

The quantum measurement problem: Nature of the apparatus

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By considering the measurement of a quantum spin by two different models of the measuring apparatus, we try to throw some light on the nature of the apparatus that can perform quantum measurement. Our scheme is to consider a quantum spin in interaction with another quantum system playing the role of an apparatus, which in turn is dissipatively coupled to an environment. The coupling with the environment drives the density matrix of the apparatus to a diagonal form. However, *measurement* is performed only when the reduced density matrix of the apparatus contains correlations with the states of the spin. The two examples considered here show that this happens when the quantum apparatus has a well-defined classical limit. If the apparatus does not have a well-defined classical limit, the reduced density matrix of the apparatus is not correlated with the states of the measured system.

In spite of the remarkable success of quantum theory, its measurement aspect remains a profound interpretational difficulty. Quantum mechanics, it seems, fails to provide a natural framework to accommodate our familiar classical perceptions. At the heart of quantum mechanics is the state vector $|\psi\rangle$, a mathematical entity, which contains all possible information about the system to which it is attributed. The time evolution of the state vector is via the Schrödinger equation. However, in spite of this apparent determinism, a knowledge of $|\psi\rangle$ does not ensure a precise knowledge of the 'observable' properties of the system, the kind we are familiar with in the 'classical' world, e.g., position, momentum, etc. These dynamical variables are represented in quantum mechanics by linear hermitian operators, which act on the state vector. An operator \hat{A} corresponding to a dynamical quantity A is associated with eigenvalues a_i 's and the corresponding eigenvectors $|\alpha_i\rangle$, the latter forming a complete orthonormal set, so that any arbitrary state vector $|\psi\rangle$ can be expanded as a linear superposition of these eigenvectors, i.e., $|\psi\rangle = \sum_i c_i |\alpha_i\rangle$. The basic postulate of quantum mechanics is that a measurement of A can yield only one of the eigenvalues a_i , but the result is not definite in the sense that different measurements for the same quantum state $|\psi\rangle$ yield different eigenvalues. Quantum mechanics predicts the

probability of obtaining the eigenvalue a_i to be $|c_i|^2$. This also implies that if the state $|\psi\rangle$ was one of the eigenstates $|\alpha_i\rangle$, then the result of the measurement is definitely a_i . The general state $|\psi\rangle$ can well be expanded in any basis which may form the complete eigenfunction set of any other 'observable', whose observed values and their probabilities can be calculated similarly. Thus, we see that the state vector contains familiar 'classical properties' only as 'potentialities' which emerge only when a measurement is performed. *Familiar classical perceptions are only potential outcomes of a measurement on a quantum system, they are not a part of the quantum mechanical framework.* We now consider the actual measurement process, in which a system is brought in contact with an apparatus. This process is visualized in the following way. The interaction with an apparatus purporting to measure A forces the state $|\psi\rangle$, which, in general, is a linear superposition of the eigenstates $\{|\alpha_i\rangle\}$, to go into one of the states $|\alpha_k\rangle$ and yield a_k as the measured quantity. Clearly, such a collapse cannot occur if the apparatus is also another quantum system, since under those conditions one would have the unitary evolution of the combined quantum state of the system and the apparatus. This should again result in a quantum state which can be regarded as a superposition of states in any number of ways. This means that the act of measurement which inevitably requires a sudden collapse of the state vector to one of the eigenstates of the dynamical operator falls outside the realm of quantum mechanics.

There have been several attempts to understand and explain the measurement process¹⁻⁶. Among the earliest explanations is the Copenhagen interpretation proposed by Niels Bohr², which requires the presence of an external classical apparatus to cause the 'collapse' and hence a measurement. The theory cannot explain the actual mechanism of the collapse and the ambiguous border between 'quantum' and 'classical' world makes it unsatisfactory. The many-worlds interpretation of Everett⁴ and De Witt⁵ treats the entire universe as a quantum system, having one state vector which 'branches out' with every interaction. The theory does away with classical concepts completely and 'potential outcomes' are accommodated by the branches of the wave function, the observer being conscious of only one

branch. The theory, however, fails to provide a convincing answer as to when the branching occurs and remains highly controversial. The statistical interpretation⁶ of quantum theory treats the quantum-mechanical formalism as merely a description of our information about the 'ensemble' in question. Although the theory provides an alternative way of looking at quantum mechanics, it fails to explain the behaviour of a single, specific system. Thus, one can see that in spite of intensive efforts, there have been severe difficulties with most theories. Can the measurement problem be understood and resolved within the framework of quantum mechanics, thus making quantum theory truly self-contained?

