Turbulence: waves or events?*

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Turbulent flows, whether in technology or nature, are generally characterized by shear in the mean velocity; indeed the mean flow is often driven by turbulent fluxes of momentum, energy, etc. To describe the time series of such fluxes, the stochastic tools in current use, such as generalized harmonic analysis and (more recently) wavelets, are inadequate as they do not provide assessments of contributions to the mean (rather than the mean square) of a sign-indefinite quantity like the flux. Recent analysis of momentum flux in the atmospheric surface layer has shown however that an episodic description is both feasible and more natural for the flux process, which to a first approximation can be described as a 'chronicle' of signed two-parameter events. In this view flux is generated in short intense bursts; a parameter called burstiness, which is a measure of temporal compactness in contributions to the mean (and varies from zero when flux generation is evenly distributed in time to unity when the chronicle is a sequence of delta functions), has a value of 70–80% in a neutral atmospheric surface layer. Furthermore, it is found that the motions are productive (for flux generation) about 35% of the time, counter-productive for about 15% of the time, and essentially idle (with weak motions cancelling each other out) at other times.

THE classical description of a turbulent flow field is in the language of generalized harmonic analysis, developed in particular by Norbert Wiener[1], whose birth centenary we observe this year. Elements of this approach had informed earlier work on turbulence by G. I. Taylor[2], but the formalism established by Wiener (and by Khintchine in the USSR) in the 1930s had a decisive influence on later work; it led to an intense effort over several decades to develop a statistical theory of turbulence. The seminal work of Kolmogorov[3], who argued that the power spectral density of the turbulent energy should depend on the wave number k like $k^{-5/3}$, in the so-called inertial subrange of wave numbers, and the spectacular support it received from careful measurements in the oceans (Grant *et al.*[4]), appeared to confirm the usefulness of generalized harmonic analysis as a tool for understanding turbulence. The books by Batchelor[5] and by Monin and Yaglom[6] made a large body of results on such a statistical theory widely available.

The heart of this description is the expression of any turbulent fluctuation, e.g. the velocity $\mathbf{u}'$, as the integral

$$\mathbf{u}'(\mathbf{x}, t) = \int \exp\{i(\mathbf{k}\cdot\mathbf{x} - \omega t)\}\, dZ(\mathbf{k}, \omega), \qquad (1)$$

where the exponential represents a wave of frequency ω and wave number $\mathbf{k}$, and dZ represents (the Lebesgue measure of) an infinitesimal amplitude corresponding to given ω, $\mathbf{k}$. Basically the field is seen as a superposition of waves, whose amplitudes however have to obey certain conditions to permit us to define a meaningful 'spectrum', which tells us the contribution to the turbulent energy $\frac{1}{2}\langle \mathbf{u}'^2\rangle$ from each infinitesimal range of waves. In single-point measurements (which are themost common kind), it is the temporal frequency representation (in terms of ω) that is directly acquired. When the field is statistically homogeneous in space, a description in terms of the wave-number vector $\mathbf{k}$ has been preferred.

The insights of Kolmogorov and Taylor, while illuminating (see Narasimha[7] for a critical discussion), did not however lead to any real solution of the problem, and eventually the reservations that had always been felt about the spectral mode of description came to the fore. In the first place, each of the elementary 'waves' that add up to describe $\mathbf{u}'$ in equation (1) extends all the way from $-\infty$ to $+\infty$ along each coordinate axis. Although it is common to speak of 'eddies' as being associated with each $\mathbf{k}$, the infinite extent of each wave goes against the intuitive notion of an eddy as a structure of some kind that has a finite extent in space and a finite life-time. Secondly a spectral description, with its emphasis on contributions to energy, averages over the phase differences between waves. Both these reservations have grown stronger over the last two decades, as evidence has accumulated that in turbulent shear flows there can be a considerable degree of order, and that 'coherent structures' in the flow are of great importance in understanding its dynamics[8,9]. We shall return to the subject shortly, but must note here that the homogeneous isotropic turbulence that is such a

