Neural network models for spatio-temporal associative memory

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General requirements in modelling spatio-temporal associative memory and learning complex sequences are discussed in the context of human memory. A formal definition of the problem of spatio-temporal association employing neural networks is given. A survey of the known models is presented and the models are examined for their closeness to human memory in their ability to learn and recall spatio-temporal patterns. Based on the nature of their recall dynamics, the models are classified as synchronous or asynchronous. In both cases models involving plastic synapses as well as those involving transmission delays are discussed. Sentences in natural languages are viewed as special cases of complex sequences, and learning of rules and constraints of a language is commented upon. A discussion on possible directions for future work concludes the paper.

MODELLING associative memory by means of an assembly of interacting neurons is an active area of research in neural networks. From the time Hopfield[1] proposed a model of a completely interconnected assembly of neurons with associative memory capabilities, a lot of work has been done in analysing his model as well as in extending it[2-4]. Most of this refers to storing and associatively recalling time-invariant or spatial patterns using the standard Hopfield model and its variants[5,6]. Storing a pattern would involve creating a corresponding local minimum in the state space of the model; recalling would mean the model's dynamical descent to and settling in the nearest local minimum. A model's configuration is usually decided by the choice of encoding of the patterns to be stored.

The main problem in such modelling is to calculate the strengths of synaptic interconnections as a function of patterns to be stored. The solutions are called learning rules, the important ones among them being Hebb[7], pseudo-inverse[8], iterative[9] and linear programming[3,10]. In the case of time-invariant patterns the main interest is to maximize both the number of patterns to be stored and the basins of attractions in the state space around each one of the patterns. The linear programming rule helps one achieve that, though unlike Hebb's rule it is not biologically plausible.

Associative memory of time-invariant patterns is interesting and important in its own right; human and animal visual memory falls in this category. However, the more interesting case is that of time-varying or what are known as spatio-temporal patterns. The former can even be considered as a special case of the latter. Human musical memory, motor skills as well as the faculty of speech and language all come under the category of spatio-temporal patterns. Modelling neural networks with spatio-temporal associative capabilities is, therefore, a very challenging and active area of current research. A number of models have been proposed by workers in this field. Some are based on transmission delays while others are based on short-term modifiable or plastic synapses. Though many of these models have been demonstrated to function satisfactorily, their scope is at present rather limited and one is still far away from modelling the prodigious spatio-temporal associative capabilities of an average human brain. Much work needs to be done and new ideas forged before one can do that.

In this paper we follow the development in this field since the early eighties and review most of the known models. The context of human memory and biological plausibility is kept in the background while discussing their capabilities and limitations. To begin with, general requirements of a model for spatio-temporal association are discussed. It is followed by a formal, though restricted, definition of the problem of modelling spatio-temporal association. The known models are classified as synchronous and asynchronous based on their dynamics. In both classes, models based on transmission delays as well as those based on plastic synapses are discussed. The next section discusses some syntactical aspects of natural languages and explores the connection between complex sequences stored in neural networks and valid sentences of natural languages. Some ideas for a new model are suggested for learning rules and constraints of a language from a set of sample sentences. The last section concludes with comments on possible directions for future research in this field.

General requirements

Modelling spatio-temporal associative capabilities of human memory is a problem which is as interesting as it is difficult. Solutions to this problem would help us understand such complex human faculties as making and enjoying music, playing physical games, and hearing and speech. In view of the enormous number of neurons

and their interconnections in an average brain, modelling by simulation is restricted to only toy-level scales. Also, research work in this area has begun only recently. Therefore, most of the known models solve only limited versions of this problem. In fact, the definition of this problem given in the next section is also somewhat restricted. However, here we shall take a broad look at spatio-temporal association from the viewpoint of full range of human memory's capabilities and organization. Besides the spatio-temporal, the spatial aspect will also be kept in view. With this brief prelude, the general requirements are stated below:

(i) The model should be able to store spatial patterns in its state space and recall any one among them with only a content of it being given as initial condition.

(ii) It should store directed sequences of distinct patterns in the state space and recall any sequence step by step with only a content of any one of its patterns being given as initial condition.

(iii) At each pattern in a sequence being recalled, it should persist for a known finite period of time. The last pattern in a sequence may be a fixed point of recall dynamics or may lead to a transition to the first pattern (the former case corresponds to an open sequence and the latter to a closed one).

(iv) It should store and recall combinations of open and closed sequences which share one or more patterns (such combinations are referred to as complex sequences).

(v) It should have two modes of dynamics, namely learning and recall. One need not exclude the other, i.e. both may operate concurrently on the model.

(vi) Special or predesigned events should trigger the model from one mode to the other. In particular, a new input pattern should set the model in learning mode while a content of a pattern learnt previously should set it in recall mode. Also, a successful recall should reinforce learning of the recalled output.

(vii) From the viewpoint of speech and language, the model should be able to learn classes of patterns, such as nouns, verbs, etc., as well as the associated syntactical constraints from a set of sample sentences as sequences. It should generate new valid sequences of members of these classes satisfying the constraints.

(viii) The model's recalling ability should exhibit recency and primary effects. That is, recently learnt patterns must be recalled with greater ease and tolerance; also some patterns should be stored with greater basins or valleys of attractions compared to others, a choice left to the designer.

(ix) The model should be robust, i.e. any damage to a small part of it should not affect its performance drastically. Furthermore, it should be able to work with incomplete and noisy input.

(x) As far as possible the model should be biologically plausible.

With the current level of research and understanding of neural networks, constructing a model to meet all the above requirements will be a tall job. However, many ideas proposed so far in the literature suggest steps in the right direction. In the following sections we will discuss them in the background of the requirements enlisted here.

Before we begin, an introduction to the common ideas and concepts as well as the notations involved in the study of neural networks is in order.

The basic unit in models of neural networks is a neuron, denoted by σ_i, where i is its index. It is either in a firing or in a non-firing (quiescent) state denoted by $+1$ or -1, respectively. The synaptic interaction strength from neuron j to neuron i is denoted by J_{ij}. Positive value for J_{ij} means the synapse is excitatory and negative value means inhibitory. The number of neurons in a model is denoted by N while p denotes the number of patterns in a temporal sequence. The transmission delay from neuron j to neuron i is denoted by τ_{ij}. Some models involve modifiable or plastic synapses, which are denoted by J_{ij}^m. To incorporate architecture-related features, most models have a weight function dependent on delays, denoted by $\varepsilon(\tau_{ij})$.

Coming to the input pattern sequences, Ξ^μ denotes the μth pattern vector among the p patterns; ξ_i^μ denotes the ith bit of the μth pattern. Depending upon the context, bold-faced English letters and capital Greek letters will denote either vectors or matrices. The overlap between μth and νth patterns is denoted by $O_{\mu\nu}$. While recalling a temporal sequence, the model is expected to persist in each pattern for a known period of time denoted by Δ. When the input pattern is indicated as a function of time as $\xi^\mu(t)$, it means that the input corresponds to the μth training session of duration D_μ.

One important aspect in the interface between neurons and their input is the input sensitivity. Denoted by η, it may take any value between 0 and 1 and it could be a function of time.

Lastly, there is the concept of order of a sequence of patterns. Denoted by g, the order of a sequence signifies the minimum number of previous patterns (excluding the current one) required to specify the transition to the next pattern in the sequence. In other words, if the recall dynamics is given a description in terms of a differential equation, then g is nothing but the order of the differential equation minus one. Examples of sequences of order one and two are given in one of the subsequent sections. Sequences of order zero are sometimes referred to as simple sequences.

Definition of the problem

Briefly, the problem of spatio-temporal association may be stated as follows. Given a set of time-dependent

vectors (or patterns) of a specific dimension and duration, how to devise a dynamical model such that the attractors in its state space correspond to these vectors identically? The dynamical model in question is to be designed with two-state neurons as its building blocks. The central problem is, therefore, to compute the synaptic interaction strengths between the neurons with or without introducing time delays in their interactions. A more detailed description of the problem may be obtained by referring to Figure 1.