An interesting line of investigation to resolve this issue was initiated by Zeh⁷, who observed that the measurement apparatus, being macroscopic, is never isolated from its environment. The closely-spaced levels of a macroscopic apparatus make it very susceptible to the influence of the environment. This is an important observation and forms the basis of the 'environment-induced decoherence' theory⁷⁻⁹ for the measurement problem. At this point one should clarify the dissipative role of the environment. When the interaction of a small quantum system with a large quantum system is considered, and one monitors the density matrix of the small system only, one sees the monotonic time decay of the off-diagonal matrix elements of the reduced density matrix. These are actually arising due to the superposition of a large number of harmonic terms, viz., $\sum_{j=1}^n a_j \exp(i\omega_j t)$, which, under the condition that ω_j are closely spaced and that the number of terms n is large, gives rise to apparent decays in the short time limit $t \ll T$, where T is the characteristic 'recurrence' time for the periodic function. However, T can be astronomically large even under mild conditions of $n \sim 1000$, and $\Delta\omega \sim 10^{-3}$. Thus, a quantum system coupled to an environment consisting of a large number of degrees of freedom behaves like a classical system in the sense that *at time scales of interest* its density matrix is diagonal. Zurek^{8,9} has applied this environment-induced decoherence idea to consider measurement by allowing the apparatus to interact with an environment.

In the following, we take another look at Zurek's full quantum-mechanical treatment of the measurement problem^{8,9}. The model consists of a spin-1/2 quantum system interacting with a two-level quantum apparatus, which in turn is interacting with an environment modelled by a large number of degrees of freedom. An analysis of the model reveals difficulties with Zurek's treatment, and we show that his model does not really accomplish a *measurement* of the spin state of the system. We then analyse¹⁰ a Stern-Gerlach-type apparatus, in which a spin measurement is made by measuring the trajectory variables of a particle obtained

by letting the position variable of the particle interact with an environment. We feel that the main point of difference in the two examples considered is that in the first case the apparatus does not have a classical limit, while in the latter it has a classical limit.

We now note some general features of both the models. The Hamiltonian of the entire system can be written as

$$H = H_S + H_A + H_E + H_{SA} + H_{AE}, \quad (1)$$

where the subscripts S, A, E stand for system, apparatus and environment, respectively, H_S etc. denote the Hamiltonians for the individual components, and H_{SA} etc. denote the interaction Hamiltonians between the components. If a dynamical variable O belonging to S is to be measured, then O commutes with H . Further, O appears linearly in H_{SA} .

Analysis of Zurek's model

The system-apparatus-environment model considered by Zurek^{8,9} consists of a spin-1/2 system, a spin-1/2 apparatus and an environment which contains a large number of spin-1/2 systems. $|\uparrow\rangle, |\downarrow\rangle; |\pm\rangle, |\mp\rangle$; and $\{|\pm\rangle_k, |\mp\rangle_k\}$ represent the basis states of the system, apparatus and the environment, respectively. σ, L and $\{J_k\}$ denote the Pauli spin operators for the system, apparatus and environment spins, respectively. The mechanism of measurement is via a two-stage process. In the first stage the system and the apparatus are allowed to interact, the time evolution being via the following Hamiltonian:

$$H_{SA} = g \sigma_z L_y, \quad (2)$$

where g is the strength of the coupling. One can see that the Hamiltonian evolution transforms an initial direct product state of the system-apparatus

$$|\psi(0)\rangle = (a|\uparrow\rangle + b|\downarrow\rangle) \otimes (c|\pm\rangle + d|\mp\rangle), \quad (3)$$

where the system and the apparatus are in general superposition states $(a|\uparrow\rangle + b|\downarrow\rangle)$ and $(c|\pm\rangle + d|\mp\rangle)$, respectively, to

$$|\psi(t)\rangle = a|\uparrow\rangle (c|\pm\rangle e^{-i\omega t} + d|\mp\rangle e^{i\omega t}) + b|\downarrow\rangle (c|\mp\rangle e^{-i\omega t} + d|\pm\rangle e^{i\omega t}). \quad (4)$$