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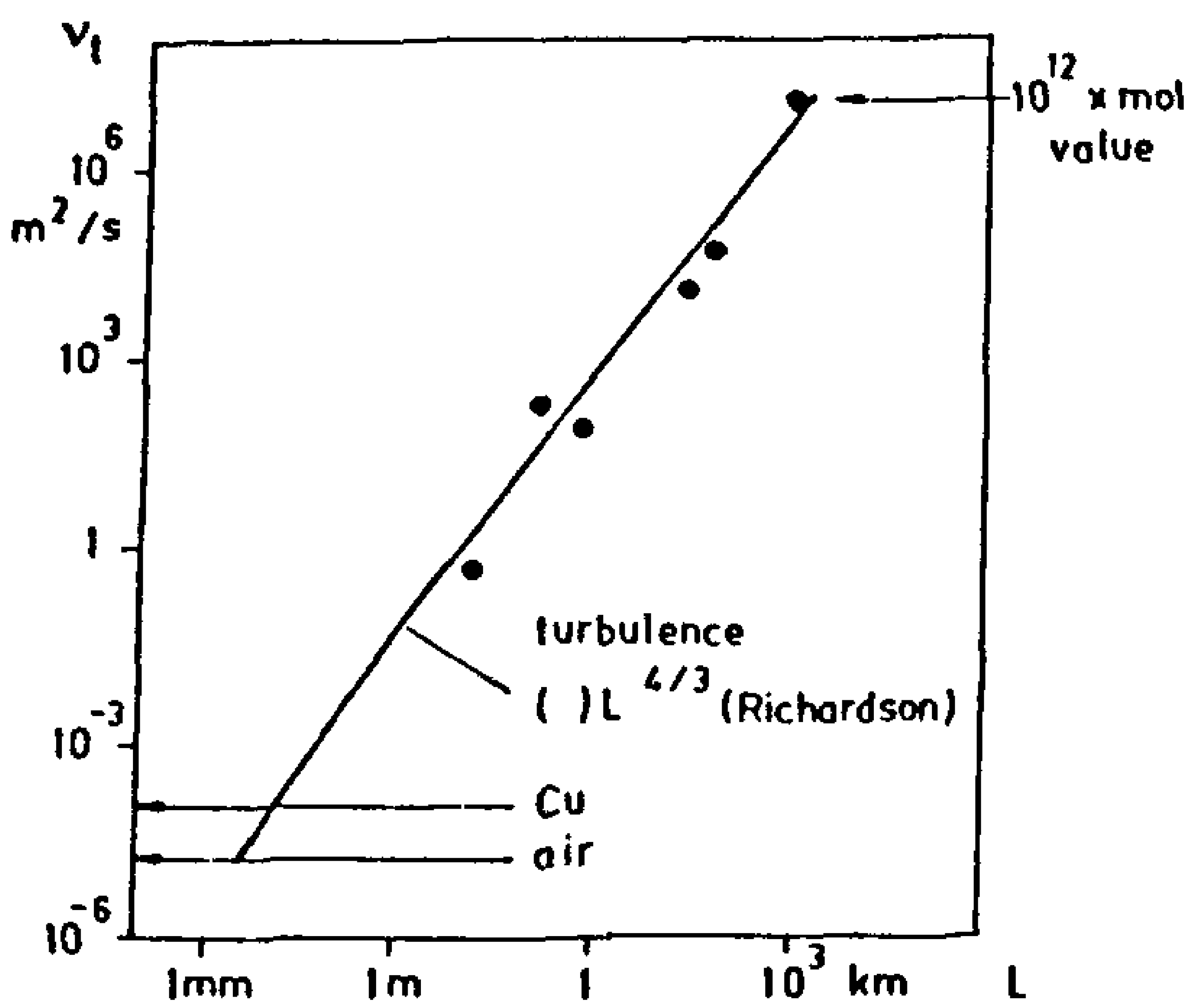

Figure 1. Effective eddy viscosity for turbulent motions in the atmosphere (based on the data of Monin[6]).

favourite idealization in much statistical theory is never encountered in nature or technology. Indeed some ingenious attempts have been made to 'manufacture' isotropic turbulence in the laboratory[10], but it has been found that it is hard to *preserve* isotropy even when it has been *created* at one time or location.

Most turbulent flows that one has to deal with in real life actually possess shear, i.e. are characterized by gradients of the mean velocity. This is true as much in technological applications as in the atmosphere and the oceans. The dynamics of the mean flow is in general dominated by turbulent transport: the mean velocity, e.g., may be thought of as being driven by the Reynolds stresses, which are proportional to mean values of products of velocity fluctuations. In the case of a simple two-dimensional shear flow in a thin (boundary) layer, the momentum equation for the streamwise mean velocity in an incompressible fluid may be written as

$$\frac{\partial \bar{u}}{\partial t}+\bar{u}\frac{\partial \bar{u}}{\partial x}+\bar{w}\frac{\partial \bar{u}}{\partial z}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x}+\nu\frac{\partial^2 \bar{u}}{\partial z^2}-\frac{\partial}{\partial z}\overline{u'w'}, \quad (2)$$

where $\bar{u}, \bar{w}$ are the mean flow velocity components along x and z, $\bar{p}$ the mean pressure, ρ the density, ν the viscosity and u', w' the velocity fluctuations along and normal to the dominant mean flow (x and z directions respectively). The $\overline{u'w'}$ term represents the Reynolds stress or the eddy momentum flux, which dominates the viscous term preceding it except near any solid surface that may be present in the flow. In shear flows it is the Reynolds stress term that characterizes turbulence: and it is the associated *fluxes* that we are after, more than anything else.

It is necessary to realize that these fluxes can be enormous. An *ad hoc* way of handling them is to assume a flux-gradient relation of the same type that, in the

