. As mentioned earlier, for an interaction to take place with appreciable probability, the phase-matching conditions (energy and momentum conservation laws) are to be satisfied. If the two down-converted signal and idler beams are recombined at some distant point from which the pump is excluded, we may take the resulting two-photon state to be a linear superposition state. In that state the single-photon detection probability $P_1(x, t)$ does not exhibit interference fringes and this simply reflects the absence of a phase relation between the signal and idler waves.

The quantum features of the electromagnetic field are exhibited through the intensity–intensity correlation function. The joint probability of detecting one photon at x_1 and another at x_2 in the interference plane is given by the fourth-order correlation function (normally-ordered), and when signal and idler photons are degenerate and similarly polarized, it comes out to be of the form[8]

$$P_2(x_1, t_1; x_2, t_2) \equiv \langle : I(x_1, t_1)\; I(x_2, t_2) : \rangle \approx [\,1 + \cos\{\,2\pi(x_1 - x_2)/L\,\}\,], \quad (9)$$

where L is the spacing of the interference fringes as before. There is a cosine modulation of P_2 (or the joint probability of two-photon detection) with the separation $(x_1 - x_2)$, with periodicity L. The joint probability vanishes when $|x_1 - x_2|$ is an odd integral multiple of half fringe-spacing, and the relative modulation amplitude or 'visibility' σ of the fringe pattern obtained from equation (9) is 100%, unlike the classical situation described by equation (8) where $\sigma \leq 50\%$.

The first observation of this nonclassical effect was reported[9] in 1987 in an interference experiment involving the down-converted photon pairs. The results supported the quantum mechanical theory, violating the classical inequality (8) with 92% confidence level.

Photon antibunching

When light falls on a photodetector, there is a probability $P_1(t)$ that a photoelectron is emitted at time t within a short interval δt, and there is an intensity correlation or a joint probability $P_2(t, t+\tau)$ that one photoelectron is emitted at time t within δt and another one at time $t+\tau$ within δt. When the light field is describable classically, it follows from the laws of classical probability that

$$P_2(t, t) \geq P_2(t, t+\tau). \tag{10}$$

If we plot the joint probability $P_2(t, t+\tau)$ of detecting two photoelectric pulses separated by time τ against τ, the resulting graph can fall below its initial value, but can never rise above its initial value. This fall of $P_2(t, t+\tau)$, or the tendency of photoelectric pulses to *bunch in time*, has been observed in many experiments since the pioneering work of Hanbury Brown and Twiss[19], and this phenomenon is understandable in terms of fluctuating electromagnetic waves, without field quantization. Thus for classical fields, the joint probability is maximum at $\tau = 0$, and it either decreases as τ increases (e.g. thermal light), or is constant for all τ (e.g. coherent light).

Antibunching[20] is the opposite of bunching, and describes a situation in which fewer photons appear close together than further apart, i.e. the joint probability at $\tau = 0$ is smaller than that for larger τ. We define the normalized intensity correlation function $g_2(\tau)$ as

$$g_2(\tau) \equiv \langle T : \Delta I(t+\tau)\, \Delta I(t) : \rangle / \langle I(t) \rangle \langle I(t+\tau) \rangle,$$

or,

$$g_2(\tau) + 1 \equiv \langle T : I(t)\, I(t+\tau) : \rangle / \langle I(t) \rangle \langle I(t+\tau) \rangle, \tag{11}$$

which is independent of t for a stationary field. Here T stands for time-ordering and the colons : : for normal ordering of the operators, i.e. $\langle T : I(t)\, I(t+\tau) : \rangle = \langle a^+(t)\, a^+(t+\tau)\, a(t+\tau)\, a(t) \rangle$. For thermal fields, $g_2(0) = 1$, and $\lim_{\tau \to \infty} g_2(\tau) = 0$; and for a coherent field, $g_2(\tau) = 0$, for all $\tau \geq 0$. Antibunching occurs when

$$g_2(0) < g_2(\tau). \tag{12}$$

This condition violates the Schwarz inequality for classical fields, and thus antibunching is a quantum phenomenon without classical description. Historically, antibunching was first observed in the process of resonance fluorescence from a single atom, when coherently excited close to resonance by a laser beam[21]. It was later observed by the use of a detection-triggered optical shutter in parametrically down-converted light[22], and more recently, in pulsed squeezed coherent light generated via degenerate parametric amplification[23].

