Norbert Wiener's place in the history of science and philosophy*

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The mathematical tradition in philosophy

Following his early mentors Bertrand Russell and Alfred North Whitehead, Norbert Wiener began research as a mathematical philosopher, but then, unlike his mentors, he went on to become a great mathematician. All three men were part of what may be called the mathematical tradition in philosophy, i.e. they subscribed to the maxim that:

> . . the utmost abstractions are the true weapons with which to control our thought of concrete facts [W, p. 41],

to use Whitehead's words. When Galileo spoke of 'God's second book, the book of Nature' as being 'written in mathematical characters', he was speaking of this very tradition. It was initiated by Pythagoras and to it belong many great philosophers from Plato to Whitehead, and of course all practitioners of mathematical sciences from Euclid to Einstein.

A very fine example of the proximity of the abstract or ideal to the practical is provided by the very early Pythagorean discovery of the relatedness of musical sonority and arithmetical simplicity. What this entails becomes clear from the following words of Wiener:

> Ideally, a simple harmonic motion is something that extends unaltered in time from the remote past to the remote future In a certain sense it exists *sub specie aeternitatis*. To start and to stop a note involves an alteration of its frequency composition which may be small, but which is very real A note lasting only over a finite time is to be analysed as a band of simple harmonic motions, no one of which can be taken as the only simple harmonic motion present. [56g, p. 106]

But all physically realizable notes do have only a finite duration. This also holds for the 'pure tones' produced by a tuning fork, e.g. the pure note, middle C. Nevertheless, the most efficient way of dealing with a pure tone in acoustical engineering, is to treat it as a simple harmonic motion (more accurately, as a simple harmonic wave of definite frequency or wavelength), i.e. as Wiener points out, to view it *sub specie aeternitatis*.

Wiener's place in this tradition

The place or niche where Wiener fits into this critical-practical tradition is indicated by the titles of the opening chapters of two of his very important books:

'Newtonian and Bergsonian Time' (Chapter I of *Cybernetics*)
'The idea of a contingent universe' (Preface to *The Human Use of Human Beings*).

To understand this, a good starting point is a 1914 paper [14d], written when Wiener was turning 20, in which he questions the pragmatic maxim of William James that the 'meaning' (or 'value') of a conception lies in its 'practical consequences'. Whether a consequence is practical or impractical will depend on the *purpose* or *goal* we have in mind. Hence pragmatism fails, Wiener argued, unless the concept of purpose is clearly analysed, and scientifically demarcated.

The concept of purpose hinges on our ability to tell past from future, i.e. it presupposes that time has a direction from past to future. The fundamental equations of Newtonian physics, however, do not change when t is replaced by $-t$, i.e. they are blind to the discrimination between past and future. Newton and his colleagues were not troubled by this, for to them purposes fell in the realm of the spirit, and in this domain, where Scripture is paramount, time is directed from past to future. But Wiener was influenced by the philosophy of Leibniz, whom he later described as the 'patron-saint of cybernetics' [61c, p. 12]. The Leibnizian thrust, as Wiener saw it, was to fuse the life of the spirit with the remaining aspects of life.

Some early evidence that inanimate nature is not averse to the unidirectional flow of time came with Immanuel Kant's outline of the formation of planets in 1755. However, the first major headway in the scientific recognition of the distinction between past and future came after the advent of the steam engine, when the studies of the French engineer–mathematician Sadi Carnot (c. 1840) led him to the proposition that *the efficiency of every steam engine is less than 1*. This proposition, on steam engine deficiency, was recognized by Lord Kelvin and others as carrying the germs of a new universal Law of Nature. This is the Second Law of Thermodynamics. A thermodynamic quantity, called

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entropy, was demarcated. It was found that *entropy of a thermally insulated system spontaneously increases or remains constant*. It so transpired that there are physical phenomena that are not blind to the distinction between past and future. Thereby this distinction was scientifically vindicated.

To return to the concept of purpose, unidirectional time, while necessary for its scientific demarcation, is not enough. Obviously, the nature of the *world's orderliness* or *causality* is also involved. As Whitehead has stated:

> ... there can be no living science unless there is a widespread instinctive conviction in the existence of an *Order of Things*, and in particular of an *Order of Nature* ... [W, p. 4].

But how tight is this order? If everything that is happening now and will happen tomorrow is predetermined by the events of yesterday, and these in turn, predetermined by the events occurring the day before, then the notion of purpose loses its meaning. And the collapse of the concept of purpose entails the collapse of the very concept of moral responsibility that sustains human society.

Clearly, so tight a conception of cosmic orderliness does not jibe with human experience, and is unrealistic. Science itself soon provided a loosening. The atomic theory of John Dalton, the great chemist, suggested to physicists that the heat of a body is just the manifestation of the sum of the mechanical energies of its molecules, its temperature being the mean mechanical energy per molecule. Next arose the question as to the laws governing these molecular motions, and how from them could be derived the macroscopic attributes of bodies, such as, for instance, the steady pressure exerted by a gas on the walls of its container.

This marked the beginning of a new trend in physics in which the mathematical concept of *probability* began to loom larger and larger. The main pioneers in the movement were James Clerk Maxwell, Ludwig Boltzmann and Josiah Willard Gibbs, the founders of *statistical mechanics*, and Albert Einstein, whose theory of the Brownian motion (1905) resulted in an independent statistical mechanical evaluation of Avogadro's constant by Jean Perrin, and revealed atomism as an integral part of the cosmic structure. We must also mention H. Lebesgue of France, whose purely mathematical theory of measure provided a firm support for probability theory and for its applications to statistical mechanics.

The philosophical significance of the work of Maxwell, Boltzmann and Gibbs was clearly seen by Charles Sanders Peirce, America's great philosopher, and fine scientist to boot. In 1883 he first advanced his doctrine of *Tychism*, named after Tyche, the Greek goddess of fortune or chance. Tychism is a weakening of strict determinism. Its central idea was lucidity stated by William James during a lecture in 1884:

> Actualities seem to float in a wider sea of possibilities, from out of which they are chosen [J, p. 151].
> Possibilities are in excess of actualities [J, p. 150].