Three basic components in the model are shown in the figure. They are, respectively, the time-dependent sensory input vectors $\{\xi_i(t)\}$, the set of neural state variables $\{\sigma_i(t)\}$ denoted as internal state, and the set of synaptic interaction strengths $\{J_{ij}(t)\}$. As one can see, there are two types of dynamics which are involved in the design. The first one, which modifies $\{J_{ij}(t)\}$ as a function of $\{\xi_i(t)\}$ and $\{\sigma_i(t)\}$ is called the *learning dynamics*. This dynamics helps in computing the synaptic interaction strengths $\{J_{ij}(t)\}$. The second one, called the *recall dynamics*, changes the internal state of the model from current time step to the next. The learning dynamics has to be designed such that new sensory input patterns become the attractors of the recall dynamics. During recall, the input to the model, which is some part of one of the memorized patterns, determines towards which attractor the recall dynamics will drive the model. Once the model settles down in an attractor, that attractor is said to be recalled for whatever input was impressed on the model to initiate the recall dynamics.

Symbolically, a spatio-temporal pattern (STP) may be defined as STP $\equiv \{\xi_i(t): i = 1, \ldots, N; t \in [0 - D]\}$. A part of an STP, called content, is then $C \equiv \{ \xi_i(t): i = n_1, \ldots, n_2; t \in [t_1 - t_2]\}$, where both n_1 and n_2 are less than N. Both t_1 and t_2 lie between 0 and D, where D is the duration of the pattern. After modifying $\{ J_{ij}\}$ as a function of one or more STPs using the learning dynamics, the model must be capable of recalling the required STP given only a content of it as the initializing input for the recall dynamics. In some models of neural networks an explicit phase of learning may not be present. Instead, one computes the synaptic strengths as functions of STPs using specific formulas.

The recall dynamics in neural network models is simple and well known and is defined as follows, ignoring delays in interactions:

A local field $h_i(t)$ at neuron i is computed as

$$h_i(t) = \sum_{j \in C(i)} J_{ij}(t) \sigma_i(t), \tag{1a}$$

where $C(i)$ denotes the subset of neurons which are connected to neuron i.

The new state at neuron i is then obtained using

$$\sigma_i(t + \delta t) = \text{sgn}(h_i(t)). \tag{1b}$$

If all the neurons are updated simultaneously at periodic times, the dynamics is called synchronous. On the other hand, if the neurons are picked at random times for updating, the dynamics is asynchronous. Equation (1b) applies to the case of zero 'temperature'. When the temperature T is not zero, one may use Glauber dynamics as indicated below instead of (1b):

$$\text{Prob}\,[\sigma_i(t + \delta t) = \pm 1] = (1/2)\,[\,1 \pm \tanh(\beta\,h_i(t))], \tag{1c}$$

where $\beta = 1/kT$ and k is a suitable scaling factor. In the context of human memory, a non-zero temperature can be used to model the effects of synaptic noise. The class of subsets $\{C(i)\}$, which are functions of neuron i, is dependent on the architecture of the model. In the standard Hopfield model each $C(i)$ is equal to the entire set of neurons excluding neuron i.

As shown in Figure 1, the main variables which influence the outcome of the recall dynamics are the synaptic strengths $\{J_{ij}(t)\}$. Therefore, it is the design of the learning dynamics as well as the architecture which is the central problem in the design of the model. There are a few general guidelines which one may follow in its solution. Some of these guidelines are restrictive while others enhance the options for a designer. Firstly, the modification of J_{ij}, the synaptic connection
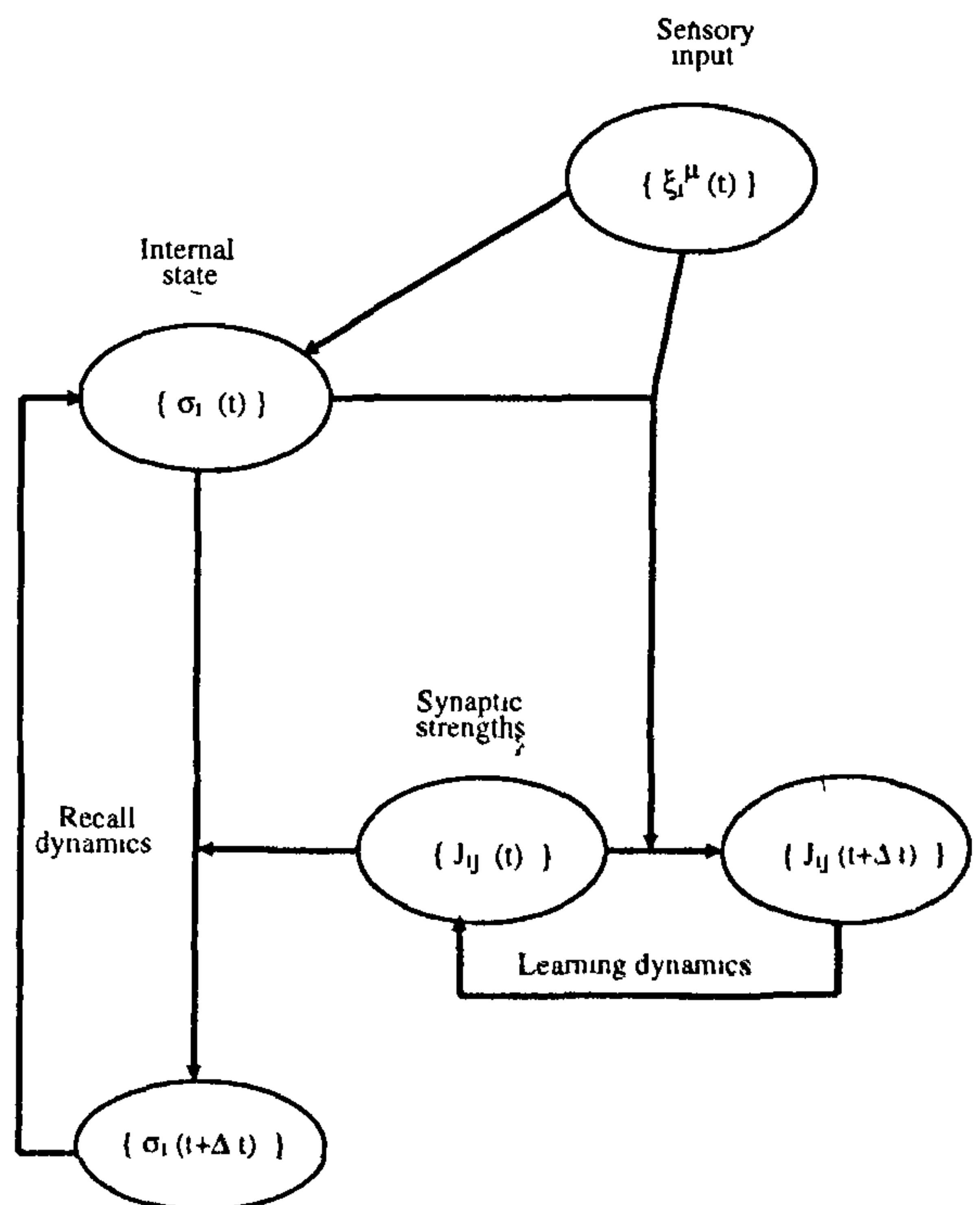

Figure 1. Schematic dynamic model of spatio-temporal associative memory. See text for a detailed explanation

from neuron j to neuron i, may be determined purely by what happens in neurons i and j. This is known as Hebb's hypothesis[7]. Secondly, some of the J_{ij}'s may be presumed to change during recall dynamics, implying that synapses need not remain hard during recall. Thirdly, delays in interactions do exist in natural neural networks either due to transmission or due to the presence of the so-called interneurons. To learn and recall correlations between patterns at various instances of time, such delays may be made use of. One may even consider the possibility of modifying the amount of delays as part of the learning dynamics.

To sum up, the problem of designing a model of spatio-temporal association is one of computing the synaptic interactions $\{J_{ij}\}$ as a function of STPs, which need to be embedded as attractors in the model's state space. The process of computing $\{J_{ij}\}$ is called the learning dynamics, which may follow certain guidelines. Given the contents of any STP as initial conditions, the recall dynamics should drive the model into the corresponding attractor. We begin with a survey of synchronous models.