Zurek considered the special case in which $c = d = 1/\sqrt{2}$, i.e., the apparatus is initially in the known state $(|\pm\rangle + |\mp\rangle)/\sqrt{2}$. At a specific time $t = t_f = \pi/4g$,

$$|\psi(t_f)\rangle = (a|\uparrow\rangle |\pm\rangle + b|\downarrow\rangle |\mp\rangle)/\sqrt{2}. \quad (5)$$

It should be noted that $|\psi(t_f)\rangle$ contains definite correlations between the system and the apparatus states, but at any other time $|\psi(t)\rangle$ contains overlaps of the apparatus states. The first stage of Zurek's treatment terminates at the special state (5), after which the system–apparatus interaction is switched off. In the second stage the apparatus–environment interaction is switched on via the Hamiltonian

$$H_{AE} = \sum_{k=1}^n g_k L_z J_{ky} \prod_{j \neq k} \mathbb{1}_j, \quad (6)$$

where $\mathbb{1}_j$ denotes a unit operator, g_k 's are the coupling strengths and n is the total number of two-state systems constituting the environment. The initial state is taken to be the correlated state (5) of the first stage and a direct product with a general 'environment state'⁸. The density matrix is a convenient formal tool for studying the time evolution of the system. Zurek⁸ shows that in the large-time limit and as $n \rightarrow \infty$, the density matrix describing the system–apparatus combination reduces to

$$\rho = |a|^2 |\uparrow\rangle\langle\uparrow| |\pm\rangle\langle\pm| + |b|^2 |\downarrow\rangle\langle\downarrow| |\mp\rangle\langle\mp|, \quad (7)$$

which is a diagonal density matrix with one-to-one correspondence between the spin and the apparatus states. An up-spin corresponds to the $|\pm\rangle$ apparatus state with a probability $|a|^2$ and a down-spin corresponds to the $|\mp\rangle$ apparatus state with probability $|b|^2$.

It is obvious that the success of the above scheme is crucially dependent on the precise time at which the interaction of the first stage is terminated. Our approach is to look at the combined time evolution of the system–apparatus–environment setup. This approach is more realistic as it does not require the termination of any interaction at a specific time. The complete interaction Hamiltonian for looking at the combined evolution is $H = H_{SA} + H_{AE}$. This Hamiltonian can be diagonalized. If $s = \pm 1$, and $\mu_k = \pm 1$ denote the eigenvalues of σ_z and J_{ky} , then the eigenvalues of H are given by $E(s, \{\mu_k\}) = \pm \sqrt{(g^2 + f^2)}$; $f = \sum_k g_k \mu_k$. The reduced density matrix of the system and the apparatus after tracing over all the environment variables is given as

$$\begin{aligned} \rho_R = \sum_{\mu} \frac{|C(\mu)|^2}{2} & \{ (|a|^2 |\uparrow\rangle\langle\uparrow| + |b|^2 |\downarrow\rangle\langle\downarrow|) (|\pm\rangle\langle\pm| + |\mp\rangle\langle\mp|) \\ & + \sin\theta_{\mu} \sin(2E_{\mu} t) (|a|^2 |\uparrow\rangle\langle\uparrow| - |b|^2 |\downarrow\rangle\langle\downarrow|) \\ & \times (|\mp\rangle\langle\mp| - |\pm\rangle\langle\pm|) + (|a|^2 |\uparrow\rangle\langle\uparrow| + |b|^2 |\downarrow\rangle\langle\downarrow|) \\ & \times [(\cos(2E_{\mu} t) - i \sin(2E_{\mu} t) \cos\theta_{\mu}) |\pm\rangle\langle\mp| + \text{C.C.}] \end{aligned}$$

$$\begin{aligned} & + [ab^* |\uparrow\rangle\langle\downarrow| e^{-i\theta_{\mu}} \{(\cos\theta_{\mu} + i \sin\theta_{\mu} \cos(2E_{\mu} t)) \\ & \times (|\pm\rangle\langle\pm| + |\mp\rangle\langle\mp|) + (i \sin\theta_{\mu} + \cos\theta_{\mu} \cos(2E_{\mu} t)) \\ & \times (|\pm\rangle\langle\mp| + |\mp\rangle\langle\pm|) + i \sin(2E_{\mu} t) \\ & \times (|\mp\rangle\langle\pm| - |\pm\rangle\langle\mp|) + \text{C.C.}] \}, \quad (8) \end{aligned}$$