James illustrated this to his audience by pointing out that there were at least two possible ways for him to walk home, (i) via Oxford Street or (ii) via Divinity Avenue. Both possibilities accord thoroughly with the laws of nature, but only one of these was going to become an actuality, and the harmony of the cosmos would not be affected by the free choice of one rather than the other.

Further developments in the thermodynamics of radiation, resulting in Planck's discovery of a new universal constant of Nature, to wit, *the elementary quantum of action h*, more or less clinched the tychistic conception of cosmic orderliness. Our quest for such orderliness cannot go beyond a certain threshold marked by the constant *h*. The exact formulation of this law is the celebrated Heisenberg Indeterminacy Principle.

The upshot of this great intellectual movement was a new conception of cosmic causality that Wiener referred to as 'the contingent universe' or the 'stochastic cosmos'. It was a crystallization of the doctrine of Peirce and James, in which the vague, and strictly speaking, nonsensical idea of chance is eliminated, and the mathematical concept of probability brought in. Its content is summed up in the following display:

The Stochastic Cosmos

1. The laws of Nature do not govern individual events; they govern the probabilities with which these events occur.
2. The orderliness of the world is incomplete: Possibilities are in access of actualities; There is scope for freedom.

Wiener's own comment on the stochastic organization or causality is worth quoting:

> My early work on probability theory, as exemplified in my studies of the Brownian motion, had convinced me that *a significant idea of organization cannot be obtained in a world in which everything is necessary and nothing is contingent*.
>
> I was forced to consider causality as something of which there can be either more or less rather than as something which is either there or absent [56g, pp. 322, 323]

The liberating effects on science of this concept of a stochastic cosmos were clearly discerned by Peirce as early as 1892, when he wrote:

> Variety is a fact which must be admitted, and the theory of chance merely consists in supposing that this diversification does not antedate all time [P, 6 65]

In a strictly deterministic cosmos, the understanding of variety would be beyond human reach, since the ultimate cause of variety would reside in events that occurred at the initial moment $t = 0$ or at $t = -\infty$,

depending on whether the cosmos had or had not a moment of creation. In the stochastic cosmos, on the other hand, we can scientifically investigate variety in a piecemeal way. Moreover, in such a cosmos it becomes possible to bridge many of the gaps between science and reality that the great French philosopher Henri Bergson thought were unbridgeable – the concept of purpose being just one among many. It is for this reason that Wiener spoke of *Bergsonian time*. This time has two features, absent in Newtonian time, to wit: (i) unidirectionness or anisotropy, (ii) innovativeness.

The stochasticity of the cosmos and the Bergsonian nature of time brings within the ambit of science the following concepts that the earlier sciences could deal with, but could not satisfactorily explain:

1. Memory; 2. Language and communication; 3. Controlled experiment; 4. Purpose; 5. Learning and growth; 6. Intelligence; 7. Disturbance, noise, disorder; 8. Recovery of message from a noisy signal; 9. Good, evil, wickedness.

These were then the topics towards which Wiener's research was propelled by the natural course of intellectual events, rather than by any deliberate design. But sustaining this research, and by far his greatest work, was his idealization of Einstein's theory of the Brownian motion in the early 1920s. Almost the entire theory of stochastic processes, as developed by Academician A. N. Kolmogorov about 10 years later in Russia, is based on this work of Wiener. Almost all of Wiener's own later work was molded by his work on the Brownian motion in one way or another.

Wiener's mathematical work

Here we cannot delve into Wiener's mathematical research. For our purposes, his greatest contribution may be summed up as follows.

After his work on the Brownian motion and on the theory of the potential, the subject called *harmonic analysis*, the origins of which go back to Pythagoras' discovery of the relationship between musical tones and wavelength, engaged Wiener's attention. This subject had received its systematic development in the 1790s when Jean Batiste Fourier extended it to cover all purely periodic signals over the interval $(-\infty, \infty)$. During the later 1920s and early 1930s Wiener showed that this Fourier analysis could be further generalized so as to cover the extremely erratic, non-periodic signals that emanate from random phenomena such as the Brownian motion, thermionic emissions, atmospheric noise, and also from erratic signals of human origin, such as brain-wave encephalograms. Briefly, Wiener showed that the natural noise, inevitable in a stochastic cosmos in which entropy reigns, reveals a certain beauty when subjected to generalized harmonic analysis.

The following two examples should elucidate Wiener's work:

Example 1. Consider the electro-encephalograms emitted by the human brain during the waking state, say. Early observers were dumbfounded by their extreme erraticity. But when subjected to generalized harmonic analysis, they reveal patterns, which tell a valuable story about the brain, worthy of serious physiological investigation.

Example 2. Consider sunlight. Each radiating solar atom emits a simple harmonic wave of definite frequency. What is observable, however, are *rays* of light, i.e. highly chaotic mixtures of *billions* of such individual monochromatic radiations. We say 'chaotic' (the technical term is '*incoherent*'), because the solar atoms fire, quite independently of one another. However, such a ray, of *white light* so-called, yields a beautiful generalized harmonic analysis, and Wiener was able to explicate mathematically the concepts of optical intensity, interference and polarization, and deduce many of the fine conclusions that physicists such as Lord Kelvin had drawn intuitively.

In 1928 Wiener wrote a paper entitled 'Coherency matrices and quantum theory' [28d]. This received scant attention until Sir Dennis Gabor rediscovered it in the 1950s. It provides the mathematical foundation for the theory of *coherent light*, the basis of the field of lasers, masers and holographs.

Antiaircraft fire control

A deeper insight into Wiener's way of thinking is gained by considering his solution of the problem of directing antiaircraft (AA) fire on fast incoming enemy bombers. Direct aiming will miss, since the bomber will move away during the time the shell takes to arrive in its vicinity. Consequently, to score a hit, the gun must be pointed at a future position of the bomber. This future position has therefore to be extrapolated from the observed path of the bomber in the past. This must be done before the shell is fired. Since the pilot is free to zigzag and take other innovative evasive action, the problem belongs to Bergsonian time. The US Air Force wanted the entire operation carried out in 30 seconds, of which 10 seconds were for the radar tracking of the plane as it approached the target, and 20 seconds for the computing of its predicted position, the aiming and firing of the gun. These constraints were dictated by the type of equipment available in 1940.