The quantum character of the field can also be exhibited by photon counting measurements, rather than by measurements of time intervals between detected photons. But there is a distinction between these two nonclassical effects as shown below.

Nonclassical statistics

When completely coherent light falls on a photoelectric detector, the number of photoelectric counts n (proportional to the light intensity I) registered in some finite time interval obeys Poisson statistics for which the variance $\langle (\Delta n)^2 \rangle$ of n equals the mean number $\langle n \rangle$. For classical waves, in general,

$$\langle (\Delta n)^2 \rangle > \langle n \rangle, \tag{13}$$

as a consequence of intensity fluctuations. However, there exist quantum states of the electromagnetic field for which the photon statistics is sub-Poissonian[24], i.e.

$$\langle (\Delta n)^2 \rangle - \langle n \rangle < 0,$$

or,

$$\langle n^{(2)} \rangle - \langle n \rangle^2 < 0, \tag{14a}$$

where $\langle n^{(2)} \rangle \equiv \langle n(n-1) \rangle$ is the second factorial moment of the photon number. These states have no classical description, because

$$g_2(0) < 0. \tag{14b}$$

Under conditions of $g_2(\tau) < 0$, photon antibunching [condition (12)] implies a sub-Poissonian distribution [condition (14b)], but in general, one may occur without the other. In the atomic beam experiments[21] demonstrating antibunching, the photon counts were not sub-Poissonian as a result of fluctuations in the number of atoms. On the other hand, for a single-mode quantum field in a Fock state[1], the photon statistics are clearly sub-Poissonian, but the field would show no antibunching as $g_2(\tau)$ is negative but independent of τ. Sub-Poissonian photon statistics was experimentally observed[25] in resonance fluorescence from a single atom.

Condition (14a) for nonclassical photon statistics of a single-mode radiation can be generalized for the case of a two-mode radiation in the following way

$$\langle n_1^{(2)} \rangle + \langle n_2^{(2)} \rangle - 2 \langle n_1 n_2 \rangle < 0. \tag{15}$$

In case of two-photon devices, such as parametric amplifiers and four-wave mixers producing nonclassical states, each mode by itself may have a classical distribution obeying (13), but the correlation $\langle n_1 n_2 \rangle$ between the two modes plays a key role in establishing the nonclassicality condition (15)[26].

We discuss some other nonclassical features of photon statistics in the context of squeezing in the following section.

Squeezing

The coherent state[27] of radiation field is the closest counterpart to a classical electromagnetic field and is defined as that whose uncertainty product $\Delta E \Delta B$ for the electric and magnetic fields is minimum for all time when subject to the simple harmonic potential characteristic of the field. The corresponding wave-packet 'coheres' (does not spread) in time. The coherent state $|\alpha\rangle$ is the right eigenstate of the annihilation operator a

$$a\,|\alpha\rangle = \alpha\,|\alpha\rangle, \tag{16}$$

where α is the complex amplitude. A coherent state may be generated by applying a unitary displacement operator $D(\alpha)$ to the ground state $|0\rangle$ of a simple harmonic oscillator, i.e.

$$|\alpha\rangle = D(\alpha)\,|0\rangle, \tag{17a}$$

where

$$D(\alpha) \equiv \exp(\alpha a^{+} - \alpha^{*} a). \tag{17b}$$

For a coherent state $|\alpha\rangle$, $g_2(\tau) = 0$, for all $\tau \geq 0$ in equation (11).

An ideal laser operating in a pure coherent state would possess quantum noise due to photon-number fluctuations (shot-noise). The electric field operator associated with a mode of angular frequency ω at a given position is

$$E(t) = E_0 [X_1 \cos(\omega t) + X_2 \sin(\omega t)], \tag{18}$$

where E_0 is a constant, and the quadrature field operators,

$$X_1 \equiv (a^{+} + a), \tag{19a}$$

$$X_2 \equiv i(a^{+} - a). \tag{19b}$$

are analogous to the position and momentum operators of a simple harmonic oscillator with $[X_1, X_2] = 2i$. The fluctuations in X_1, X_2 obey the uncertainty relation

$$\langle(\Delta X_1)^2\rangle \langle(\Delta X_2)^2\rangle \geq 1. \tag{20}$$

The equality sign holds for the minimum uncertainty states (e.g. the coherent state), for which

$$\langle(\Delta X_1)^2\rangle = \langle(\Delta X_2)^2\rangle = 1. \tag{21}$$

Squeezed states[2] of the electromagnetic field are a unique set of quantum states (which may or may not be minimum uncertainty states) with less fluctuations in one (ith) quadrature phase than a coherent state at the expense of increased fluctuations in the other quadrature phase, i.e.