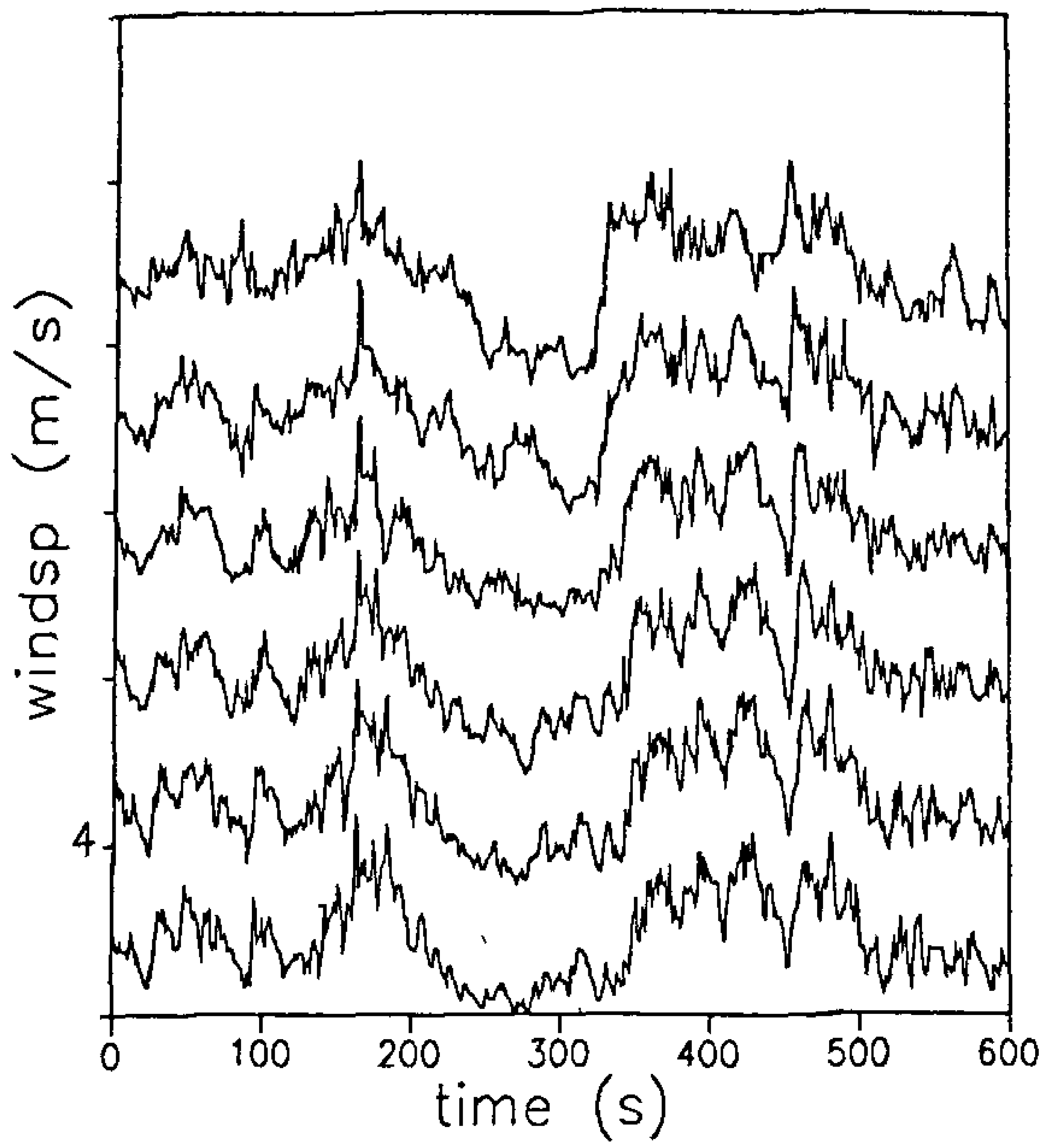

Figure 2. Typical traces of horizontal velocity from cup anemometers at different levels on a 10 m mast[11] (The vertical separation between anemometers is approximately logarithmic in height)

kinetic theory of gases, leads to the familiar molecular viscosity ν. Thus we may put

$$-\overline{u'w'}=\nu_t\frac{\partial \bar{u}}{\partial z}$$

in a thin shear flow (without taking too seriously for the moment the implied localness hypothesis that we know to be generally wrong in turbulent flows). Figure 1 shows values of the eddy viscosity ν_t estimated by Monin[6] as a function of the scale of motion; these data are from the atmosphere. It is seen that the eddy diffusivity is strongly scale-dependent, and at the large scales may be 10^{12} times the molecular value! Large-scale turbulent motions are thus superconductors of momentum (and heat): if there were no turbulent transport, temperature contrasts on the globe would be much stronger than they are.

Given that the fluxes are so important, it is surprising how little work has been done to understand the structure of flux time series. Generalized harmonic analysis does not appear to be a good tool here, for one fundamental reason. Our interest is in finding out how turbulent motions contribute to the *mean* flux $-\overline{u'w'}$, which can in general be positive or negative. (It is essential to note that the fluxes are *not* sign-definite, unlike quantities like kinetic energy and dissipation)