Let us break the 30 second time interval into two parts,

$$(-10, 0) \text{ and } (0, 20),$$

the first for the radar tracking and the second for the remaining operations. The first thing that Wiener did was to consider the problem *sub specie aeternitatis*. He replaced the first interval $(-10, 0)$ by

$$(-\infty, 0),$$

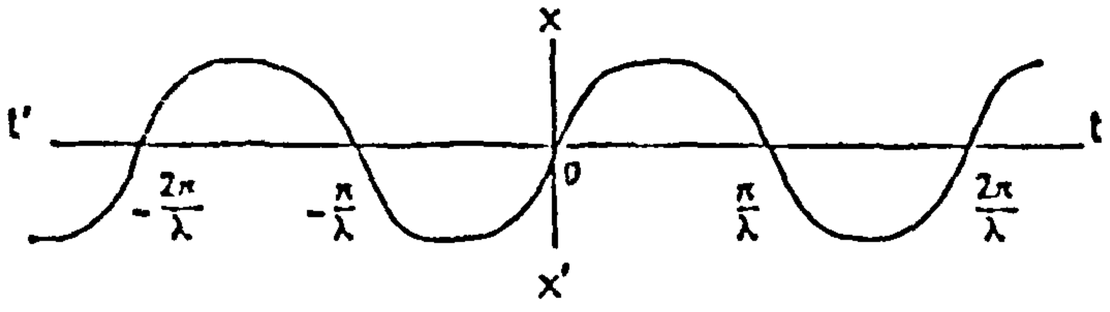

Figure 1. Pure tone, *sub specie aeternitatis*

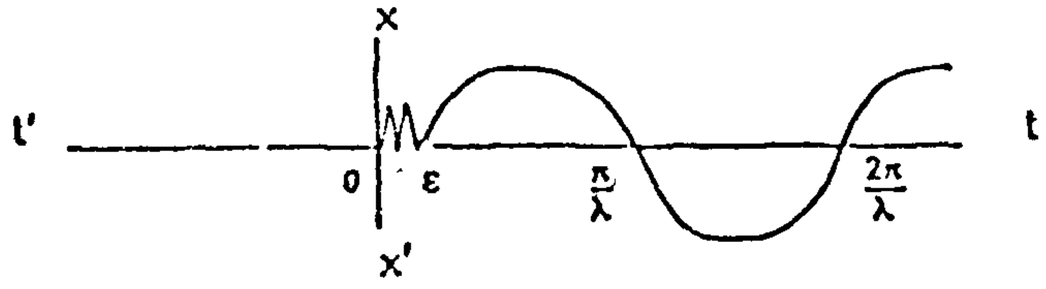

Figure 2. Pure tone, as produced by a tuning fork struck at $t = 0$

i.e. assumed that the plane had been flying since the dawn of the universe. Thus Wiener proceeded pretty much as an accoustician would in dealing with the sound produced by a tuning fork. This is clear from Figure 1 *a* wherein $t = 0$ is the moment the fork is struck. The high frequencies in the time-interval $(0, \varepsilon)$ die out so quickly that they do not register on the human consciousness, and what we hear is the eternal pure tone of Figure 1 *b*. Wiener hoped that the transients in his AA problem too might be of so short a duration as not to affect the apparatus used for prediction and for firing.

Wiener's hopes did not materialize. His situation was rather like that of the driver of an old lorry, who is told that a bomb is going to explode within a minute. He loses 20 seconds in furiously cranking the handle, but the ignited engine does not settle down within the remaining 40 seconds, and he cannot drive away. In Wiener's case the '*cranking errors*' or transients created by suddenly fixing the radar on the incoming airplane, by suddenly starting the machinery to move the gun's barrel, etc., did not die down within the allotted 20 seconds. Here are his own words:

> Our original theoretical development was based on *infinite runs for the determination of auto correlation coefficients as well as on infinite runs for prediction.* Of course, we have known from the beginning that these assumptions are not strictly true and we have found out that the practical consequences of their falsity are not always negligible. This is particularly true of the error in determining the autocorrelation coefficient. *The transients due to a sudden starting and a sudden stopping of our catalogued data may completely mask the true behavior of our statistical system for high frequencies.* [VII, p. 8, para 1 (June 1942)] (emphasis added)

Thus from an immediate military angle, Wiener's researches were a failure, but from a long-range viewpoint, his researches were of tremendous importance, a point not at all lost on the Department of Defense, as is clear from their final summary reports. Let us look into some of these deeper ramifications of this work of Wiener.

Ramifications of the antiaircraft fire control problem

A. A new chapter in the theory of stochastic processes was opened up, viz. that of statistical or stochastic prediction. It is nowadays called the *Prediction Theory of Stationary S.Ps.* Unknown to Wiener, the same theory had also been worked out by Kolmogorov in the USSR a few months before Wiener. It is therefore referred to as the Kolmogorov–Wiener theory.

B. In his war work Wiener went beyond Kolmogorov's work, by considering the noise mixed into the past data from which the prediction is to be made, and deriving optimum ways in which it can be mitigated or filtered out. Thereby he initiated a new branch of engineering that goes by the name of *The Filtration of Noise.*

The parts A and B of Wiener's work were first covered in the Chapters II and III of Wiener's book on Time Series [49g], which first appeared as a classified document in 1942. The two chapters have the headings:

II. The linear prediction of a single time series (p. 56)
III. The linear filter for a single time series (p. 81).

To read a little from the second:

> *Let $f(t)$ and $g(t)$ be two complex time series. Let $f(t)$ represent a* message and $g(t)$ a disturbance We wish to determine that linear operator which, when applied to $f(t) + g(t)$, will give us *the best approximation to $f(t + \alpha)$.* In order to do this, we need statistical information concerning $f(t)$ and $g(t)$, which will be given by their auto-correlation and cross-correlation coefficients. We shall suppose that sufficient auxiliary conditions are satisfied to permit the free shifting of the time origin of the series as justified by the lemmas of the last chapter Of course, the results of this chapter will include those of the last in the particular case in which $g(t)$ vanishes. [49g, p. 81]

As is clear from this paragraph, it is now the *disturbance* or *noise* $g(t)$ that is central. We can, as Wiener says, recover the noise-free theory of Chapter II by putting $g(t) = 0$.