where E_{μ} is the short-hand notation for $E(s, \{\mu_k\})$, $\tan\theta_{\mu} = g/f$, and $C(\mu)$'s are constants dependent upon the state of the environment given by the variables $\{\mu_k\}$. The summation is over all possible 2^n values of the μ_k variables. In the large-time limit the reduced density matrix becomes:

$$\rho_R = \sum_{\mu} \frac{|C(\mu)|^2}{2} (|a|^2 |\uparrow\rangle\langle\uparrow| + |b|^2 |\downarrow\rangle\langle\downarrow|) (|\pm\rangle\langle\pm| + |\mp\rangle\langle\mp|). \quad (9)$$

Result (9) is significantly different from Zurek's result (7) in the sense that although the environmental influence does drive the density matrix of the system and the apparatus to become diagonal (overlaps between apparatus states disappear), there is no one-to-one correlation between the states of the system and those of the apparatus. The *measurement* of spin is clearly not accomplished by using a two-state quantum apparatus coupled to a bath of many two-state systems. The above treatment demonstrates that for the model considered, 'environment-induced decoherence' is not enough to perform a measurement in the sense of a one-to-one correlation between the system and the apparatus states. We believe that the failure of this scheme has to do with the fact that the measuring apparatus is itself a quantum two-level system which has no well-understood classical distribution for the range of 'pointer states'. For such models, the correlations between the system and apparatus states are transitory as seen above.

A model for spin measurement – The Stern–Gerlach apparatus

Our analysis of Zurek's model motivates us to examine an apparatus which has a well-understood classical limit. We consider a model of the Stern–Gerlach apparatus for spin-1/2 measurements. The measurement of spin here is done via the trajectory (position, momentum) of the spin-carrying particle. The spin constitutes the system and the position/momentum degrees of freedom of the particle, the apparatus. Further, the particle is coupled to the environment via its position. The Hamiltonian of the combined setup is¹⁰

$$H_{SAE} = p^2/(2m) + \lambda\sigma_z + \epsilon x\sigma_z + H_{AE} + H_E. \quad (10)$$

Here x and p denote the position and momentum (taken in one dimension for convenience) of the particle, $\lambda\sigma_z$ the Hamiltonian of the system, ε the product of the field gradient and the magnetic moment of the particle, H_{AE} the interaction of the environmental degrees of freedom with x , and H_E denotes the Hamiltonian for the environmental degrees of freedom. The $\varepsilon x\sigma_z$ term gives the force on the particle, whose direction depends upon the spin direction. The particle spin (system) does not interact with the environment. As before, our aim is to study the time evolution of the reduced density matrix of the system, from which the environmental degrees of freedom have been traced out. Such an equation for the reduced density matrix has been derived in a number of ways earlier¹¹⁻¹⁶. The heat bath (environment) can be visualized as a collection of harmonic oscillators and the Feynman-Vernon influence functional technique¹⁷ can be used to obtain the expression¹⁵ for the reduced density matrix. In the limit of high temperature the expression for the reduced density matrix is then seen to be a solution of the above-mentioned density matrix equation. The high-temperature limit is necessary since we are looking for classical features to emerge as a *measurement*, and earlier work^{13,15} has established the emergence of classical solutions at higher temperatures. The time evolution of the density matrix is studied¹⁰ in the $|s, x\rangle$ representation, where $|s\rangle$ refers to the eigenstates of σ_z and $|x\rangle$ are the position eigenstates. Corresponding to the four elements of the spin space ($\uparrow\uparrow, \downarrow\downarrow, \uparrow\downarrow, \downarrow\uparrow$), the equations for the elements of the reduced density matrix $\rho(x, y, t)$ for our Hamiltonian are:

$$\begin{aligned} \frac{\partial \rho_d(x, y, t)}{\partial t} = & \frac{-\hbar}{2im} \left[\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right] \rho_d(x, y, t) \\ & \pm \frac{i\varepsilon(x-y)}{\hbar} \rho_d(x, y, t) \\ & - \gamma(x-y) \left[\frac{\partial}{\partial x} - \frac{\partial}{\partial y} \right] \rho_d(x, y, t) \\ & - \frac{D}{4\hbar^2} (x-y)^2 \rho_d(x, y, t), \end{aligned} \quad (11)$$