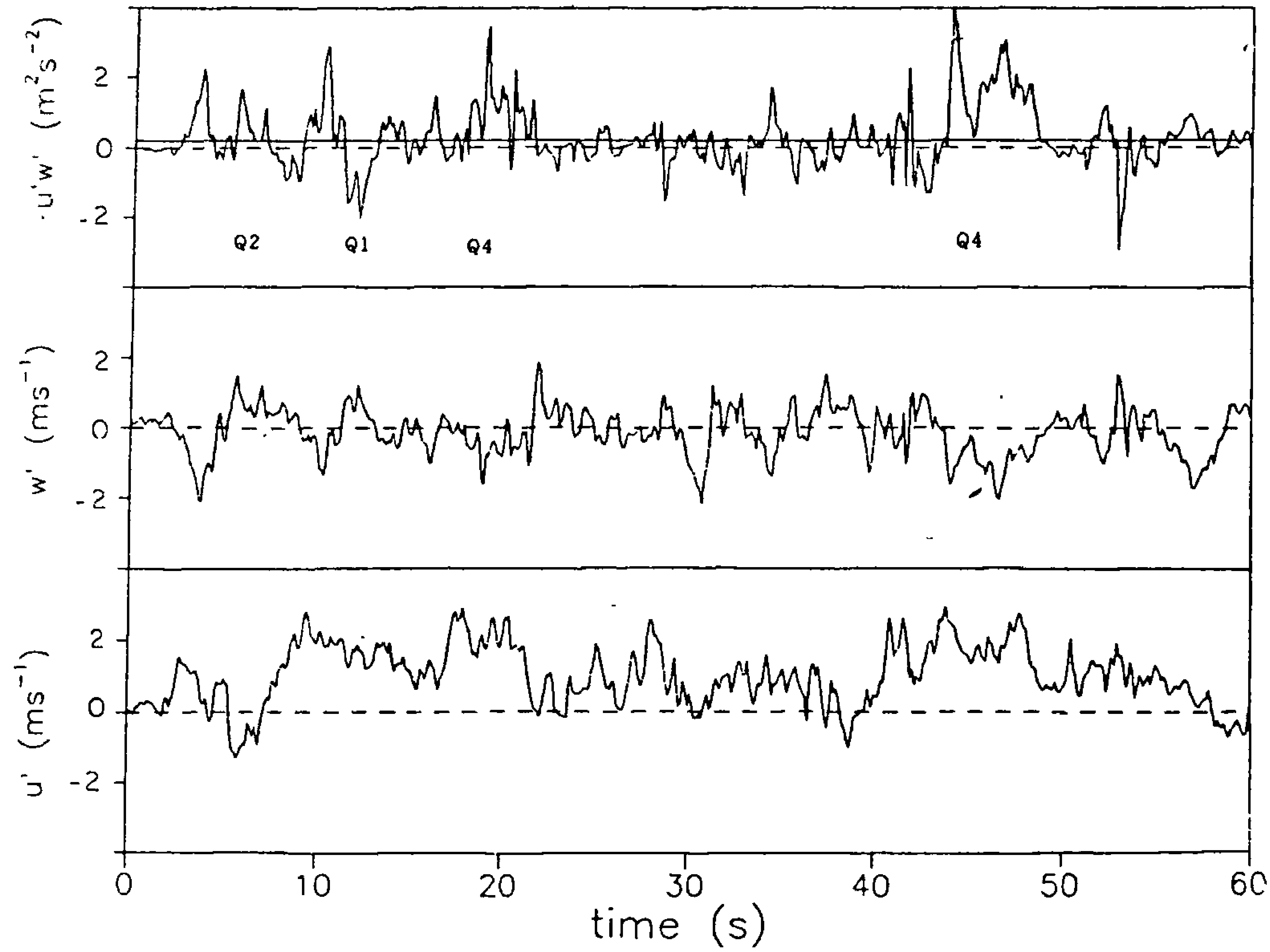

Figure 3. Typical traces of horizontal and vertical velocity fluctuations and their product[11]; the mean value of the product is shown by the full line in the top panel.

The spectral approach is able to describe contributions to positive-definite quantities like $\overline{u'^2}$; it might be of some use if we wished to analyse the quantity

$$\overline{(u'w' - \overline{u'w'})^2},$$

but not the quantity $\overline{u'w'}$ itself.

I want to describe here an attempt that we have recently made[11] that enables us, for the first time, to represent the flux time series as a different kind of stochastic process.

We have already mentioned briefly the importance of coherent structures in turbulent shear flow. Although single-point measurements do not usually suggest any obvious order in the signal, simultaneous measurements at several points can often unmistakably reveal the presence of coherent motions. An example is shown in Figure 2, which presents a set of six traces of the streamwise velocity from sensors strung along a mast of 30 m height to probe the atmospheric surface layer. (These measurements were made in 1990 at Jodhpur as part of a national project called MONTBLEX – for the Monsoon Trough Boundary Layer Experiment[12].) These data leave no doubt whatever about the strong coherence of the motions, at least over a 30 m height! (The coherent scales are often even larger.)

Given such coherence in turbulent motions, we first seek to identify any 'patterns' that may be present in the flux signal. After the earlier mention of eddies of finite size and life, it might seem that wavelet analysis might provide just the tool we are looking for. Wavelets have been discussed at length recently[13,14], and have some very attractive properties for use in stochastic analysis. But there are several difficulties with wavelets as far as flux analysis is concerned. First of all, the shape of the wavelet, or the mother wavelet, can have widely varying forms, subject only to the satisfaction of certain admissibility conditions, one of which is that its mean must be zero. It therefore follows that wavelets cannot be expected to depict a contribution to the *mean* flux. Furthermore, if the true pattern in the time series of interest – assuming one exists – is different from admissible mother wavelets, we cannot expect an efficient or optimum description to emerge. Finally, wavelets also obey a Parseval theorem, so irrespective of the mother wavelet chosen, a superposition like (1) will add up to the energy of the motion, and in fact we are back to looking at sign-definite quantities: as we have already seen, such a tool cannot answer the question raised earlier about sign-indefinite fluxes.