$$\langle(\Delta X_i)^2\rangle < 1, \quad i = 1 \text{ or } 2. \tag{22}$$

The component $X_1 = (a^{+} + a)$ will be squeezed if the inequality (22) holds with $i = 1$, i.e. if $\langle\Delta(a^{+} + a)^2\rangle < 1$.

A single-mode squeezed coherent state $|s;\alpha\rangle$ is obtained by operating a unitary squeeze operator $S(s)$ on the coherent state $|\alpha\rangle$, i.e.

$$|s;\alpha\rangle = S(s)\,|\alpha\rangle = S(s)\,D(\alpha)\,|0\rangle, \tag{23a}$$

where

$$S(s) \equiv \exp(s a^{+2} - s^{*} a^{2}), \tag{23b}$$

s being a complex parameter, and $D(\alpha)$ is given by equation (17b). The squeeze operator creates and destroys photons in pairs. If the order of the two operators $S(s)$ and $D(\alpha)$ in equation (23a) is reversed, a coherent squeezed state $|\alpha; s\rangle$ is obtained.

A phase-sensitive nonlinear interaction in a medium is required to generate squeezed states. A parametric amplifier can produce squeezing in either one mode or two modes of radiation. For a two-mode squeezed state, the generalization of $S(s)$ and $D(\alpha)$ is straightforward

$$S(s) \rightarrow \exp(s a_1^{+} a_2^{+} - s^{*} a_1 a_2), \tag{24}$$

$$D(\alpha) \rightarrow \exp(\alpha_1 a_1^{+} + \alpha_2 a_2^{+} - \alpha_1^{*} a_1 - \alpha_2^{*} a_2), \tag{25}$$

where the subscripts 1, 2 refer to the two modes. Two-mode squeezed states can be generated by applying these two operators to the two-mode vacuum $|0, 0\rangle$.

The first experiment[3] to successfully generate squeezed states (two-mode) employed the process of degenerate four-wave mixing. In this technique, the nonlinear medium essentially takes some photons from two strong pump waves and feeds them into two weaker conjugate beams. When these two correlated signal beams are combined by some optical means, viz. at a beam-splitter, the resulting light exhibits the amplified and reduced quadrature fluctuations. Slusher *et al.*[3] used an atomic sodium beam pumped near the D_2 resonance in a cavity to generate the squeezed light with noise reduced by 7–10% below the vacuum limit. Wu *et al.*[4] reported a 63% reduction in the noise by employing a parametric amplifier with a lithium niobate ($LiNbO_3$) crystal as the $\chi^{(2)}$ nonlinear medium. In an optical cavity the three-wave mixing process of parametric down-conversion was used to generate pairs of subharmonic photons of half the pump frequency. The pump amplifies a coherent subharmonic field when the two are in step, and it deamplifies when they are 90° out of step. The associated fluctuations at the subharmonic are amplified and deamplified in the same way.

Recently, we have proposed a theoretical model for a two-photon squeezed laser[5], in which the ordinary gain medium inside the laser cavity is replaced by a suitable active nonlinear medium. An intense pump laser beam causes two-photon excitations in the medium and generates two radiation fields due to four-wave mixing (FWM). The generated photons can get reabsorbed by a two-photon absorption (TPA) process. A strong competition among FWM, TPA and linear cavity losses leads to lasing action above a certain threshold determined by the nonlinear mixing and the linear damping constants. This is an example of a laser where amplification is obtained without population inversion. The two photons generated inside the cavity in this process are strongly coupled, as they are either produced simultaneously in FWM or absorbed simultaneously in TPA process, and the phase correlations between them leads to a narrower than usual linewidth of the two-photon laser far above threshold. The spectrum of fluctuations in the intensity difference between the two output modes shows evidence of strong squeezing, as the photon-number fluctuations of the two modes try to balance each other.