C. *The clinical aspect.* In a lecture Wiener gave to medical doctors in 1960 [60a], he pointed out that the AA fire control problem was one in *clinical rather than natural science.* Indeed the elements of clinical science, viz. *diagnosis, prognosis* and *therapeutics* are easily discernible; we have:

> The radar tracking of the bomber, the lethal agent = the diagnosis.
>
> The statistical* estimation of its future position = the prognosis.
>
> The aiming and firing of the gun and shooting down the bomber = therapeutics.

*Statistical because the course is not fixed but subject to the pilot's decisions

As in medicine, the time-span between diagnosis and therapeutic implementation is crucial, although in most medical problems it is much more than 30 seconds. However, the cranking errors and transients, which are major impediments in the firing problem, are quite negligible in medicine, except perhaps in psychiatry.

D. Another field that benefited from Wiener's work on AA fire control was *medical or physiological cybernetics*. The enemy pilot of course tries to dodge ground fire by zigzagging and other evasive action. But with incoming flak, he has neither the time nor the ability to indulge in a contest against adversaries located somewhere on the ground. In short, the circumstances are such that the pilot becomes an appendage to the aircraft, and has little scope to exercise real initiative. To quote Wiener's words from a military memorandum:

> ... the 'randomness' or irregularity of an airplane's path is introduced by the pilot, that in attempting to force his dynamic craft *to execute a useful maneuver*, such as a straight-line flight, or a 180 degree turn, *the pilot behaves like a servo-mechanism*, attempting to overcome the intrinsic lag due to the dynamics of his plane as a physical system, in response to a stimulus which increases in intensity with the degree to which he has failed to accomplish his task A further factor of importance was that the pilot's kinaesthetic senses would normally lead him to expect, so that for precision flying, he must disassociate his kinaesthetic from his visual sense [VII, p 6 (1942)] (emphasis added)

The crucial assertion here that 'to execute a useful maneuver ... the pilot behaves like a servo-mechanism ... ' marked a very important advance in medical cybernetics. Prior to the 1940s medical cybernetics was essentially the study of equilibrium, or *homeostasis*, based on the work of Claude Bernard and Walter Canon. Wiener's work brought in the *dynamics*. How, for instance, is the act of picking up a pencil executed? The general answer is: in accomplishing its goals the animal nervous system follows the principles of servo mechanics, resting on negative feedbacks.

Wiener was quite interested in this field, especially on the light it shed on sensory and muscular-skeletal prostheses – i.e. the building of artificial limbs and organs for the maimed. He was especially involved in a project called the 'Boston arm', cf. Figure 2.

Figure 3. The patient can control both the Elbow and the Hand myoelectrically with one pair of muscles. [Photograph courtesy of Mr T. Wall Williams, Liberty Mutual Insurance Company, Boston, Massachusetts]

Inquiry versus contest

To grasp the further ramifications of Wiener's work we must understand the very crucial distinction that he drew between two human activities: (i) inquiry, and (ii) contest.

In an inquiry to fulfil our goal, we make a conjecture, then try it out by observation and experiment or ratiocination. If it passes, we build upon it. If it does not, we try to modify it, or perhaps discard it. On the other hand, in a contest, whether friendly as in a game of chess, or unfriendly as in litigation or warfare, we have an adversary whose goal is to frustrate our objective, just as it is our goal to frustrate his own. The moves we make are governed by the previous moves of our adversary and by our guessing what move or moves he may make in the future.

Wiener on noise and evil

The important question arises as to whether in human contests there is anything akin to the cranking errors and persistent noise that inevitably arises in engineering undertakings such as AA fire control. We owe to Wiener the important realization that the disturbing role played by evil in human conflicts and other purposive undertakings is vaguely analogous to that played by noise in natural processes. He wrote:

> The gap between Gibbs-Lebesgue way of thinking and Freud's intuitive but somewhat discursive method is too large Yet in their recognition of a fundamental element of chance in the texture of the universe itself, these men are close to one another and close to the tradition of St Augustine For this random element, this organic incompleteness, is one which without too violent a figure of speech we may consider evil, the negative evil which St. Augustine characterizes as incompleteness rather than the positive malicious evil of the Manichaean. [50], p. 11]

Unfortunately, there are inaccuracies in this statement, both as to Manichaeanism and as to St. Augustine's concept of evil. St. Augustine, basing himself on the words of St. Paul that 'Every creature of God is good' (I, Timothy 44), wrote:

> No nature, therefore, as far as it is nature is evil, but to each nature there is no evil except to be diminished in respect of the good [A, Ch. 17]

Thus, Wiener is right in affirming the negativity of Augustinian evil. But St. Augustine also accepted the Judeo-Christian doctrine of the *Fall of Man*, which Wiener bypasses. When the Fall is also taken into

account, human evil has to be characterized in more diabolical terms than as a mere privation, such as blindness, the loss of vision. Elephants do not sexually molest their infants, humans do – and this human perversion, far from being a deprivation such as blindness, is a new propensity of which the elephant has not a trace. The avaricious exploitation of conspecifics is a human propensity not found in other mammals.

The addition of the Fall of Man to Wiener's thought on evil, if anything, gives additional credence to the analogy he clearly saw between the lingering all pervasive noise in natural processes, and the disturbing presence of evil in human undertakings. Furthermore, one can transform Wiener's theory of phylogenetic learning into one of phylogenetic corruptibility, and thereby gain an insight as to how the Fall may have occurred, cf. [M, p. 321].

It is well worth noting that the doctrine of the Fall of Man is there in all the great world religions. It is certainly there in Hinduism, as the opening pages of A. K. Coomaraswamy's masterpiece *Hinduism and Buddhism* [C1, pp. 6–9] clearly show (see Appendix).

Although the Hindu and Christian Falls have marked differences, there are basic similarities. Christianity too has an atemporal side as is evident from the passage, 'Before Abraham was, I am' (John 9:58) and from the opening words of the St. John Gospel: In the beginning was the word and the word was with God and the word was God (John 1:1). As Coomaraswamy points out, 'in the beginning' (in Latin *in principio*) means 'In the first principle of things'. However, the Judeo-Christian emphasis on the Fall of Man, as a specific event within the pluralistic setting, has tended to discourage escapism and encourage socio-economic change.