Synchronous dynamical models

Extension to the standard Hopfield model

One of the first attempts to store simple sequences of unbiased patterns in a neural network was made by Hopfield[1]. Let

$$J_{ij}^{(1)} = \frac{1}{N} \sum_{\mu=1}^{p} \xi_i^\mu \xi_j^\mu \tag{2}$$

provide stability to the μth pattern. The transition to the $(\mu+1)$th pattern can be effected by the 'forward projecting' synapses

$$J_{ij}^{(2)} = \frac{\varepsilon}{N} \sum_{\mu=1}^{p} \xi_i^{\mu+1} \xi_j^{\mu} \tag{3}$$

The local field is then given by

$$h_i(t) = \sum_{j} (J_{ij}^{(1)} + J_{ij}^{(2)}) \, \sigma_j(t) \tag{4a}$$

$$= \sum_{\mu=1}^{p} (\xi_i^\mu + \varepsilon \xi_i^{\mu+1}) \, m_\mu(t), \tag{4b}$$

where

$$m_\mu(t) = \frac{1}{N} \sum_{i} \xi_i^\mu \sigma_i(t). \tag{5}$$

Assuming that the model is in state Ξ^ν and $p \ll N$,

$$h_i(t) \cong \xi_i + \varepsilon \xi_i^{\nu+1}. \tag{6}$$

Therefore for $\varepsilon > 1$, the model will make a transition to $\Xi^{\nu+1}$. The dynamics of recall must, of course, be synchronous. With asynchronous dynamics, transitions to adjacent patterns are known to get mixed up, leading to a failure in recalling any stored sequence. In fact, some models which we shall discuss under the asynchronous category later are essentially borne out of attempts to remedy this problem[5].

The pseudo-inverse method

In this section we discuss the model proposed by Guyon et al.[11]. First we consider the case of simple sequences, wherein the patterns in a given sequence are all distinct. Let the patterns be $\{\Xi^\mu : \mu = 1, \ldots, p\}$. The required transitions are

$$\Xi^\mu \rightarrow \Xi^{\mu+1} \equiv \Xi^{\mu+} \text{ (say)} \tag{7}$$

for $\mu = 1, \ldots, p$. To store an open sequence, $\mu + 1$ may be set to p when $\mu = p$. For a closed sequence, $\mu + 1$ may be set to 1 when $\mu = p$. The transitions (7) are ensured if J satisfies

$$J \Xi^\mu = \Xi^{\mu+} \quad \text{for } \mu = 1, \ldots, p \tag{8a}$$

or, equivalently,

$$J \Xi = \Xi^+, \tag{8b}$$

where Ξ is the pattern matrix whose columns are Ξ^μ's. The general solution of (8) is

$$J = \Xi^+ \Xi^I + B(I - \Xi^I \Xi), \tag{9}$$

where Ξ^I is the pseudo-inverse[12] of Ξ and B is an arbitrary matrix of dimension $N \times N$.

$$\Xi^I = (\Xi^T \Xi)^I \Xi^T.$$

Therefore, irrespective of the dimension of Ξ one needs only to find the inverse of $(\Xi^T \Xi)$, which is a $p \times p$ square matrix. In fact, $\Xi^T \Xi = O$ (overlap matrix). Thus,

$$J = \Xi^+ (O)^I \Xi^T \tag{10}$$

taking B to be the null matrix. Using this approach a maximum of N transitions can be stored. The N transitions may all correspond to a single simple sequence or many smaller simple sequences. The fact that no two Ξ^μ's may be identical ensures that O will be nonsingular.

In complex sequences of order one or more, transition from the current to the next pattern cannot be decided

by current pattern alone. Two examples of complex sequences are given in Figure 2.

The most general formulation to learn and recall complex sequences is

$$h_i(t) = \sum_{l=0}^{g} \sum_{j} J_{ij}^{(l)} \sigma_j(t-l)$$

$$+ \sum_{l,l'} \sum_{j,j'} J_{i,jj'}^{(l,l')} \sigma_j(t-l) \sigma_{j'}(t-l')$$

$$+ \cdots$$

$$+ \sum_{j_0,\ldots,j_g} J_{i,j_0,\ldots,j_g}^{(0,\ldots,g)} \sigma_{j_0}(t) \sigma_{j_1}(t-1) \cdots \sigma_{j_g}(t-g).$$
(11)

For order $g = 1$ equation (11) reduces to

$$h_i(t) = \sum_{j} J_{ij}^{(0)} \sigma_j(t) + \sum_{j} J_{ij}^{(l)} \sigma_j(t-l)$$

$$+ \sum_{j,j'} J_{i,jj'}^{(0,0)} \sigma_j(t) \sigma_{j'}(t)$$

$$+ \sum_{j,j'} J_{i,jj'}^{(1,1)} \sigma_j(t-1) \sigma_{j'}(t-1)$$

f [a b c d a e f] a

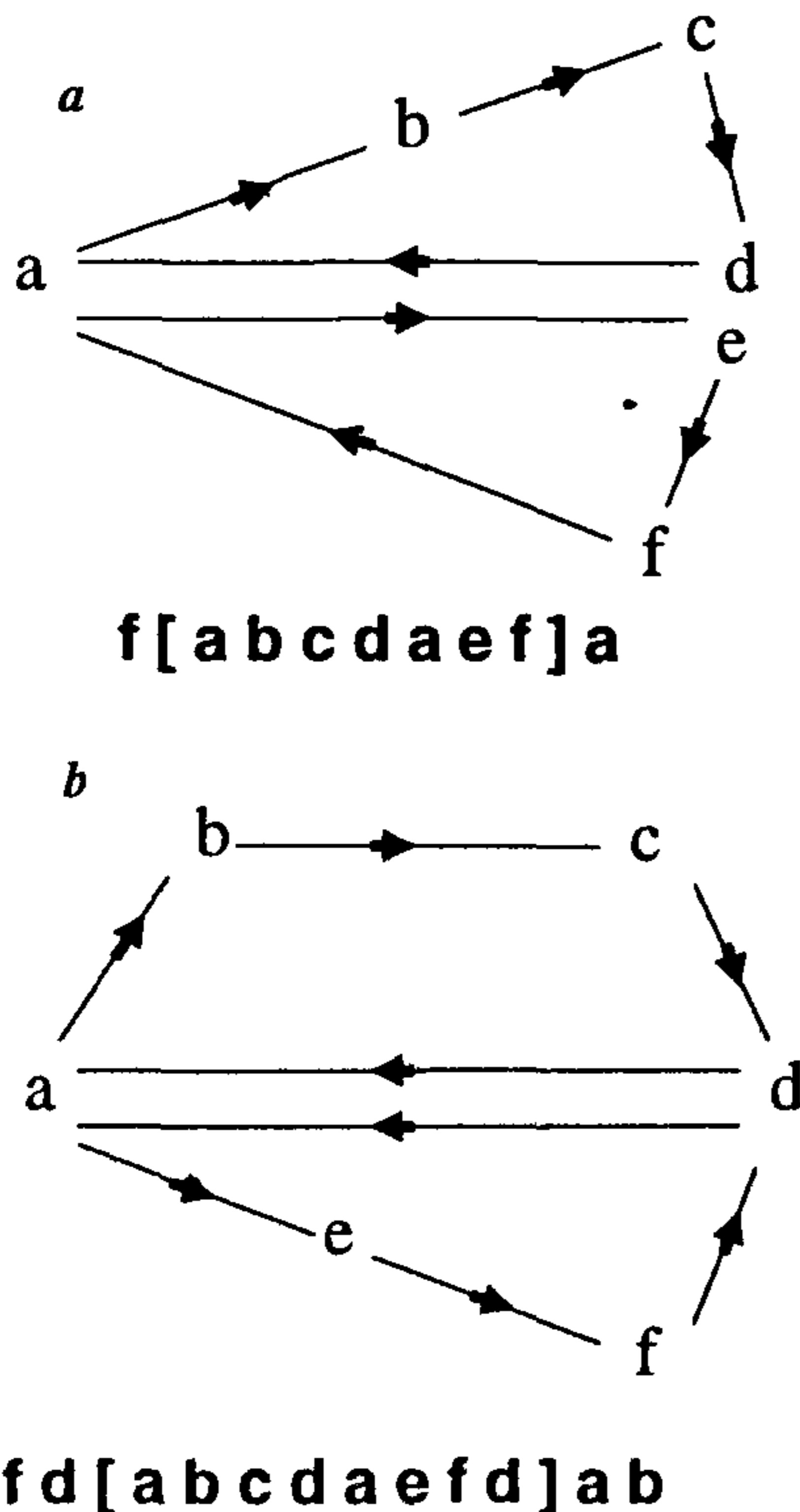**f d [a b c d a e f d] a b**

Figure 2. *a*, Complex sequence of order one with six different patterns; *b*, complex sequence of order two.

$$+ \sum_{j,j'} J_{i,jj'}^{(0,1)} \sigma_j(t) \sigma_{j'}(t-1).$$
(12)

Two simple solutions concerning (12) may be obtained by selectively ignoring some summands in its right-hand side (RHS).