Before proceeding, let us look at a typical flux signal, which is shown in Figure 3, along with its mean over a

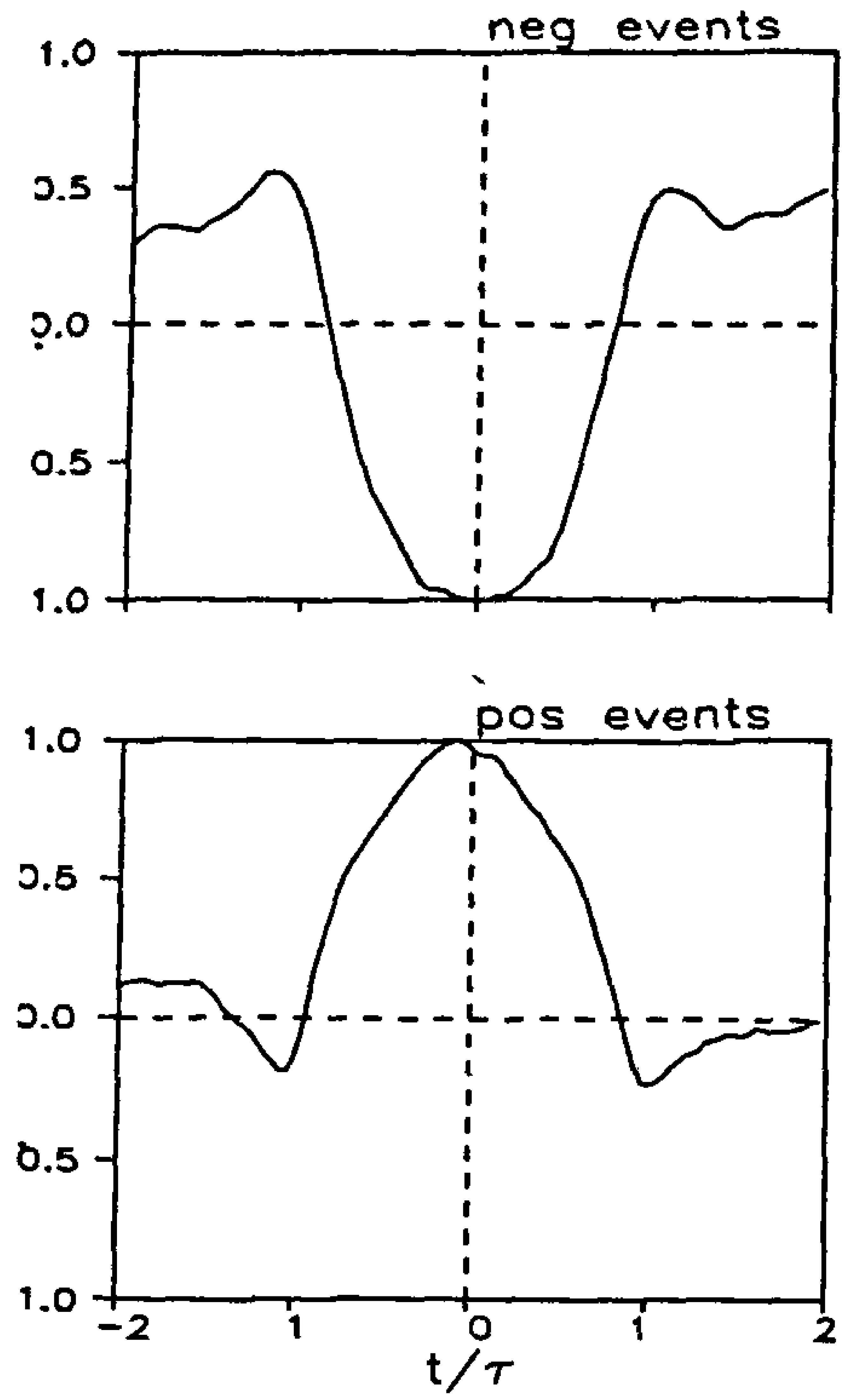

Figure 4. Profiles of positive and negative flux events, in normalized variables[11].

period of 10 min. The first striking fact is that the fluctuations in the instantaneous flux are violent: values that are twenty times the mean are common, and those a hundred times the mean are not rare! Recalling how large the mean fluxes involved are, we see that the atmosphere has to work very hard indeed to produce the superconduction of Figure 1!

Are there any patterns in the signal of Figure 3? After trying several techniques for detecting events – including wavelets, variable interval time averaging, quadrant analyses etc. – we have recently come to the conclusion[11] that the best procedure is a very simple one indeed – namely that of looking for fluctuations that are intense in some sense. That is, we select a threshold, look at the fluctuations above this threshold, and average over all of the intense events that stand out to see if any shape or pattern is revealed. This kind of 'conditional' averaging makes sense especially if the final results are not sensitively dependent on the threshold level chosen; and in the present case this indeed appears to be so, as any threshold below that corresponding to one standard deviation in both velocity components leaves the final picture unaltered. We thus arrive at the conclusion that, when appropriately