Wiener's deviations from the thought of Thomas Hobbes, Adam Smith and John von Neumann

Wiener's rather unusual approach to the problem of evil, brought on some significant divergences between his positions on politics, economics and warfare and those of Thomas Hobbes, Adam Smith and John von Neumann. The last three thinkers recognized the selfish and exploitative element in men, but based their social doctrines on the hope that human rationality would tend to constrain individual *self*ishness in ways that would bring about some stability in man's political life (Hobbes), in man's economic life (Smith), and in the waging of contests (von Neumann).

Wiener was not so uncritical and wishful in his thinking. Here is what he had to say about this aspect of the views commonly but incorrectly attributed to Adam Smith:

> There is a belief, current in many countries ... that free competition is itself a homeostatic process: that in a free market the individual selfishness of the bargainers, each seeking to sell as high and buy as low as possible, will result in the end in a stable dynamics of prices, and will redound to the greatest common good. This is associated with the very comforting view that the individual entrepreneur, in seeking to forward his own interest, is in some manner a public benefactor and has thus earned the great rewards with which society has showered him. *Unfortunately, the evidence, such as it is, is against this simple-minded theory.* [61c, pp. 158–159] (emphasis added)

Simple-minded it is because it assumes that the market place is unaffected by the Fall of Man. The evidence is against this as Wiener says: much trading is unfair, if not exploitative.

Speaking of the applicability of the von Neumann–Morgenstern game theory to the capitalist market, Wiener continued:

> Even in the case of two players, the theory is complicated, although it often leads to the choice of a definite line of play. In many cases, however, where there are three players, and in the overwhelming majority of cases, *when the number of players is large, the result is one of extreme indeterminacy and instability.* The individual players are compelled by their own *cupidity* to form coalitions; but these coalitions do not generally establish themselves in any single, determinate way, and usually terminate in a *welter of betrayal, turncoatism, and deception* .... In the long run, even the most brilliant and *unprincipled huckster* must expect ruin; but let the hucksters become tired of this and agree to live in peace with one another, and the great rewards are reserved for the one who watches for an opportune time to break his agreement and betray his companions. There is no homeostasis whatever.
>
> It is rare to find a large number of thoroughly clever and unprincipled persons playing a game together. Where the *knaves* assemble, there will always be *fools*; and where the fools are present in sufficient numbers, they offer a more profitable object of exploitation for the knaves. The psychology of the fool has become a subject well worth the serious attention of the knaves. Instead of looking out for his own ultimate interest, after the fashion of von Neumann's gamesters, the fool operates in a matter which, by and large, is as predictable as the struggles of a rat in a maze. [61c, p. 159] (emphasis added)

In this passage the important points are:

(i) the game is highly volatile when the number of players is large;
(ii) the rich rewards awaiting the traitor who breaks an agreement, lowers the prospect of any long lasting peace;
(iii) where knaves assemble there will be fools, and von Neumann's underlying premise of 'rational players' is undercut in practice.

On the role of dishonesty in commercial advertising, Wiener wrote:

> This *policy of lies* – or rather, of statements irrelevant to the truth – will make him [the gullible consumer] buy a particular brand of cigarettes .... A certain precise mixture of religion, pornography, and pseudo science will sell an illustrated newspaper. [61c, pp 159–160] (emphasis added)

The only human contest in which Wiener explicitly refers to *evil* as such is one that would employ nuclear bombs. His response to this danger was to give an objective, unsentimental demonstration that atomic bombs are bad weapons from a sound military stand-

point. It will suffice to quote just one sentence from an interesting lecture he gave to the Officers in the Industrial College of the Armed Forces:

> One shot weapons don't combine with reconnaissance and are exceedingly dangerous to everybody. They are guns which kick as much with the butts as they do with the bullet. [XII, p. 6] (1953)

Feedback, as Wiener explained to his military audience, is the engineer's word for reconnaissance. The less the reconnaissance, the more foolishly violent the fighting will be. Wiener introduced a hierarchical order of military feedbacks, the executions of which demand knowledge of the enemy over longer and longer periods of history. As the order of the feedback increases, the war tends to become less violent. If this process continues, a stage may be reached at which the understanding of the enemy will be great enough to induce the general to sit down with this adversary, and settle the dispute over the table instead of the battlefield.

The objectivist attitude to evil

Wiener was more courageous than most of his colleagues in not underestimating the role of human evil in the political and economic life of man, and giving the devil his due. Words he used, such as

> 'cupidity', 'welter of betrayal, turncoatism and deception', 'unprincipled huckster', 'knaves', 'fools', 'policy of lies'

will not be found in the scholarly work of von Neumann. Their appearance in Wiener's scholarly writings shows an awareness of the need to deal with human evil objectively in scientific terms. For instance, it will not do for managerial scientists to push under the rug the central evil of bureaucracy. On the contrary, they must take full account of Parkinson's Laws on bureaucracy in all administrative flow diagrams.

Wiener's attitude of not flinching from evil should sound a sympathetic chord in India, where there has been an old tradition of extraordinary honesty in the depiction of human evil, beginning with the *Mahabharata*. It is echoed in the animal fables of the *Hitopedesa*. The active role of evil in government is depicted thoroughly in Kautiliya's *Arthasastra*, written around 300 BC. It will suffice to cite the following interesting excerpt:

> 3. The Administrator should station in the country (secret agents) appearing as holy ascetics, wandering monks, cart-drivers, wandering minstrels, jugglers, tramps, fortune-tellers, soothsayers, astrologers, physicians, lunatics, dumb persons, deaf persons, idiots, blind persons, traders, artisans, artists, actors, brothel-keepers, vintners, dealers in bread, dealers in cooked meat, and dealers in cooked rice. 4. They should find out the integrity or otherwise of village-officers and heads of departments 5 And whom-so-ever among these he suspects of deriving a secret income he should cause to be spied upon by a secret agent. *Arthasastra* (c 300 B C) [K, p 265]

In the light of this, a good present from India to Norbert Wiener on his 100th birthday would be a modern version of the *Arthasastra*, written with the same candor as Kautiliya:

> ARTHASASTRA, 1994 A.D.
>
> A Tribute to Norbert Wiener
> on the occasion of his
> First Centenary
>
> by
>
> _ _ _ _ _ _ _
>
> Society of Management Science and
> Applied Cybernetics
>
> NEW DELHI, INDIA

Wiener and the Enlightenment philosophy. Duty

The recognition and objective treatment of the problem of evil brings into focus the paramountcy of love in the redemptive effort of man. One's ability to so recognize and objectify evil enhances, if anything, the ability to love.