Linear local field: In this solution, all summands which are quadratic in σ's in (12) are ignored. As a result, equation (12) can be written in a compact form using $\gamma_a(t)$ to denote the remaining two linear terms on its RHS as follows:

$$h(t) = \sum_{a} J_{ia} \gamma_a(t)$$
(13)

or

$$\mathbf{h}(t) = \mathbf{J} \Gamma(t),$$
(14)

where

$$\Gamma(t) = \begin{bmatrix} \Sigma(t) \\ \Sigma^-(t) \end{bmatrix}.$$

The required transitions are still represented by (7); therefore, $\mathbf{J}$ may be computed by solving

$$\mathbf{J} \Gamma^\mu = \Xi^{\mu+1} \quad \text{for } \mu = 1,\ldots,p,$$
(15a)

where

$$\Gamma^\mu = \begin{bmatrix} \Xi^\mu \\ \Xi^{\mu-1} \end{bmatrix}.$$

An equivalent matrix formulation of (15a) is

$$\mathbf{J} \Gamma \equiv \mathbf{J} \begin{bmatrix} \Xi \\ \Xi^- \end{bmatrix} = \Xi^+.$$
(15b)

A particular solution of $\mathbf{J}$ from (15) is

$$\mathbf{J} = \Xi^+ \Gamma^I$$

$$= \Xi^+ (\Gamma^T \Gamma)^I \Gamma^T,$$
(16)

where Γ^I and Γ^T are the pseudo-inverse and transpose of Γ, respectively.

Quadratic local field: All terms linear in σ's in the RHS of (12) are ignored in this solution. Besides, the synchronous correlations between σ's, i.e. terms such as $\sigma_j(t) \sigma_{j'}(t)$, are also ignored. Therefore, (12) reduces to

$$\mathbf{h}(t) = \mathbf{J} \Gamma(t) = \mathbf{J} [\Sigma(t) \otimes \Sigma(t-1)],$$
(17)

with $\otimes$ denoting the termwise product. Using transitions (7)

$$\mathbf{J} \Gamma^\mu \equiv \mathbf{J} [\Xi^\mu \otimes \Xi^{\mu-1}] = \Xi^{\mu+1}$$
(17a)

or

$$J\Gamma \equiv J[\Xi \otimes \Xi^-] = \Xi^+. \tag{17b}$$

As in the case of linear field, a particular solution of (17) is

$$J = \Xi^+\Gamma^i = \Xi^+[\Gamma^T\Gamma]^i\Gamma^T. \tag{18}$$

The above approach can be generalized for higher-order sequences in a straightforward manner.

Though lacking in biological plausibility, this model solves the problem of spatio-temporal association in a most direct manner. It does not have any explicit learning dynamics; the synaptic strengths are calculated directly by formulas. It can store and recall complex sequences, and would be acceptable for anyone looking for an engineering solution. Guyon et al.[11] point out a problem in identifying the basic units in a sequence. In learning any verse as a complex sequence, one has four choices of basic units, viz. letters, words, lines and sentences. The larger the basic unit, the smaller is the number of transitions, the smaller the order sometimes but larger becomes the network size. Words, perhaps, are the natural basic units in the case of natural language sequences, as we argue in a subsequent section.

Optimal stability

The pseudo-inverse approach ensures large, but not necessarily maximal, basins and valleys of attraction around sequences to be stored. Employing the method of linear programming is one way of maximizing the sizes of these basins and valleys of attractions.

Given the p patterns and the associated transitions, the conditions for learning them can be set down as[11]

$$\sum_a J_{ia}\gamma_a^\mu \xi_{ai}^{\mu+} > k \quad \text{for } i = 1, \ldots, N \text{ and} \\ \mu = 1, \ldots, p, \tag{19}$$

where k represents the size of the basin or valley of attraction. The pN inequality constraints (19) are linear in $\{J_{ia}\}$. Therefore, the method of linear programming can be applied[3,10] to find the optimal value for k. As a by-product one can obtain $\{J_{ia}\}$.

Alternatively, one may use an iterative approach à la Gardner[9] to compute $\{J_{ia}\}$ starting from a desired value for k.

Modelling with plastic synapses

In majority of models the synaptic strengths get modified only during the operation of learning dynamics. During recall they stay invariant. However, there is evidence that some synapses do change even during recall dynamics. Alternatively, one can say that the model

learns even while it is recalling. Dehaene et al.[13] propose a model for spatio-temporal association based on synapses which get modified during recall.

In their model, the basic units are clusters which are made up of several neurons. Within a cluster, these neurons are interconnected by excitatory synapses. The activity level of a cluster is a continuous variable $\zeta_i \in [0\text{--}1]$. In practical applications, clusters may denote words, notes, etc. Clusters are interconnected by inhibitory and modulating bundles of synapses. Thus, J_{ij} is inhibitory when $i \neq j$ and excitatory otherwise; J_{ij}^m denotes a modulated synapse from cluster j to the cluster i. The local field is computed using

$$h_i(t) = \sum_j J_{ij}\zeta_j(t) + \sum_{j \in m(i)} J_{ij}^m(t)\,\zeta_j(t), \tag{20}$$

where $m(i)$ denotes a set of clusters having modulated connections with cluster i. The updating of $\zeta_i(t)$ is done by

$$\zeta_i(t + \Delta t) = \frac{1}{1 + \exp(-h_i(t))}. \tag{21}$$

The modulated synapses themselves undergo the following dynamics depending on the activity of the cluster (here k) which influences the modulation:

$$J_{ij}^m(t + \Delta t) = \begin{cases} \alpha_p\,J_{ij}^m(t) + (1 - \alpha_p)\,L_{ij}(t) & \text{if } \zeta_k(t) > 0.5, \quad (22a) \\ \alpha_d\,J_{ij}^m(t) & \text{if } \zeta_k(t) \le 0.5, \quad (22b) \end{cases}$$

where

$$\alpha_p = \exp(-\Delta t/T_p)$$

is called the rate of potentiation and

$$\alpha_d = \exp(-\Delta t/T_d)$$

is called the rate of decay. T_p and T_d are parameters of the model. Physically, equation (22) means that $J_{ij}^m(t)$ tend towards a current maximum $L_{ij}(t)$ if activity of the modulating cluster is high, else it decays to zero.

The current maximum $L_{ij}(t)$ is also subject to change as follows. If

$$J_{ij}(t - 2\Delta t)\,\zeta_j(t - 2\Delta t) > 0.5\,L_{ij}(t)$$

then

$$L_{ij}(t + \Delta t) = \begin{cases} \beta_1 L_{ij}(t) + (1 - \beta_1)L & \text{if } \zeta_i(t) > 0.5, \quad (23a) \\ \beta_2 L_{ij}(t) & \text{if } \zeta_i(t) \le 0.5, \quad (23b) \end{cases}$$

where β_1 and β_2 are the normalized rate constants and are parameters of the architecture. L is the absolute limiting value for modulated synaptic strengths. Physi-

cally, (23) means that if J_{ij} and ζ_j at two time steps earlier were high and the activity of i is high now, then increase L_{ij}, else decrease L_{ij}. This is indeed a kind of Hebb's principle.