Four-wave mixing systems offer wider applications in noise reduction problems than their three-wave mixing counterparts. In the case with no absorptions and detuning, the quantum noise reduction below the vacuum (complete darkness) noise level, or squeezing[2], in the intensity *difference* of the two output beams is the same for the cases of intracavity three-wave and four-wave mixing. However, in the case of FWM, because *two* pump photons are absorbed to generate the output pair, as opposed to *one* for the down-conversion, it is possible to produce noise reduction in the intensity *sum* of the two outputs, i.e. generate output beams with *individually* reduced intensity fluctuations[5].

The probability distribution $p(n \mid \alpha, s)$ for a single-mode squeezed state to contain n photons shows nonclassical oscillations with n (ref. 28). This is additional to the expected even-odd modulation of probability distribution for the squeezed vacuum state ($\alpha = 0$), given by $p(n \mid \alpha = 0, s) \equiv 0$ for odd n, as the squeeze operator [equation (23b)] creates and destroys photons in pairs. The other nonclassical oscillations remain for $\alpha \neq 0$, and are interpreted in terms of interference in the phase space of the harmonic oscillator which models the single-mode optical field [equation (18)].

The nonclassical photon-number oscillations in a single-mode squeezed state were detected in an experiment using pulsed squeezed light[29]. A $\chi^{(2)}$ nonlinear medium was pumped by a series of nonoverlapping pulses, produced by frequency-doubling the light from a mode-locked Q-switched laser. The output of the nonlinear medium is a series of nonoverlapping wave packets, each of which is excited into a single-mode squeezed vacuum state. If the successive squeezed states are identical, an ideal photodetector counting the number of photons in successive pulses would build up the photon-number probability distribution for a single-mode squeezed state.

Nonlocality

As is well known, the wave-function description of quantum mechanics does not provide the detailed space-time behaviour of a system between the initial preparation and the interaction with the measurement apparatus. This aspect of the quantum measurement process was first discussed by Einstein, Podolsky and Rosen $(E-P-R)$[10] who concluded that quantum mechanics fails to give an adequate description of physical reality and that in quantum mechanics the motion of a particle must be described in terms of probabilities only because some 'hidden parameters' that determine the motion have not yet been specified.

Quantum theory makes certain predictions that are incompatible with any realistic, local theory. Realism assumes existence of an objective reality independent of whether someone observes it or not. Locality assumes that forces or information can only travel between bodies at speeds less than or equal to that of light. Using essentially the same postulates as those of $E-P-R$, Bell and several other workers formulated some inequalities obeyed by every realistic, local theory and violated by quantum mechanics. These provide a way to test experimentally the predictions of the local deterministic hidden variable theories against the predictions of quantum mechanics[11]. The possibility of violation of Bell inequalities in correlated states produced by nonlinear processes, such as multimode parametric amplifiers and four-wave mixing, have been studied[15]. The nonlocal character of the generated quantum fields is considered by superposing them with the help of a beam-splitter and then performing a polarization correlation experiment.

Let a_1 and a_2 be two correlated modes coming out of a nonlinear material. These are made to fall from opposite sides on a beam-splitter. a_4 and a_3 are the mixed beams which arrive at the detectors placed at points x_1 and x_2 with two polarizers set at variable angles Θ_1 and Θ_2 in front of them respectively. The Bell inequality in this case has the following well-known form[11]:

$$S \equiv P(\Theta_1, \Theta_2) - P(\Theta_1, \Theta_2') + P(\Theta_1', \Theta_2) + P(\Theta_1', \Theta_2') - P(\Theta_1', -) - P(-, \Theta_2) \leq 0. \quad (26)$$