Wiener was a great lover of small town American democracy. Speaking of the small New England town in which he grew up, he wrote:

> I have the impression that my friends in this small industrial town represent a sort of stability without snobbishness which is universal rather than provincial, and that the structure of their society compares well with the best that a similar place in Europe would have to offer. [53h, p. 101]

His small-town friends were descendants of emancipated farmers, who after a hard day's work, would read Thomas Paine at night and take an intelligent interest in community affairs. They exemplified the success of the age of Enlightenment, in which political science was intended to bear positively on the practical political life of ordinary people. In his *Cybernetics*, Wiener explained what it is that sustains the homeostasis of such small communities. His words are worth quoting:

> The average man is quite reasonably intelligent concerning subjects which come to his direct attention and quite reasonably altruistic in matters of public benefit or private suffering which are brought before his own eyes. In a small country community which has been running long enough to have developed somewhat uniform levels of intelligence and behavior, there is a very respectable standard of care for the unfortunate, of administration of roads and other public facilities, of tolerance for those who have offended once or twice against society ... in such a community, it does not do for a man to have the habit of over reaching his neighbors [61c, p 160]

Thus, Wiener shared and admired the outlook of the enlightenment thinkers. But Wiener was not taken in by

such successful instances of enlightenment. The age of enlightenment was also the age of one of the worst forms of chattel slavery, of acute exploitation of women and children in factory labor, of exploitation and genocide of colonial people, and of the opium and white slave traffics.

More fundamentally, the Enlightenment philosophy is marred by its basic belief in the inevitability of human progress. Of course there has been a steady progress in human knowledge and technology: the physics of today is definitely superior to the physics we had in the days of Galileo. But there is not a shred of evidence of any corresponding improvement in the arenas of the arts, or of individual morality or of social justice or of the peaceful settlement of disputes. As Wiener cogently observed, '*The simple faith in progress is not a conviction belonging to strength, but one belonging to acquiescence and hence to weakness*' [50j, p. 47].

To base the system of ethics on the notion of striving for 'progress' is thus to place it on a shaky footing. An alternative is offered by the objective rationalistic ethics of Spinoza. However, Wiener was too thermodynamically-conscious and too aware of the role of exploitation in human phylogeny to accept the equating of evil with ignorance, à la Spinoza or Socrates. Wisdom does not come from being learned, but rather from being able, instinctively so-to-speak, to keep out noise. A stuttering, illiterate peasant can be wiser than an articulate college professor.

Wiener built his ethics on the concept of duty-for-its-own-sake. This of course is the position taken in India around 400 BC. Stanza after stanza in the *Gita* elucidate the basic theme:

> Now, if thou this duty-required
> Conflict wilt not perform,
> Then thine own duty and glory
> Abandoning, thou shalt get thee evil.
>
> *Bhagavad Gita*, II, 33 [E. pp. 20–21]

Classical doctrines on duty, whether Hindu, Buddhist, Judeo-Christian or Islamic, are all posited on an other-worldly eschatology. Wiener, the scientist, bypassed the eschatology. But this apart, his writings on duty are close in spirit to those in the *Gita*, and are worth noting:

The Duty of the Intellectual

It is a commonplace that *every profession*, every function in life, involves its own *peculiar responsibilities*. The civilian will not be greatly censured if he runs away from a stricken battlefield The *soldier* has accepted a very special responsibility for physical bravery and moral bravery as well, and above all the officer in charge of troops. I do not think that any honest man looks forward with any pleasure at the prospect not merely of dying in action but of having to face a situation in which the *only honorable thing for him to do is to die in action*. Nevertheless, in becoming a soldier, and especially in becoming an officer, he must accept the contemplation of this possibility.

You or I, not being *doctors or nurses*, could leave a *plague-stricken city* without reproach. The doctor or nurse must remain until the last moment at which his or her services have any possibility of usefulness. Whatever our legal duties are, nobody would make too much of it against us if we refused to aid in the capture of a *dangerous and armed criminal*. The risk of being killed in such an encounter is implicit and accepted, however, by every *policeman* no matter how honorable or important his status. Similarly, the *fireman* knowingly and willingly assumes the risk of being crushed or *burned to death* in the flames of a falling building. *These responsibilities are of the same order as that of the early Christian in accepting death in the arena* rather than undergoing the disgrace and humility of burning a pinch of incense before the gods. [60e, p. 758] (emphasis added).

Thus, Wiener's thinking was more mature than that of the philosophers of the enlightenment. Aware of thermodynamic reality, he saw the superstitious element in the idea of progress. At the same time he saw the paramount importance of doing one's duty come what may.

The long-time State

In a paper read at a conference on national planning in 1954, Wiener analysed the problem of planning, *sub specie aeternitatis*, much as he had done with the AA fire control problem during the war.

The *multiple time-series* with governmental variables such as the national income, the gross agricultural produce, the size of the population, the levels of education, the indices giving the distribution of wealth, the measures of cohesiveness in the society, strength of the armed forces, etc., has some intrinsic differences with those encountered in the AA fire control problem. Nevertheless, here too Wiener was able to discern the 'eternal', steady-state, and 'short-time' transient components.