With these dynamics, let us see the action of a synaptic triad illustrated in Figure 3. Let cluster j's activity be high ($\zeta_j(t) \cong 1$), i's activity be low ($\zeta_i(t) \cong 0$) and k's activity be medium high ($\zeta_k(t) > 0.5$). Also, let $L_{ij}(t) > J_{ij}$ and J_{ii} and J_{jj} be small. Then as time increases, $J_{ij}^m(t)$ will increase towards $L_{ij}(t)$. After a certain period, $J_{ij}^m(t) > J_{ij}$. Consequently, $\zeta_i(t)$ will tend towards 1. Through inhibitory J_{ji}, $\zeta_j(t)$ will tend towards zero. Thus, activity will be propagated from cluster j to cluster i. Simple sequences can be stored and recalled with a ring of such triads. Sequences of higher order can also be stored and recalled as demonstrated by Dehaene et al.[13] using suitable interconnections between clusters. However, general guidelines for configuring an architecture for specific complex sequences are yet to be identified.

The major criticism against this model is that it does not genuinely learn any STP[14]. If the input matches with any one of the latent attractors, then the model merely performs a selection. Inputs which do not match any of the latent attractors are neither learnt nor recalled. However, the concept of modifiable synapses used in this model is interesting and merits further studies.

Approach based on transmission delays

One way of learning temporal relationships in time-varying sensory input is to compute autocorrelations of the input with respect to time. Herz et al.[15] point out that in the presence of delays due to axonal transmission,

among other types, Hebb's principle indeed leads to the computation of such autocorrelations. As a result, the network can learn to store both spatial and temporal information about the input in its synapses. The central idea in a model proposed by Herz et al.[15] is associating an integral delay with each synaptic connection. They also assume that there may be more than one synaptic connection between any given pair of neurons.

The application of Hebb's rule in the presence of delays is stated as follows:

$$J_{ij}(t+1; \tau) = J_{ij}(t; \tau) + \frac{\varepsilon(\tau)}{N} \sigma_i(t+1) \sigma_j(t-\tau). \quad (24)$$

After p sessions of learning, each of duration D_μ, one may obtain

$$J_{ij}(\tau) = \frac{\varepsilon(\tau)}{N} \sum_{\mu=1}^{p} \sum_{t_\mu=1}^{D_\mu} \sigma_i(t_\mu+1) \sigma_j(t_\mu-\tau). \quad (25)$$

In both (24) and (25) $\varepsilon(\tau)$ is a weight factor useful for incorporating architectural features into the model.

The local field at neuron i is computed using

$$h_i(t) = (1-\eta) \sum_{j=1}^{N} \sum_{\tau=0}^{\tau_{max}} J_{ij}(\tau) \sigma_j(t-\tau) + \eta \xi_i(t). \quad (26)$$

The constant η is the input sensitivity mentioned earlier. Though the inner summation, indexed by τ, refers to all integral values from 0 to τ_{max}, it is not necessary that all $J_{ij}(\tau)$ are nonzeros. In the so-called 'clamped' learning mode η is set to 1 and so
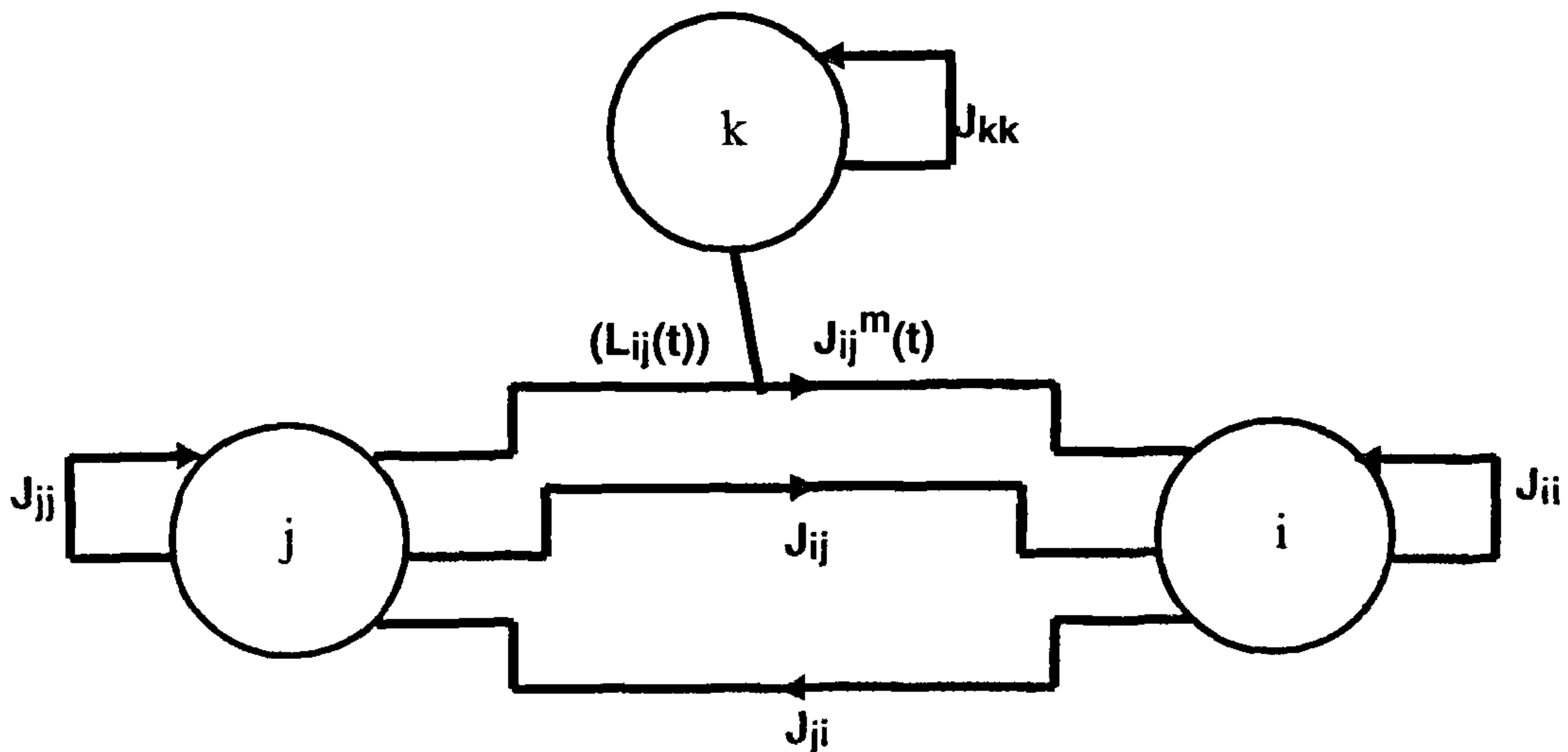
$$\sigma_i(t_\mu) = \xi_i(t_\mu-1). \quad (27)$$

Figure 3. A synaptic triad involving a modulated synapse $J_{ij}^m(t)$ from cluster j to cluster i, with k denoting the modulating cluster

Once the training session, where the J_{ij}'s are updated, is over, the network may be used for recalling stored spatio-temporal patterns.