Here $P(\Theta_1, \Theta_2)$ is the joint probability density of detecting two photons for polarizer settings of Θ_1 and Θ_2 measured by the coincidence counter. $P(\Theta_1, -)$ stands for the probability when the second polarizer is removed. Now the joint probability density of detection of two photons is given as

$$P(\Theta_1, \Theta_2) = K\langle a_4^+ a_3^+ a_3 a_4 \rangle, \tag{27}$$

where K is a constant characterizing the detectors. One may write the fields at the detectors as

$$a_4(x_1, \Theta_1) = X_1 a_1 + X_2 a_2, \tag{28a}$$

$$a_3(x_2, \Theta_2) = Y_1 a_1 + Y_2 a_2, \tag{28b}$$

where

$$|X_1|^2 + |X_2|^2 = |Y_1|^2 + |Y_2|^2 = 1. \tag{28c}$$

The probability density when the second polarizer is removed is calculated using unitarity

$$P(\Theta_1, -) = P(\Theta_1, \Theta_2) + P(\Theta_1, \Theta_2 + \pi/2). \tag{29}$$

For comparison of the different probabilities, all of them should be scaled by the joint probability density when both polarizers are removed

$$P(-, -) = P(\Theta_1, -) + P(\Theta_1 + \pi/2, -). \tag{30}$$

For a symmetric 50/50 beam-splitter and with the choice of angles $\Theta_1 = \pi/8$, $\Theta_1' = 3\pi/8$, $\Theta_2 = \pi/4$, $\Theta_2' = 0$, the Bell inequality [equation (26)] is violated[16] whenever

$$\frac{\langle n_1 (n_1 - 1)\rangle + \langle n_2 (n_2 - 1)\rangle}{\langle n_1 n_2 \rangle} < 0.48, \tag{31}$$

where $n_1 \equiv a_1^+ a_1$ and $n_2 \equiv a_2^+ a_2$ are the photon-number operators for the two beams incident on the beam-splitter. For optimum choice of angles, the right-hand-side of equation (31) can be made equal to 0.5.

For the parametric down-conversion process, the photon statistics is nearly Poissonian with mean $\langle n \rangle$

$$\langle n_1 (n_1 - 1)\rangle = \langle n_2 (n_2 - 1)\rangle = \langle n \rangle^2, \tag{32a}$$

$$\langle n_1 n_2 \rangle = \langle n \rangle + \langle n \rangle^2. \tag{32b}$$

Hence from equation (31) we get

$$\frac{\langle n \rangle^2}{\langle n \rangle + \langle n \rangle^2} < 0.24, \tag{33a}$$

or

$$\langle n \rangle < 0.32. \tag{33b}$$

Violation of the Bell inequality in correlation measurements of mixed signal and idler photons produced in the process of parametric down-conversion has been experimentally observed[14]. Similar violation is predicted[16] in the output of the two-photon squeezed laser described above[6].

Summary

Two-photon processes are extremely interesting in quantum optics, because of their potential in the generation of nonclassical states of light, such as number states, or squeezed states, or in violation of classical inequalities in interference experiments. These nonclassical properties are readily revealed in the intensity correlation which is proportional to the probability of two photons being detected at a time interval. These quantum effects have been observed experimentally in dissipative nonlinear systems producing correlated two-photon states.

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RESEARCH ARTICLES

Post-Gondwana tectonics of the Indian Peninsula

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The concept of plate tectonics implies that the plate boundaries are sites of tectonic activity and that regions within the plate are stable. Exceptions to this rule are known from the Indian Ocean where the oceanic crust is undergoing folding and fracturing. Examples of intraplate deformation within the continental crust are rare. The uplift of landmass in the region of 13°N of the Indian peninsula is one such unusual occurrence.

THE peninsular India, which is a Precambrian shield area, is considered to be a stable landmass. Hence it is generally believed that no major earth movements are possible in this region. This view gains strength from the fact that the Peninsula is an intraplate region bound by passive margins on all sides except in the north. Although this picture is generally true, there is evidence to indicate that certain regions have been active over a considerable period and have left their imprints on the landforms[1,2]. Most of the major geomorphic features owe their genesis to the post-Gondwanaland plate tectonic regime.

The following picture emerges when the history of the Peninsula is traced over the past 150 Ma. It is then that the East Coast emerged due to its separation from Antarctica and Australia. The West Coast came into being much later, due to separation of India from Madagascar. This event can be placed at 93 Ma if the columnar rhyodacitic volcanics of St. Mary islands off Karnataka coast[3,4] represent the rift stage volcanism.