Institutions such as cities, churches, universities and academies have life-spans several times that of a human generation. The engineering infrastructure that ensures their survival, water supply, dams, sewage, cathedrals, mosques, temples, buildings and roads, also have correspondingly long-life spans. The sustenance of these vital long-time institutions, and this includes science, is the responsibility of what Wiener has called the *long-time State*. In his words:

> The State itself is a long-time institution. Even in those cases where the state has been subject to frequent changes of regime, the internal continuity of the State has been much greater than the external changes of regime might indicate. [62c, p. 36]

Wiener viewed this 'State' as a long-life transducer taking in and giving out messages. To execute its responsibilities, this state has to adopt an approach that is markedly different from what is involved in the day-to-day operations of the political state. The plethora of political regimes that come and go are but transients in the life of the long-time State. Such a distinction between the *short-time State* and the *long-time State*, is alien to post-Renaissance political thought, especially to the thought of John Locke and his followers. But it is

clearly in tune with the more classical political perception of the duality in government, *regnum* and *sacerdotium*. Indeed, Wiener was aware of this. He wrote:

> Perhaps the best thing that has been said about the continuity of the State was formulated two centuries before Christ by the Chinese sage Mencius. Mencius said, in effect, that the rule of the emperor is from heaven, but that when a country has come through a long period of misrule and misfortune, it is a sign that the emperor and even the dynasty has lost the mandate of heaven, and that the country must seek elsewhere for its rulers. This view represents an interesting attempt to combine a certain permanency in the essentials of government with the transience of its details and in the selection of those on whom the task of government lies. [62c, p. 36]

The recognition of two components in government (the *regnum* and *sacerdotium*) was also the foundation of Indian and early Christian and medieval polity. Indian polity, as Coomaraswamy [C2] has explained, starts with the thought expressed in the *Rig Veda*: 'There is no rain so long as sky and earth are estranged.'

In good government, Indra, the ruler of Earth, has to work in consonance with the advice of Agni, the counselor or sky. In Augustinian terminology these two powers are the City of Man and the City of God [A1]. Indra, acting alone, can offer no more than a 'danda government', of the kind depicted in the *Arthasastra*. The confluence of Indra and Agni engenders a benevolent government of a virtuous Chakravartin monarch. In Indian history an approximation to this would be the reign of Ashoka after his conversion to Buddhism and the reign of Akbar. The counselor's advice is that government be for the people. As Mencius put it:

> Heaven sees according as the people see,
> Heaven hears accordingly as the people hear.

These considerations suggest that secular constitutions that do not refer to the long-time arm of government are inadequate, and in need of amendment that specify the long-time functions. Wiener was convinced that whatever affected the long-time interests of humanity, such as education (scientific, aesthetic and moral) and all modes of communication had to be entrusted to these long-time institutions. For instance, the communication networks should not measure the value of their operation by monetary profit, as in the present system in the United States, where the channels are leased to commercial companies. For the dissemination of communal information is, like education, not a profit-making business*. This means in effect that the channels have to be entrusted to long-time institutions outside the capitalist market, such as the cities, the churches, the universities and the academies.

This line of thought led Wiener to an interesting conclusion, which we may express crudely by the formula

Science $\rightarrow$ Religion; as $T \rightarrow \infty$.

Science tends towards religion as the time span increases indefinitely. As Wiener wrote:

> For if religion purports to deal with the eternal, its vocabulary should be very suitable when we wish to treat with those matters which, although not eternal, are of very long duration. [62c, p. 35]

Wiener was led to this conclusion by his fine understanding of the prediction of stochastic time-series. In long-time prediction (say over a 500-year time span) events of very low probability but very high import become appreciably more significant than in short-time prediction. Such events fall into two classes: (i) the excessively malevolent (for instance, a catastrophic earthquake), so-called acts of God in insurance parlance; (ii) the excessively benevolent (a Mother Teresa, a new Einstein, or a great sage, or a fortunate natural event), the so-called acts of Grace, a term Wiener picked up from the political scientist Karl Deutsch. Long-time planning is posited on the faith that the effects of such 'acts of Grace' will more than balance the total due to the 'acts of God' and the acts of human perversity. This faith is more religious than the faith in the Uniformity of Nature, on which all science depends. In the former faith, the Lord God is Spinoza's God, not capricious but indifferent to human concerns. Under the new faith he is no longer indifferent to the needs of mankind. This faith still of course falls far short of the faith in the personal God, to whom one could pray, that Gandhi and other religionists share.

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As one scans the array of great men and women of this century, one comes across examples of towering stature that surpass Norbert Wiener in sheer strength of genius and dedication. In the realm of theoretical science Einstein outstrips him by far. In mathematics von Neumann was a greater and more thorough contributor. In the realms of the spirit and charity, we have great examples in Gandhi and Teresa. However, when we turn to the understanding of the interrelationship of these and other strands in human life, based upon a deep understanding of the stochasticity of the cosmos and of the freedom and corruption that it allows, it is hard to find anyone in this century who has shed so much coherent light, and has advanced so wholesome and catholic a point of view as Norbert Wiener.

Wiener's towering stature in the history of science and philosophy rests not only on his unusual ability to discern so much unity amid such wide apparent diversity, but in his appreciation of its continuity. He was a revolutionary-traditionalist in the best sense of the word. He incorporated in the edifice of human wisdom the new stochastic storey without impairing the total architecture.

*More fundamentally information or negentropy is not an additive quantity like weight or volume, and it is impossible to assign to it a fair price.

APPENDIX

The myth

Like the Revelation (*sruti*) itself, we must begin with the Myth (*itihäsa*), the penultimate truth, of which all experience is the temporal reflection. The mythical narrative is of timeless and placeless validity, true nowhere and everywhere: just as in Christianity, 'In the beginning God created' and 'Through him all things were made', regardless of the millennia that come between the dateable words, amount to saying that the creation took place at Christ's 'eternal birth'. 'In the beginning' (*agre*), or rather 'at the summit', means 'in the first cause': just as in our still told myths, 'once upon a time' does not mean 'once' alone but 'once for all'. The Myth is not a 'poetic invention' in the sense these words now bear: on the other hand, and just because of its universality, it can be told, and with equal authority, from many different points of view.