Herz et al.[15] consider a special case of patterns, i.e. simple cycles of duration D, where

$$\xi_i(t) = \xi_i(t - D). \tag{28}$$

If the weight factors $\varepsilon(\tau)$ are selected such that

$$\varepsilon(\tau) = \varepsilon(D - 2 - \tau) \tag{29}$$

then it is shown that there exists a Hamiltonian for their model as given below:

$$H(t) = -\frac{1}{2} \sum_{i,j=1}^{N} \sum_{a=0}^{D-1} \sum_{\tau=0}^{\tau_{max}} J_{ij}(\tau)\, \sigma_i(t - a)$$
$$\times\, \sigma_j(t - (a + \tau + 1) \bmod D)). \tag{30}$$

The Hamiltonian is a function of states between time $t + 1 - D$ and t; therefore, it is possible that solutions of constant H need not necessarily be fixed points. Indeed one can see that

$$-\Delta H(t) = \sum_{i=1}^{N} [\sigma_i(t) - \sigma_i(t - D)]\, h_i(t - 1). \tag{31}$$

Since $\sigma_i(t) = \mathrm{sgn}[h_i(t - 1)]$, $-\Delta H(t)$ is always positive. Consequently, as $t \to \infty$, $\Delta H(t)$ should tend to zero because H is bounded. The end result is that the model will have to settle in a state with

$$\sigma_i(t) = \sigma_i(t - D) \quad \text{for } i = 1, \ldots, N.$$

The existence of a Hamiltonian enables one to apply statistical mechanics to the study of the model. The application is simplified by defining a higher-dimensional model with ND neurons so that the simple cycles $\{\xi^\mu(t): t = 1, \ldots, D\}$ can be mapped to points in the higher-dimensional model. For that purpose let

$$J_{ij}^{ab} \equiv J_{ij}((b - a - 1) \bmod D) \quad \text{for } a \neq b$$

and

$$J_{ij}^{aa} = 0 \quad \text{for all } a.$$

Then the Hamiltonian of the larger model is given by

$$H = -\frac{1}{2} \sum_{i,j=1}^{N} \sum_{a,b=0}^{D-1} J_{ij}^{ab}\, \sigma_{ia}\, \sigma_{jb}. \tag{32}$$

With the above form for the Hamiltonian, the model

resembles the standard Hopfield kind with ND neurons. Analytical results concerning the critical capacity (ratio of p to N) as a function of D can then be obtained following the approach of Amit et al.[2]. Results from numerical simulations are also reported by the authors. Both are given in Table 1 for a comparison, which is indeed good. Studies imply that uniform distribution of $\varepsilon(\tau)$ leads to optimal values of α_c and m_c.

Overall, the model described here is very promising though storing and recalling complex sequences needs to be examined further. Despite the dynamics being synchronous, learning is biologically plausible. Both spatial and dynamic patterns can be stored and recalled based on one and the same learning principle. Further studies and extensions of this model will indeed be very fruitful.

Asynchronous dynamical models

Models based on asynchronous dynamics are rather limited in depth and variety compared to those based on synchronous dynamics. The chief reason is that the former are more difficult to construct and analyse compared to the latter. Here we shall discuss two different models for storing and recalling simple cycles, one of which is based on modifiable synapses. Both these models are in a way extensions of the first attempt by Hopfield to the case of asynchronous dynamics. Models based on transmission delays will be considered next, followed by some observations on synchronization in the presence of delays.

Transitions induced by noise

Using the idea of forward projection suggested by Hopfield[1], Bhumann and Shulten[16] proposed a model with (0–1) type of neurons. Three types of interactions are computed in their model, namely, excitatory, inhibitory and projectional as shown below:

$$J_{ij}^E = \sum_{\mu=1}^{p} \varepsilon^\mu\, \zeta_i^\mu\, \zeta_j^\mu, \quad \text{where } \varepsilon^\mu = \sum_{i=1}^{N} \zeta_i^\mu, \tag{33a}$$

Table 1.

D	α_c		m_c
	Analytical	Numerical	
3	0.110	0.12	0.95
4	0.116	0.125	0.96

Results from analytical and numerical studies of the model described by equation (32). Analytical results were obtained from an application of equilibrium statistical mechanics. The constant D denotes the duration of simple cycles while α_c and m_c represent the critical capacity and the corresponding threshold overlap.

924

$$J_{ij}^{I} = -\eta \frac{p}{N} \sum_{\mu=1}^{p} \sum_{\nu=1}^{p} \zeta_i^{\mu} \zeta_j^{\nu}, \tag{33b}$$

$$J_{ij}^{P} = \sum_{\mu=1}^{p} (\epsilon^{\mu-1} \alpha^{\mu-1} \zeta_j^{\mu-1} - \epsilon^{\mu+1} \beta^{\mu+1} \zeta_j^{\mu+1}) \zeta_i^{\mu}. \tag{33c}$$

The projective type consists of excitatory forward and inhibitory backward interactions. The final $\{J_{ij}\}$ are computed using

$$J_{ij} = \begin{cases} J_{ij}^{E}, & \text{if } J_{ij}^{E} \neq 0, \\ J_{ij}^{P}, & \text{if } J_{ij}^{E} = 0 \text{ and} \\ \quad \sum_{\mu=1}^{p} (\zeta_j^{\mu-1} + \zeta_j^{\mu+1}) \zeta_i^{\mu} \neq 0, \\ J_{ij}^{I}, & \text{if } \sum_{\mu=1}^{p} \sum_{\nu=1}^{p} \zeta_i^{\mu} \zeta_j^{\nu} = 0, \\ \quad |\mu - \nu| \leq 1. \end{cases} \tag{34}$$

The $\{\sigma_i(t)\}$ are computed using Glauber dynamics adapted for (0–1) representation of neurons. After suitable tuning of α's and β's, the model is capable of transition to successive patterns in the presence of nonzero temperature. The patterns have to be unbiased. The persistence time, Δ, is naturally a function of temperature, α's and β's. Smaller temperatures imply larger Δ's. If the temperature is zero, the model would not be able to make transitions to successive patterns.

However, Nakamura and Nishimori[17] point out that for the case of orthogonal patterns, using

$$J_{ij} = \sum_{\mu=1}^{p} (\alpha \epsilon^{\mu-1} \zeta_j^{\mu-1} + \epsilon^{\mu} \zeta_j^{\mu} - \beta \epsilon^{\mu+1} \zeta_j^{\mu+1}) \zeta_i^{\mu}, \tag{35}$$

it is possible to induce transitions even at zero temperature. As the patterns are orthogonal, the inhibitory term, J_{ij}^{I} in the Bhumann and Shulten model gets excluded. Otherwise (34) and (35) are very similar except for the tuning parameters α and β, which are not functions of μ in (35).

Though these models work for simple sequences, they are rather fragile, having to depend on noise in one case and requiring orthogonality of patterns in the other. Nor are the ideas involved biologically plausible. Further studies with these models may not give rise to any interesting results.

Model with plastic synapses

In another extension to Hopfield's model, Peretto and Niez[18] propose a set of modifiable synapses $J_{ij}^{m}(t)$ besides the ones defined in (2) and (3). The $\{J_{ij}^{m}(t)\}$ are computed by

$$J_{ij}^{m}(t + \Delta t) = \left(1 - \frac{\Delta t}{T_s}\right) J_{ij}^{m}(t) - \frac{\Delta t}{T_c N} \sigma_i(t) \sigma_j(t), \tag{36}$$

where T_s is a relaxation time constant controlling decay of old contributions. T_c determines the rate of destabilization of the current state. With a given history $\{\sigma_i(t - k\Delta t);\ k$ is an integer$\}$, using (36) one can get

$$J_{ij}^{m}(t) = - \frac{\Delta t}{N T_c} \sum_{k=1}^{\tilde{}} R^{k-1} \sigma_i(t - k\Delta t) \sigma_j(t - k\Delta t), \tag{37}$$

where $R \equiv (1 - \Delta t / T_s)$.

Let $\{\sigma_i(t)\} = \{\xi_i^{\nu}\}$ for $t - \Delta t, t - 2\Delta t, \ldots, t - n\Delta t$. Then, from (37)

$$J_{ij}^{m}(t) = - \frac{T_s}{N T_c} (1 - R^n) \xi_i^{\nu} \xi_j^{\nu} - \frac{\Delta t}{N T_c}$$

$$\times \sum_{k=n+1}^{\tilde{}} R^{k-1} \sigma_i(t - k\Delta t) \sigma_j(t - k\Delta t). \tag{38}$$

The first term in (38) with a minus sign destabilizes the current state Ξ^{ν}. As n becomes large, $J_{ij}^{(2)}$, the forward projection term, will dominate over $J_{ij}^{(1)}$, thereby effecting a transition to $\Xi^{\nu+1}$. The persistence time Δ is easily obtained by setting the fields stabilizing Ξ^{ν} and $\Xi^{\nu+1}$ equal:

$$\Delta \cong \frac{(1 - \epsilon)}{p} T_c.$$

The value of ϵ has to be of course less than one.

The patterns have to be unbiased and only simple sequences can be stored and recalled. The model is biologically not plausible in spite of working with asynchronous dynamics. The concept of modifiable synapses is indeed original and needs to be explored further in combination with the ideas of Dehaene et al.[13].