This was followed by the hotspot volcanism which gave rise to Deccan Traps (67 Ma). Due to northerly

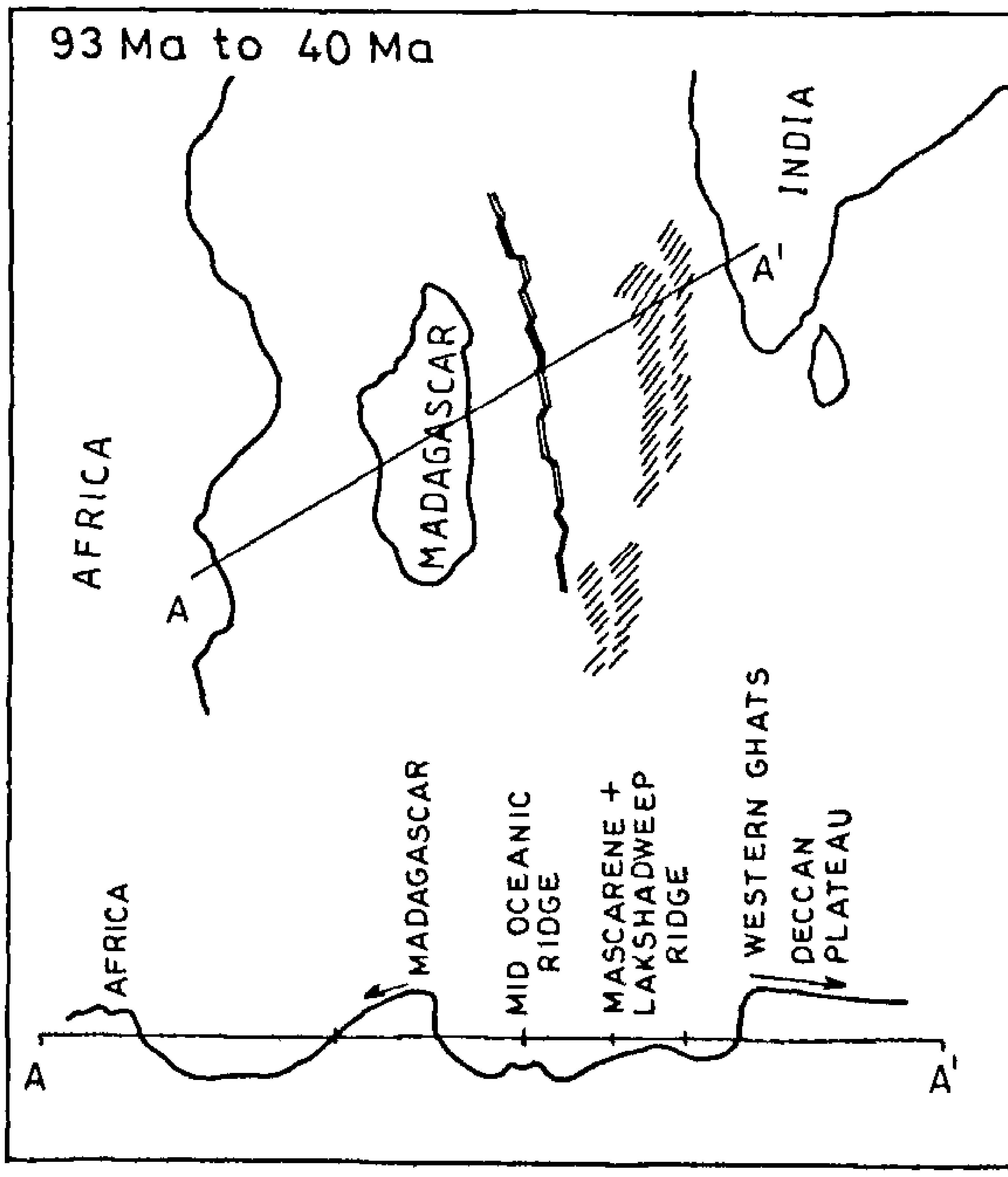

Figure 1. Separation of Madagascar from India along an MOR close to Madagascar (93 Ma). Tilting of blocks and formation of scarps along the Western margin of India (Western Ghats) and the eastern margin of Madagascar. Initiation of easterly drainage in India (indicated by an arrow on the Deccan plateau). Outpouring of large quantity of basaltic lava resulting in Deccan Traps (67 Ma) and a marginal plateau in the Arabian Sea (67 Ma to 40 Ma)