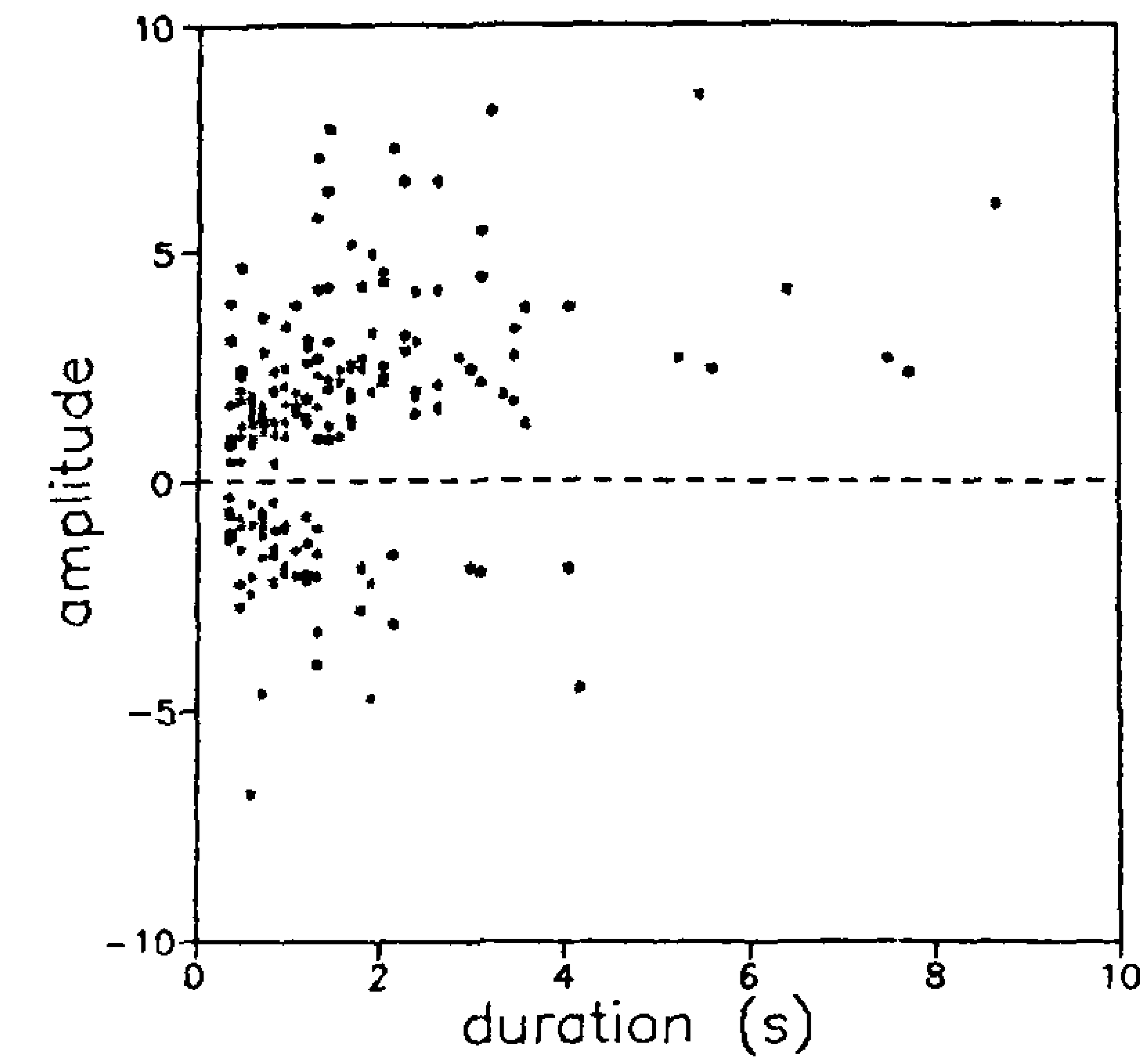

Figure 5. Amplitude of flux events as a function of their duration[11].

normalized, a well-defined flux event profile (or shape) can indeed be identified (Figure 4). An event can contribute to the mean flux positively or negatively – so we first define their profiles separately, but are pleased to discover that they are approximately mirror reflections. At the present stage of our work, it is therefore a sufficiently good approximation to say that there is only one shape – but that each event carries a sign with it.

It will be seen that the event-shape that emerges is rather like the Mexican hat that has become familiar from wavelet studies, but there is a crucial difference: unlike the Mexican hat, the profile seen in Figure 4 has a non-zero mean (as it must) – it may be called a *heavy* Mexican hat. However the wings of the profile are rather fuzzy, and we have tentatively ignored them till further work leads to more definitive data.

Two parameters then describe each event: its duration in time, and its amplitude – say the peak value of the flux during the event. These parameters seem uncorrelated with each other (Figure 5), so we need both. Using the integral under the profile curve, we can also define the magnitude of the event as the contribution made to the mean flux by the event: the magnitude can also be signed.

All of this suggests the idea that an 'episodic' description of the flux process may be feasible. A graphic way to display this episodic character is what we have called the burstiness diagram[15,16] (Figure 6). Given a stretch of data that has been analysed into events, one orders them down from the biggest (i.e. largest magnitude or contribution to flux). Then one computes the cumulative contribution to the flux down the list from the biggest events, and their cumulative duration as well, and plots them as in Figure 6. The