Models with transmission delays

The usefulness of transmission delays has been mentioned earlier while discussing synchronous dynamics models. Here we shall discuss two solutions to the problem of spatio-temporal association based on delays in signal transmission. The first one is yet another extension of Hopfield's idea of forward projection. The second is based on Herz et al.'s proposal[15] in the light of asynchronous dynamics.

Model with a single delay: Using a single delay, τ, associated with the forward projection term $J_{ij}^{(2)}$, Sompolinsky and Kanter[19] propose a model for the storage of simple sequences. The local field $h_i(t)$ is given by

$$h_i(t) = \sum_j J_{ij}^{(1)} \sigma_j(t) + \sum_j J_{ij}^{(2)} \sigma_j(t - \tau). \tag{39}$$

Using overlaps, $m_\mu(t)$'s defined by (15), (39) can be written as

$$h_i(t) = \sum_{\mu=1}^{p} (\xi_i^\mu m_\mu(t) + \varepsilon \xi_i^{\mu+1} m_\mu(t-\tau)). \qquad (40)$$

Assume that the model was in state $\Xi^{\nu-1}$ for $-\tau \le t \le 0$ and it changed its state to Ξ^ν at $t = 0$. Assuming $p \ll N$ and Ξ^μ's are unbiased,

$$m_{\nu-1}(t-\tau) \cong 1 \quad \text{for } 0 \le t < \tau$$

and

$$m_\mu(t-\tau) \cong 0 \quad \text{for } \mu \ne \nu-1.$$

The above equations imply

$$h_i(t) = (1+\varepsilon)\xi_i^\nu$$

soon after transition. That is, both the instantaneous and retarded signals stabilize Ξ^ν till t becomes τ. When $t > \tau$

$$m_\nu(t-\tau) \cong 1$$

giving

$$h_i(t) \cong \xi_i^\nu + \varepsilon \xi_i^{\nu+1}.$$

If $\varepsilon > 1$, there will be a transition to $\Xi^{\nu+1}$ at $t = \tau$. The entire scenario will repeat again to effect a transition to $\Xi^{\nu+2}$ at $t = 2\tau$ and so on.

The persistence time Δ is nothing but τ itself. If the patterns have a common average bias a, then $J_{ij}^{(1)}$ and $J_{ij}^{(2)}$ may be computed using

$$J_{ij}^{(1)} = \frac{1}{N}\left\{1 + \sum_\mu \frac{(\xi_i^\mu - a)(\xi_j^\mu - a)}{1-a^2}\right\}$$

$$J_{ij}^{(2)} = \frac{\varepsilon}{N}\left\{1 + \sum_\mu \frac{(\xi_i^{\mu+1} - a)(\xi_j^\mu - a)}{1-a^2}\right\}.$$

The assumption of a single delay may be questionable, though the model works with asynchronous dynamics. It can be viewed as a special case of the one proposed by Herz et al.[15]. In the latter the delays range between 0 and a maximum of τ_{max}, whereas Sompolinsky and Kanter[19] limit themselves to two delays in their model, namely, 0 and ♦ and, hence, two synaptic connections between every pair of neurons. It is not difficult to convince oneself that a training session employing Hebb's rule with an input $\{\Xi^{\nu(t)}: \nu(t) = \lfloor t/\tau \rfloor \bmod p\}$ will result in computing $J_{ij}^{(1)}$ and $J_{ij}^{(2)}$ automatically. The factor ε is, of course, to be put in externally. To be biologically plausible delays will have to depend on neuronal indices and also a spectrum of values must be allowed for them.

Hebbian learning with delays: In the presence of transmission delays the local field $h_i(t)$ has to be computed by

$$h_i(t) = \sum_j J_{ij}(\tau_{ij}) \sigma_j(t-\tau_{ij}). \qquad (41)$$

The above equation applies only when the signal transmission is soliton-like. If there is dispersion during transmission, then

$$h_i(t) = \sum_j J_{ij}(\tau_{ij}) \int_0^\infty dt' \, \omega(t',\tau_{ij}) \sigma_j(t-t'), \qquad (42)$$

where

$$\omega(t',\tau_{ij}) = \frac{1}{\tau_{ij}} e^{(-t'/\tau_{ij})}$$

models the dispersion. Updating of σ's is done using

$$\sigma_i(t+\Delta t) = \text{sgn}(h_i(t)) \qquad (43)$$

as usual; synaptic noise may be introduced, if necessary, using Glauber dynamics with nonzero temperature. The generalized Hebb's rule in the presence of delays is then given by

$$\Delta J_{ij}(\tau_{ij}; t) = \phi_{ij}(\sigma_i(t+\Delta t), \sigma_j(t-\tau_{ij}), J_{ij}(\tau_{ij}; t)). \qquad (44)$$

The synaptic strength at time t, $J_{ij}(\tau_{ij}; t)$, is included among the arguments of ϕ_{ij} so that saturation effects and, consequently, forgetfulness can also be modelled. Using a simple correlative function for ϕ_{ij} without any nonlinearity,

$$J_{ij}(\tau_{ij}) = \frac{\varepsilon_{ij}}{N} \frac{1}{T_0} \int_0^{T_0} \sigma_i(t)\, dt \int_0^\infty dt' \, \omega(t',\tau_{ij}) \sigma_j(t-t'), \qquad (45)$$

where T_0 is the period of learning.

Equation (45) is nothing but the continuous analogue of (25). It also includes the effects of dispersion. The usefulness of (45) depends on two critical factors: one is the statistical distribution of delays and the other is the rate of change of sensory input. On the one hand, if the sensory input is time-invariant, or varies very slowly, then we are concerned with spatial patterns only. The learning dynamics (45) can compute the synaptic connections appropriately. On the other hand, if the variation of sensory input is too fast, then (45) cannot extract meaningful autocorrelations of the input and so nothing sensible will be learnt. The most useful situation is obtained only when the delays in the models and time constants involved in the input match or resonate with each other. This should be evident from the discussion on the model by Sompolinsky and Kanter[19] in the previous section. When there is a resonance between the delay distribution and time constants of the input,

the time-varying sensory input can indeed be stored as a single spatio-temporal object and recalled later. As pointed out earlier, it is only the auditory input which is fundamentally spatio-temporal and probably well understood from a statistical point of view. The task is then to configure a matching architecture for a model for processing auditory signals. It may even be necessary to consider introducing delays which are not necessarily due to transmission.

Synchronization in the presence of delays: While Herz et al.[15] were pursuing the idea of Hebbian learning with delays, Kerzberg and Zippelius[20] also were studying the use of delays in learning spatio-temporal patterns. Their work has resulted in some important findings regarding the effect of dispersion and delays on synaptic noise. They find that any synaptic noise is effectively countered by dispersion-related integration. Also, combination of delays and the integration leads to a smoother response and resistance to various types of noise. Networks are not too sensitive to sloppy synchronization due to delays.

Complex sequences and natural language

A composition in any natural language can be viewed as a sequence in many ways: as a sequence of letters of its alphabet, or a sequence of words or phrases or even sentences. Clearly, the number of possible phrases or sentences in any natural language is too large for either of them to be considered as basic units. Letters, on the other hand, are too atomic and are also devoid of any semantic content to serve as basic units either. That leaves us with words as the only choice. In fact, research in linguistics suggests that maximal addressability and choice in framing sentences exist only at the level of words[21-23]. As the word-set in any natural language is finite, more than one occurrence of a word in a single sentence or composition is quite normal. Thus, from a limited syntactical point of view a sentence or composition is a complex sequence of words. Now, most of the models for spatio-temporal association discussed earlier concern storing and recalling entire sequences. However, such an approach to store and recall all kinds of sentences is impractical, if not downright meaningless. New models of networks capable of learning rules for choice of words based on syntax and semantics need to be constructed. A study of formal languages and attempts to build models which can recognize valid formal language statements could be useful as intermediate steps towards such an effort.