In this eternal beginning there is only the Supreme Identity of 'That One' (***tad ekam***), without differentiation of being from non-being, light from darkness, or separation of sky from earth. The All is for the present impounded in the first principle, which may be spoken of as the Person, Progenitor, Mountain, Tree, ***Dragon*** or endless serpent. Related to this principle by filiation or ***younger brotherhood***, and ***alter ego*** rather than another principle, is *the* ***Dragon-slayer***, born to supplant the Father and take possession of the kingdom, distributing its treasures to his followers. For if there is to be a world, the prison must be shattered and its potentialities liberated. This can be done either in accordance with the Father's will or against his will; he may 'choose death for his children's sake', or it may be that the Gods impose the passion upon him, making him their sacrificial victim. These are not contradictory doctrines, but different ways of telling one and the same story; in reality, Slayer and Dragon, sacrificer and victim are of one mind behind the scenes, where there is no polarity of contraries, but mortal enemies on the stage, where the everlasting war of the Gods and the Titans is displayed. In any case, the Dragon-Father remains a Pleroma, no more diminished by what he exhales than he is increased by what is repossest. He is the Death, on whom our life depends; and to the question 'Is Death one, or many?' the answer is made that 'He is one as he is there, but many as he is in his children here'. The Dragon-slayer *is* our Friend; the Dragon must be pacified and *made* a friend of.

The passion is both an exhaustion and a dismemberment. The endless Serpent, who for so long as he was one Abundance remained invincible, is disjointed and dismembered as a tree is felled and cut up into logs. For the Dragon, as we shall presently find, is also the World-Tree, and there is an allusion to the 'wood' of which the world is made by the Carpenter. The Fire of Life and Water of Life (Agni and Soma), all Gods, all beings, sciences and goods are constricted by the Python, who as 'Holdfast' will not let them go until he is smitten and made to gape and pant: and from this Great Being, as if from a damp fire smoking, are exhaled the Scriptures, the Sacrifice, these worlds and all beings; leaving him exhausted of his contents and like an empty skin. In the same way the Progenitor, when he has emanated his children, is emptied out of all his possibilities of finite manifestation, and falls down unstrung, overcome by Death, though he survives this woe. Now the positions are reversed, for the Fiery Dragon will not and cannot be destroyed, but would enter into the Hero, to whose question 'What, wouldst thou consume me?' it replies 'Rather to kindle (waken, quicken) thee, that *thou* mayst eat.' The Progenitor, whose emanated children are as it were sleeping and inanimate stones, reflects 'Let me enter into them, to awaken them'; but so long as he is one, he cannot, and therefore divides himself into the powers of perception and consumption, extending these powers from his hidden lair in the 'cave' of the heart through the doors of the senses to their objects, thinking 'Let me eat of these objects'; in this way 'our' bodies are set up in possession of consciousness, he being their mover. And since the Several Gods or Measures of Fire into which he is thus divided are 'our' energies and powers, it is the same to say that 'the Gods entered into man, they made the mortal their house'. His passible nature has now become 'ours': and from this predicament he cannot easily recollect or rebuild himself, whole and complete.

We are now the stone from which the spark can be struck, the mountain beneath which God lies buried, the scaly reptilian skin conceals him, and the fuel for his kindling. That his lair is now a cave or house presupposes the mountain or walls by which he is enclosed, *verborgen* and *verbaut*. 'You' and 'I' are the psychophysical prison and Constrictor in whom the First has been swallowed up that 'we' might be at all. For as we are repeatedly told, the Dragon-slayer devours his victim, swallows him up and drinks him dry, and by this Eucharistic meal he takes possession of the first-born Dragon's treasure and powers and becomes what he was. We can cite, in fact, a remarkable text in which our composite soul is called the 'mountain of God' and we are told that the Comprehensor of this doctrine shall in like manner swallow up his own evil, hateful adversary. This 'adversary' is, of course, none but our self. The meaning of the text will only be fully grasped if we explain that the word for 'mountain', *giri*, derives from the root *gir*, to '*swallow*'. Thus He in whom we were imprisoned is now our prisoner; as our Inner Man he is submerged in and hidden by our Outer Man. It is now his turn to become the Dragon-slayer; and in this war of the God with the Titan, now fought within you, where we are 'at war with ourselves', his victory and resurrection will be also ours, *if* we have known Who we are. It is now for him to drink us dry, for us to be his wine.

We have realised that the deity is implicitly or explicitly a willing victim; and this is reflected in the human ritual, where the agreement of the victim, who must have been originally human, is always formally secured. In either case the death of the victim is also its birth, in accordance with the infallible rule that every birth must have been preceded by a death: in the first case, the deity is multiply born in living beings, in the second they are reborn in him. But even so it is recognized that the sacrifice and dismemberment of the victim are acts of cruelty and even treachery, and ***this is the original sin (kilbisa) of the Gods, in which all men participate*** by the very fact of their separate existence and their manner of knowing in terms of subject and object, good and evil, because of which the Outer Man is excluded from a direct participation in 'what the Brahmans understand by Soma'. The form of our 'knowledge', or rather 'ignorance' (*avidya*), dismembers him daily; and for this *ignorantia divisiva* an expiation is provided for in the Sacrifice, where by the sacrificer's surrender of himself and the building up again of the dismembered deity, whole and complete, the multiple

selves are reduced to their single principle. There is thus an incessant multiplication of the inexhaustible One and unification of the indefinitely Many. Such are the beginnings and endings of worlds and of individual beings: expanded from a point without position or dimensions and a now without date or duration, accomplishing their destiny, and when their time is up returning 'home' to the Sea in which their life originated.

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The mathematical work of N. Wiener (1894–1964)

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NORBERT Wiener's intellectual activity coursed through multifarious channels; beginning with logic and philosophy, it surged through pure mathematics, mathematical physics, engineering and statistics touching on its way literary, political and social criticism. Today, Wiener is widely remembered in the general scientific audience as a founder of cybernetics. This popular appreciation of Wiener's work is not entirely wrong; however, to mathematicians, Wiener's name is always associated with fundamental advances in twentieth century analysis. If Wiener had written nothing else other than his mathematical papers, his name would still remain alive amongst modern mathematicians – even though, admittedly, his popular impact would have been much less.

The purpose of the present essay is to describe some parts of Wiener's significant contributions to mathematics in as simple and non-technical a language as possible. More detailed and technical presentations can be found in items [A], [C], [M] of the short list of references given at the end of this article; a brief chronology of Wiener's life is given there as well. References to Wiener's original papers will be given via his collected works [W]; an excellent source for acquiring further understanding of all aspects of Wiener's work is the biography [M] by Masani.

Although Wiener was extremely precocious in his intellectual development (as would be evident from even a cursory glance at the chronology given at the end), his best-known mathematical contributions were mostly the