for the spin-diagonal elements $\rho_{\uparrow\uparrow}$ and $\rho_{\downarrow\downarrow}$, with + (-) sign for $\rho_{\uparrow\uparrow}$ ($\rho_{\downarrow\downarrow}$);

$$\begin{aligned} \frac{\partial \rho_{od}(x, y, t)}{\partial t} = & \frac{-\hbar}{2im} \left[\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right] \rho_{od}(x, y, t) \\ & \pm \frac{i\varepsilon(x+y)}{\hbar} \rho_{od}(x, y, t) \\ & - \gamma(x-y) \left[\frac{\partial}{\partial x} - \frac{\partial}{\partial y} \right] \rho_{od}(x, y, t) \end{aligned}$$

$$-\frac{D}{4\hbar^2} (x-y)^2 \rho_{od}(x, y, t) \pm \frac{2i\lambda}{\hbar} \rho_{od}(x, y, t), \quad (12)$$

for the spin-off-diagonal elements $\rho_{\uparrow\downarrow}$ and $\rho_{\downarrow\uparrow}$ with + (-) for $\rho_{\uparrow\downarrow}$ ($\rho_{\downarrow\uparrow}$). Here γ is the Langevin friction coefficient and D has the interpretation of the diffusion coefficient in momentum space. In the case of a heat bath of harmonic oscillators at temperature T , $D = 2\gamma m k_B T$. The physical significance of the solution can be clearly understood if we choose the following Gaussian wave packet of mean momentum \bar{p} and width σ for the initial condition:

$$|\psi(x, 0)\rangle = \frac{1}{(\sigma\sqrt{\pi})^{1/2}} \exp[i\bar{p}x - x^2/2\sigma^2] (a|\uparrow\rangle + b|\downarrow\rangle). \quad (13)$$

This initial state corresponds to the wave packet describing the particle entering the setup. In a recent work¹⁰ we have obtained exact solutions for these equations. The solutions in momentum and position representations are of special physical significance since they are connected to what we would actually observe in the Stern-Gerlach experiment. Consider the solution in the momentum representation, u and v being conjugate to x and y . Identifying $Q = u - v$ and $q = (u + v)/2$, the solution of (11) is

$$\begin{aligned} \rho_d(Q, q, t) = & 2\sqrt{\frac{\pi}{N(\tau)}} \exp \left\{ \frac{-1}{N(\tau)} \left[q + \bar{p}e^{-\tau} \mp \frac{\varepsilon}{\hbar\gamma} (1 - e^{-\tau}) \right. \right. \\ & \left. \left. + \frac{i\hbar Q}{2\sigma^2 m \gamma} e^{-\tau} (1 - e^{-\tau}) - \frac{iQD}{4\hbar\gamma^2 m} (1 - e^{-\tau})^2 \right]^2 \right. \\ & \left. - \left[\frac{\hbar^2}{4\sigma^2 m^2 \gamma^2} (1 - e^{-\tau})^2 + \frac{\sigma^2}{4} \right. \right. \\ & \left. \left. + \frac{D\tau}{2m^2 \gamma^3} (2\tau - 3 + 4e^{-\tau} - e^{-2\tau}) \right] Q^2 \right. \\ & \left. + \left[\frac{i\bar{p}\hbar}{m\gamma} (1 - e^{-\tau}) \mp \frac{i\varepsilon\tau}{m\gamma^2} \right. \right. \\ & \left. \left. \pm \frac{i\varepsilon}{m\gamma^2} (1 - e^{-\tau}) \right] Q \right\}, \end{aligned} \quad (14)$$

where

$$N(\tau) \equiv (D/2\hbar^2\gamma) (1 - e^{-2\tau}) + (1/\sigma^2) e^{-2\tau}, \quad (15)$$