One major difference between formal and natural languages lies in the size of their terminal symbol set. We have already argued as to why words must be the terminal symbols for a natural language. Compared to the symbol sets of practical formal languages, the word-

set of any natural language is much larger. The words are also classified into so many categories whose interrelationship is indeed very hierarchical[23]. Furthermore, there is this difference between the connotation and denotation of one and the same word in natural languages. Terminal symbols in formal languages, on the other hand, have unambiguous meaning irrespective of where they occur in a sentence. Sensitivity to context is another complex characteristic of natural languages. Though there is a formal definition of languages which are context-sensitive, to apply any such definition to natural languages is beset with problems. The main difficulty is that the number of possible contextual situations in a natural language is again very large. Also, the relationship between a context and the meaning of a word may change from time to time. Therefore, one may not be able to capture a context-sensitive aspect in the form of a time-invariant rule. For instance, how is one supposed to write a rule for the simple requirement in 'good writing' that repetition of words be kept to a minimum. However, in spite of all these difficulties, using concepts from formal language theory for constructing new models of neural networks may prove to be fruitful.

In dealing with natural languages one major problem is the sheer number of basic units involved. One way to tackle the problem would be to limit oneself to a highly restricted domain of the real world. That is, start with a small set of objects and their characteristics. Let the type of sentences also be limited to one or two basic patterns. Then by a combination of syntactical and semantical inputs, it would be possible for a two-component neural network to learn and generate (not recall) meaningful sentences in a limited natural language. A small example to explain this idea is given below.

Perhaps the best way to look at natural language understanding using neural networks is to observe how children learn their mother tongues. To begin with, they learn the meaning of real life objects such as, say, a cat or tree (nouns), and their attributes (adjectives), then simple sentences linking these two types of parts of speech. Suppose that the only sentences a child has heard being spoken are: (a) 'The cat is black' and (b) 'The tree is tall'. From this elementary universe of sentences several syntactical rules can be extracted. Firstly, every sentence starts with the word 'The'. Secondly, the third word of every sentence is 'is'. Thirdly, the second word of any sentence should be from the set {cat, tree}. And, lastly, the final word is from the set {black, tall}. Already the child can learn from these two sample sentences, general concepts such as 'article', 'noun', 'linking verb' and 'adjective'. Key to the generation of meaningful sentences is a spatial type of association between visual representation of an

object and its name, as well as between a characteristic of the object and the corresponding adjective. Such an association is realizable using known models of neural networks. On one bank or component, visual representations could be stored and on another the corresponding verbal equivalent. Visual input on the first will then induce the associated word in the second. The second component has to be of a spatio-temporal nature. Having got into a state denoting a specific word, it should make a transition to the next state which should correspond to a meaningful word, and so on. Now, it is possible that the transition to the next state is uniquely decided by the current state. This situation is, in practice, an exception rather than a rule. More often, two or more branches may exist at many points in any complex sequence. In fact, if the two sample sentences are viewed as complex sequences of words, at the first word 'The' there are two branches; the third word 'is' also has two branches. Whenever the current state does not uniquely define a transition, one has two approaches. The first one is to refer to one or more antecedent states as in the case of dynamical models. Alternatively, output from an external source, which could be an independent dynamical model by itself, can be used to decide the choice among the various branches. Going back to the elementary universe of two sentences, the choice between 'cat' and 'tree' should indeed be made by such an output from the visual associative memory rather than by referring to the past. The same can be said about the choice between the modifiers 'tall' and 'black'. The interaction between spatial and spatio-temporal components required here can be as described in equation (26). The term corresponding to sensory input can play the role of output from the spatial component. One problem would be to find out when to activate this external influence. In other words, what kind of dependence should η have on time? A simple solution is to treat η as a constant. When not required, this external influence will only act as random noise which can be countered by the robustness of the spatio-temporal component. At branch points though, it can help in deciding the choice uniquely.

The above description of a two-component model for sentence generation is, admittedly, sketchy. However, the central idea is probably clear.

Conclusion

Some important concepts which emerge from the review of known models are: transmission delays, plastic synapses and complex sequences. Delays due to transmission are known to exist not only due to the time required for neural signals to travel the axonal length but also due to the presence of the so-called interneurons in their path. In fact, for temporal correlations across

durations of the order of seconds, axonal delays alone may not be sufficient; architectural mechanisms, such as the presence of interneurons or synaptic triads[13] may also be required. The second concept, plastic synapses, is a biologically meaningful one. There are at least two arguments in favour of it. One is the existence of short-term memory, which implies that some synapses get modified more often compared to others. The second argument is based on evidence for the coexistence of learning and recall processes in the human brain. Indeed, the recency effect, exhibited by our brain, may be traced to such a coexistence. Thirdly, the concept of complex sequences provides the basic and complete foundation for defining spatio-temporal patterns or attractors, at least as far as synchronous dynamics is concerned.

Evidently, it is easier to construct and analyse models based on synchronous dynamics compared to those based on asynchronous dynamics. From an engineering viewpoint, there is no reason to insist on asynchronous dynamics. However, no evidence of any synchronization mechanisms exists in living biological systems at the level of neural assemblies. Therefore, for biological plausibility, asynchronous dynamics is fundamental. Nevertheless, one may take a route via synchronous dynamics as far as model making is concerned, even for understanding natural neural systems. Such an approach is exemplified profitably by both Guyon et al.[11] as well as Herz et al.[15].

Among the models reviewed, the ones based on delay and plastic synapses are interesting. Further studies on them should focus on understanding the shapes and sizes of valleys of attractors in their state spaces. For practical applications, it may be desirable that the sizes of valleys of attractors are optimal. Using linear programming to learn patterns would be one way of optimizing the valleys of attractors.

Biological plausibility is important in its own right. Models based on delays appear to be the most interesting in that context. Numerical studies of models based on transmission delays will highlight the possibilities of resonance between the frequency distribution of the sensory input and the distribution of delays. Simulating variations in architectures in the form of limiting interactions to within a neighbourhood will be a step towards incorporating more biological realism in the models. Architectures based on a fractal code could be another variation. In fact, studies on biological growth suggest that concepts from the theory of fractals may be useful in constructing models of neural networks. Learning dynamics in currently known models change only the synaptic interaction strengths. Though it may seem far-fetched, changing even the values of delays during the learning process deserves some attention. The problem of justifying it biologically may be considered later.

A final comment concerns the problem of storing and

recalling words, basic units of natural language sentences. The number of words which an average native speaker is able to access readily is estimated[21] to be about 50,000. This number is rather small compared to the number of neurons and their interconnections in the segment of human brain which deals with speech. Therefore, the method of encoding words can be safely assumed to be sparse. Simulation studies of the two-component model outlined in the previous section may do well to adhere to this aspect of encoding.

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RESEARCH ARTICLES

Overexpression of particulate phosphofructokinase in yeast by *PFK2*

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A glucose-negative strain *pfk1 pfk2* of *Saccharomyces cerevisiae* mutated in both the genes coding for the soluble phosphofructokinase brings about nearly 30-fold overexpression of the particulate phosphofructokinase (PFK II) when transformed with multiple copies of *PFK2*. The overexpression is independent of the dosage of *PFK1*-encoded β subunit. The elevated enzyme activity correlates directly with the increased particulate association of the polypeptide encoded by *PFK2*. Multiple copies of the *PFK2* gene appear to be the only requirement for overproduction of the particulate phosphofructokinase.

THE soluble phosphofructokinase of yeast (PFK I) is a hetero-octameric enzyme, composed of 4α and 4β subunits[1]. This contrasts with the phosphofructokinase found in the human muscle or in *Escherichia coli*, where the activity resides in a single polypeptide. Mutants in both the subunits have been isolated in various laboratories. This led to the identification and isolation of two genes *PFK1* and *PFK2* coding for the β and α subunits, respectively[2-10] (note: the genes *PFK1* and *PFK2* and the encoded polypeptides have been named differently in different laboratories. The nomenclature used here is according to Gayatri and Maitra[11]). Analysis of a large number of *pfk1* and *pfk2* mutants isolated in this laboratory has shown that mutations in the *pfk1* (β) gene abolish the enzyme activity while mutations in the *pfk2* (α) gene invariably give rise to an alteration of the regulatory properties of the enzyme. Thus, the two subunits have distinct roles to play in the hetero-octamer[12,13]. Two mutants that do not conform to this rule are DFY70 (α) and DFY250 (β), which exhibit reverse phenotypes[4,11]. Recently, it has been shown that single-amino-acid changes in either of the two subunits