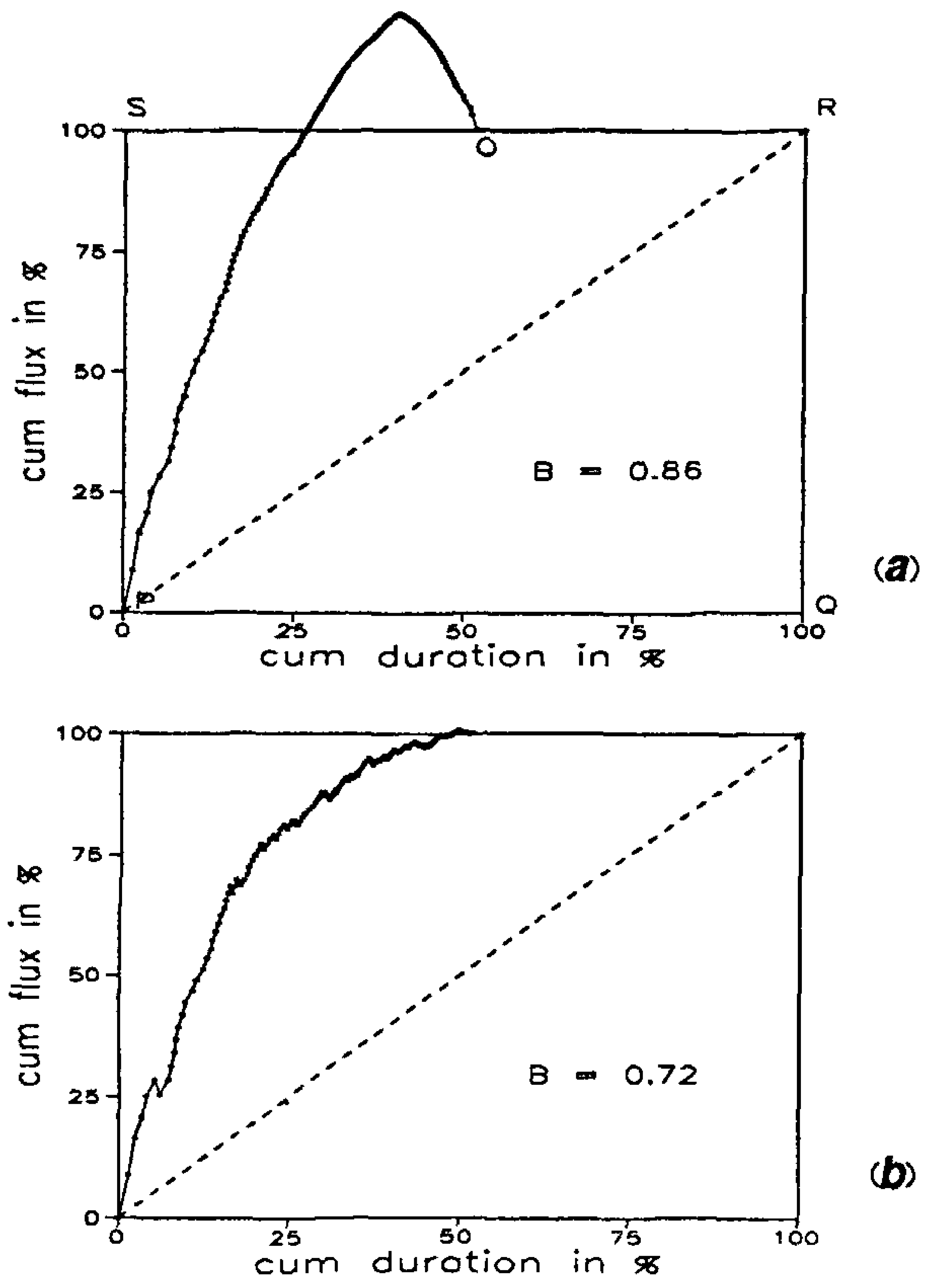

Figure 6. Burstiness diagrams[11].