and the solution of (12) in momentum Q and position $r = (x - y)$ is

$$\begin{aligned} \rho_{od}(Q, r, t) = & \exp\left[\frac{-\tau^3 \epsilon^2 D}{3m^2 \gamma^5 \hbar^2}\right] \exp\left[\pm \frac{2i\lambda t}{\hbar}\right] \\ & \times \exp\left\{-\frac{1}{4}\left[\frac{D}{\hbar^2 \gamma}(1-e^{-2\tau}) + \frac{1}{\sigma^2}e^{-2\tau}\right]r^2\right. \\ & + \left[i\bar{p}e^{-\tau} \mp \frac{\epsilon\tau e^{-2\tau}}{\gamma^2 m \sigma^2} \mp \frac{\epsilon D}{2\hbar^2 m \gamma^3}(\tau(1-e^{-2\tau})\right. \\ & \left. - 2(1-e^{-\tau})) \pm \frac{D\epsilon\tau}{\hbar^2 m \gamma}\right]r - \frac{1}{4}\left[\frac{1}{\sigma^2}(1-e^{-\tau})^2\right. \\ & \left. + \frac{D}{2\hbar^2 \gamma}(4\tau - 3 + 4e^{-\tau} - e^{-2\tau})\right]r_Q^2 \\ & + \left[ip(1-e^{-\tau}) - \frac{1}{4\sigma^2}\left(2r \pm \frac{4\epsilon\tau}{\gamma^2 m}\right)(1-e^{-\tau})\right. \\ & \left. - \frac{Dr}{4\hbar^2 \gamma}(1-e^{-\tau})^2 \pm \frac{D\epsilon}{2\hbar^2 \gamma^3 m}(1-e^{-\tau})\right. \\ & \left. \times (\tau(1+e^{-\tau}) - 2) \pm \frac{D\epsilon\tau^2}{2\hbar^2 m \gamma^2} \mp \frac{D\epsilon t}{\hbar^2 m \gamma^3}\right]r_Q\left. \right\} \\ & \times \exp\left[-\left(\frac{Q^2 \sigma^2}{4} + \frac{\epsilon^2 \tau^2 \sigma^2}{\hbar^2 \gamma^2}\right)\right], \end{aligned} \quad (16)$$

where

$$r_Q = Q\hbar/m\gamma \pm 2\epsilon\tau/m\gamma^2 \mp 2\epsilon/m\gamma^2. \quad (17)$$

First we note that $\rho_{od}(Q, r, t)$ approaches zero for large τ as $e^{-A\tau^3}$ for any value of its arguments, establishing that the spatial overlap of wavefunctions corresponding to different spin components vanishes. This happens over a time scale

$$t_s = (3m\gamma\hbar^2/2\epsilon^2 k_B T)^{1/3}. \quad (18)$$

For the diagonal part ρ_d , one sees that as $\tau \rightarrow \infty$, the momentum-off-diagonal components ($Q \neq 0, u \neq v$) vanish, reducing ρ_d to a diagonal form in the momentum space. This happens over a time scale: $t_m = m\gamma/2k_B T Q^2$, where Q is the extent of momentum space off-diagonality. However, this is a classical time scale of momentum relaxation. If one looks at the spatial transform of (14), one can see that¹⁸ the spatial off-diagonal parts rapidly decay on a time scale

$$t_r = \frac{\hbar^2}{2m\gamma k_B T r^2}, \quad (19)$$

r being the spatial separation, even though the density matrix in the position representation does not eventually

become diagonal. The measurement aspects become clear if we look at the momentum distribution function, which corresponds to the diagonal elements¹³ ($Q = 0, u = v$) of eq. (14), as $t \rightarrow \infty$:

$$\begin{aligned} \rho_d(0, u, t) \equiv |\psi(u)|^2 = & 2\sqrt{\frac{\pi}{N(\tau)}} \exp\left\{\frac{-1}{N(\tau)}\right. \\ & \left. \times \left[u + \bar{p}e^{-\tau} \mp \frac{\epsilon}{\hbar\gamma}(1-e^{-\tau})\right]^2\right\}. \end{aligned} \quad (20)$$

The momentum distribution function is centred around $\epsilon/\hbar\gamma$ ($-\epsilon/\hbar\gamma$), which clearly corresponds to up (down) spins. The density matrix of the combined system at any time is

$$\begin{aligned} \rho = & |a|^2 |\uparrow\rangle \langle \uparrow| \rho_{\uparrow\uparrow} + |b|^2 |\downarrow\rangle \langle \downarrow| \rho_{\downarrow\downarrow} \\ & + ab^* |\uparrow\rangle \langle \downarrow| \rho_{\uparrow\downarrow} + a^* b |\downarrow\rangle \langle \uparrow| \rho_{\downarrow\uparrow}, \end{aligned} \quad (21)$$

where $\rho_{\uparrow\uparrow}$ and $\rho_{\downarrow\downarrow}$ are given by (14) and $\rho_{\uparrow\downarrow}$ and $\rho_{\downarrow\uparrow}$ are given by (16). Since $\rho_{\uparrow\downarrow}$ and $\rho_{\downarrow\uparrow}$ go to zero with time as $e^{-A\tau^3}$, and $\rho_{\uparrow\uparrow}$ and $\rho_{\downarrow\downarrow}$ become diagonal in the momentum space, the density matrix (21) at large times is spin-diagonal and also diagonal in momentum. Moreover, the probability weights are in accordance with the quantum-mechanical predictions. Thus, the measurement of momentum has ensured a *measurement* of the spin state of the particle and the 'momentum basis' emerges as the 'preferred basis'^{9, 18, 19}. It can be easily seen from the Fourier transform of (14) that the density matrix in the position representation has off-diagonal elements which do not vanish at long times. However, even here the superposition effects are confined to regions comparable to the de Broglie wavelength of the particle in question, which is of little consequence for a macroscopic particle¹⁸.

The significant aspect of this model of measurement is that it occurs via environmental decoherence which does not happen instantaneously but rather over a finite time. So, it is important to analyse the time scales given by (18) and (19). Both the time scales involve two macroscopic parameters, namely, the temperature T of the environment and the dynamical friction coefficient γ . For a particle larger and more massive than the particles constituting the environment, $\gamma = 6\pi\eta a/m$, where a is the radius of the particle and η the coefficient of viscosity of the medium. So, in this situation the time scales become independent of the mass of the particle. To make an estimate of the time scales for atomic scale particles, we make the following plausible assumptions: $\gamma \cong 10^{12} \text{ s}^{-1}$, $m \cong 10^{-24} \text{ g}$, $T = 300 \text{ K}$, $Q \cong 10^5 \text{ cm}^{-1}$, $\epsilon \cong 1 \text{ eV/cm}$ and $r \cong 100 \text{ \AA}$. This gives $t_r \cong 10^{-17} \text{ s}$, $t_s \cong 10^{-10} \text{ s}$ and $t_m \cong 10^{-9} \text{ s}$.

A possible way to test these ideas is to consider the spin recombination interference experiments in which

the first Stern–Gerlach apparatus (SGA) splits the spin- $1/2$ beam and the second one recombines these split beams in a reversed magnetic field. If in this setup, a certain amount of gas is introduced to decohere the positional wavefunction of the spins, the following consequences emerge. When an x -polarized beam is passed through the first SGA, the beam splits into two z -polarized beams which have partially decohered (depending upon the time of passage). As these beams pass through the second SGA, the recombined beam will no longer be one with x -polarized particles but will contain a statistical mixture of up and down z -polarized beams as well as an x -polarized component. A quantitative study of the effects induced by varying the pressure of the gas may provide an understanding of the decoherence time scales and the decoherence theory in general.

Summary

Our analysis of two specific apparatus models reveals some important aspects of the measurement process. A full quantum-mechanical treatment of Zurek's model shows that although there is dephasing of the apparatus states due to dissipative coupling with the environment, there is no one-to-one correlation between the states of the system and that of the apparatus, and hence no *measurement*. This result is interesting because it suggests that for a measurement to take place it is not enough to have an apparatus coupled to a large environment as it does not ensure correlations between the system and the apparatus states in the reduced density matrix. The reason for the absence of a one-to-one correlation can be traced back to time-dependent state vector of the combined system–apparatus, where the system–apparatus correlations are oscillatory in nature. We feel that the reason for this feature is the absence of a classical limit for the apparatus in the sense of the correspondence principle. Our analysis of the second model, that of a Stern–Gerlach apparatus, strengthens this belief. In this case the apparatus clearly performs a measurement of spin¹⁰, the measuring variable being the particle momentum, which has a well-defined classical distribution. This conclusion is much on the lines of the Copenhagen interpretation², which requires the measuring apparatus to be necessarily

classical. Here we have been able to provide a scheme of incorporating the concept of a 'classical apparatus' in a purely quantum formalism, and demonstrate that a quantum apparatus having a classical limit, when dissipatively coupled to an appropriate environment, does perform a measurement.

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ACKNOWLEDGEMENT. AV acknowledges the financial support from the University Grants Commission, India. DK acknowledges very stimulating discussions with Dr. Ishwar Singh.

Received 1 November 1993, revised accepted 15 September 1994