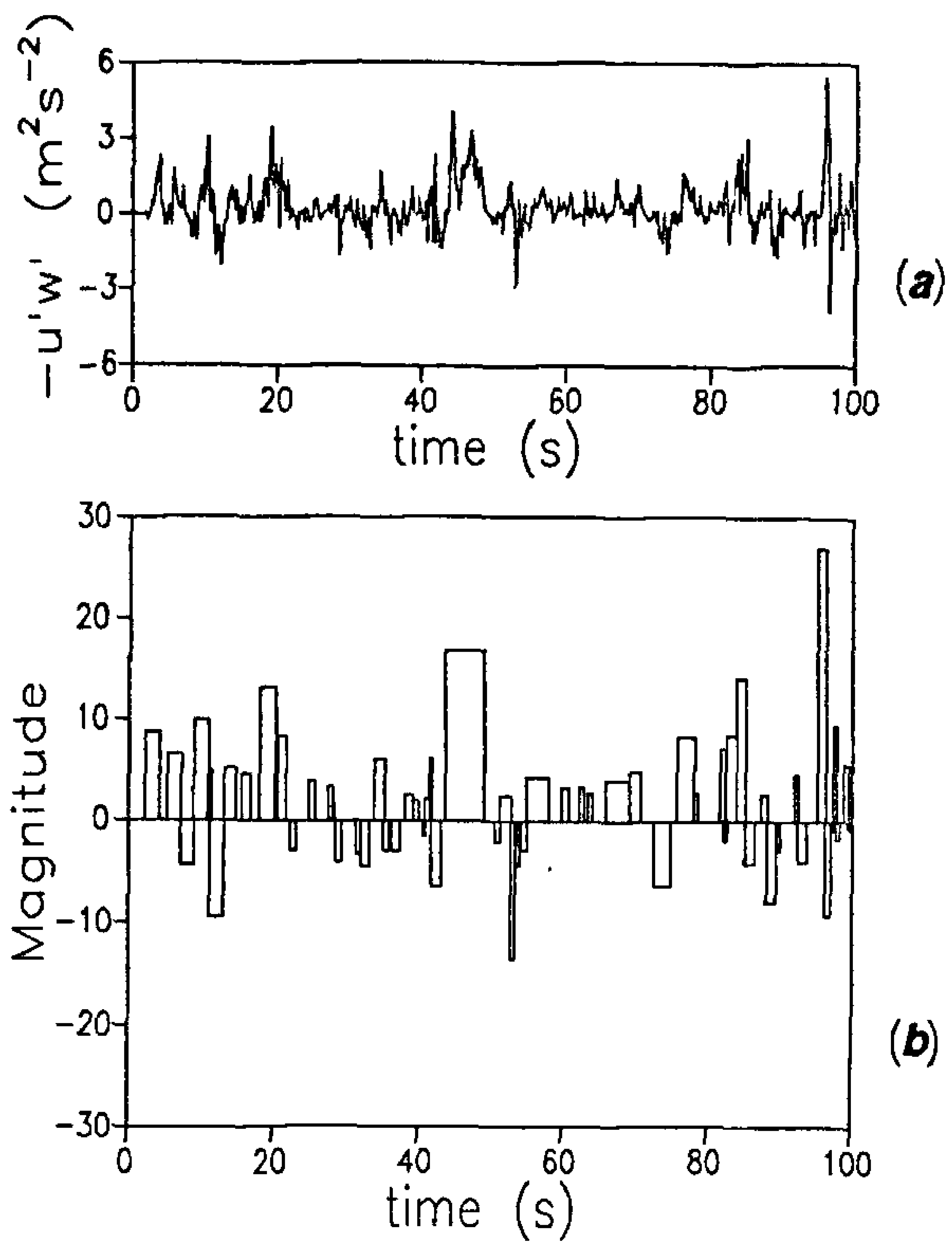

Figure 7. Trace of the raw momentum flux signal (bottom), together with the equivalent chronicle of flux events (top)[11]

curve that results (Figure 6 *a*) typically rises steeply from the origin, overshoots the 100% flux mark, reaches a peak and drops down; eventually the small events left (below the threshold) make little contribution to the flux, so the curve reaches 100% time on a flat trajectory. The interpretation is simple. Positive flux events, lasting only a fraction of the time, generate rather more than the net mean flux during the time, and negative events, lasting an even shorter fraction, cancel out the excess and reduce the net flux to nearly 100%. Over the rest of the time not much flux is generated overall. Reading numbers off Figure 6, we could say that, as far as the momentum flux is concerned, the atmosphere (under the conditions prevailing at the time when the data in Figure 6 were taken) was productive about 35% of the time, counter-productive about 15% of the time, and idle about 50% of the time – a pattern that might remind us of how many human organizations work! The numbers given are characteristic of the atmosphere when it is neutrally stable; when conditions are different the numbers do change appreciably[11].

A second type of burstiness diagram (Figure 6 *b*) is sometimes useful. Here the events are ordered down according to the *absolute* magnitude of the event (i.e. ignoring its sign). Then the overshoot seen in Figure 6 *a* is absent. The area between the burstiness curve and the 45° line, suitably normalized, yields us an index that tells us how 'bursty' the flow is. If the index (which we call the 'burstiness'[15]) is unity, all flux is produced in events of zero duration, i.e. the flux signal is a sequence of delta functions; if the index is zero, flux generation is evenly distributed in time. Under neutral conditions the burstiness in the atmosphere is found to be about 70–80%.

It is now possible to replace the raw flux signal by what we have called a chronicle of events[11,16], as in Figure 7. This brings a description of the flux signal to that of a special kind of 'point process'[17], rather than the generalized harmonic analysis that has been the traditional tool. Once we adopt such an episodic description, the questions that naturally arise become entirely different: what are the distributions of the magnitude and duration of events? what are the arrival times? how are these parameters affected by stability? how well are they correlated over the scale of the flow? And so on. Answers to some of these questions are now beginning to be available[11].

Incidentally, we appear to have here, for the first time, an objective way of distinguishing 'active' from 'passive' motion, a concept that was introduced by Townsend[18]. The very simple criterion we have found is that low fluctuations (typically less than a standard deviation in the velocity components) contribute little to the flux, and could perhaps be identified with passive motion; the more intense fluctuations are 'active' in the generation of flux. An attractive possibility is that the passive motion is best described in the language of waves, whereas the active motions – productive or counter-productive – are best seen as a series of events. So I suggest a tentative answer to the question in the title that turbulence can be both waves and events – but the waves are passive, and all the flux comes from the events, which (to a first approximation) are always members of a signed two-parameter family, with the positive events outnumbering and outlasting the negative ones in a neutrally stable flow.

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Rydberg atoms and molecules – Testing grounds for quantum manifestations of chaos

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Understanding quantum manifestations of classical chaos is of intrinsic and practical interest. In the recent years several fingerprints of chaos have been recognized in microscopic systems. In this connection Rydberg atoms and molecules, which are highly excited systems under various external interactions, are found to be testing grounds to understand quantum chaos. We trace here these recent developments and discuss their implications and future outlook.

THE deterministic randomness or chaos exhibited by generic nonlinear dynamical systems has been found to present significant practical and philosophical implications[1–5], and probably limitations[6] as well, in the quantum description of microscopic world. There is no doubt that quantum theory is a more accurate description of nature. However, Bohr's correspondence principle requires that in the appropriate limit the remnance of the signatures of (classical) chaos (of macroscopic world), namely the exponential divergence of nearby trajectories and the intrinsic uncertainty due to nonlinearity, should follow, barring unforeseen singularities in the Planck

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