chaotic, any two nearby similar trajectories are thrown apart. This introduces loss of predictability even for the low-frequency component.

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Possibility of usage of return maps to predict dynamical behaviour of lakes: Hypothetical approach

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In this article, we show how one-dimensional maps can be useful in analysing experimentally the dynamics of lake systems. We illustrate this by means of hypothetical lake systems.

ONE of the ways to make a complex system easier to analyse is by reducing the system to a simple system that still captures the important features of the original system. As the theory of one-dimensional (1-D) maps is well developed in several fields[1–4], it will be useful if an appropriate 1-D map can be constructed from the system under study. In this communication, we demonstrate how an approximate 1-D map can be used to analyse the dynamical behaviour of some simulated hypothetical lakes. The first-order difference equations and the general conditions of the water bodies, and the logistic map analysis for various possibilities are hypothetically described in successive sections.

A treatise by May[1] lucidly explained the role of the first-order difference equations, dynamical properties and bifurcation generations in the application of 'simple mathematical models with very complicated dynamical systems'.

The difference equation can be used for studying a dynamical system as a water body at different time intervals. It is represented as

$$X_{t+1} = F(X_t), \tag{1}$$

where X_t and X_{t+1} are the populations (pixel population in water body) of a natural system at time periods t and $t+1$, respectively. It indicates that output becomes an input feedback, and hence an iterative process. The following expression shows the relation between X_t and X_{t+1}:

$$X_{t+2} = F(X_{t+1}). \tag{2}$$

The magnitude of population at a definite time in a natural system is related to the magnitude of the population in the preceding generation. This can be represented applying the first-order difference equation $X_{t+1} = \lambda X_t(1-X_t)$, or $X_{t+1} = \lambda X_t - \lambda X_t^2$, in which the first term is linear and the second nonlinear. In this equation the term λ will give an idea about the magnitude of variation. This equation defines an inverted parabola with intercepts at $X_t = 0$ and 1, and a maximum value of $X_{t+1} = \lambda/4$ at $X_t = 0.5$. If $\lambda > 1$, it is an indication that the population growth rate is increasing. The parameter λ gives the entire description of the system. The steepness of the inverted parabola in the logistic map depends on λ. If $\lambda < 1$, the population death rate is said to be increasing. The strength of nonlinearity explains the temporal changes in the dynamical system. Till a certain degree of magnitude of nonlinearity, the growth in areal extent of the water body will be attracted to the equilibrium stage specific to that level of nonlinearity. For a magnitude range $\lambda = 3.57$–4, the dynamical system shows chaotic behaviour, revealing that the areal extent of the water body is repelling. All the parameters in the difference equation should be such as to fix the linear term to between 0 and 1, and the strength of nonlinearity[1] to between 0 and 4, failing which the areal extent tends to become extinct. The graphic analysis explains that the normalized status of a dynamical system as water body if starting at larger than 1, it immediately goes negative and becomes extinct at one time step. Moreover, if $\lambda > 4$, the hump of the parabola exceeds 1, thus enabling the initial population near 0.5 to become extinct in two time steps. Therefore, there is a need to restrict the analysis[1] to values of λ between 1 and 4 and values of X_0 between 0 and 1. As the areal extent X_0 of the water body is small (much less than 1 on a normalized scale, where 1 might stand for any number such as 1 million km^2), the nonlinear term can initially be neglected. Then the areal extent at time step (year) $t = 1$ will be approximately equal to λX_0. Figure 1 shows a logistic model and its essential

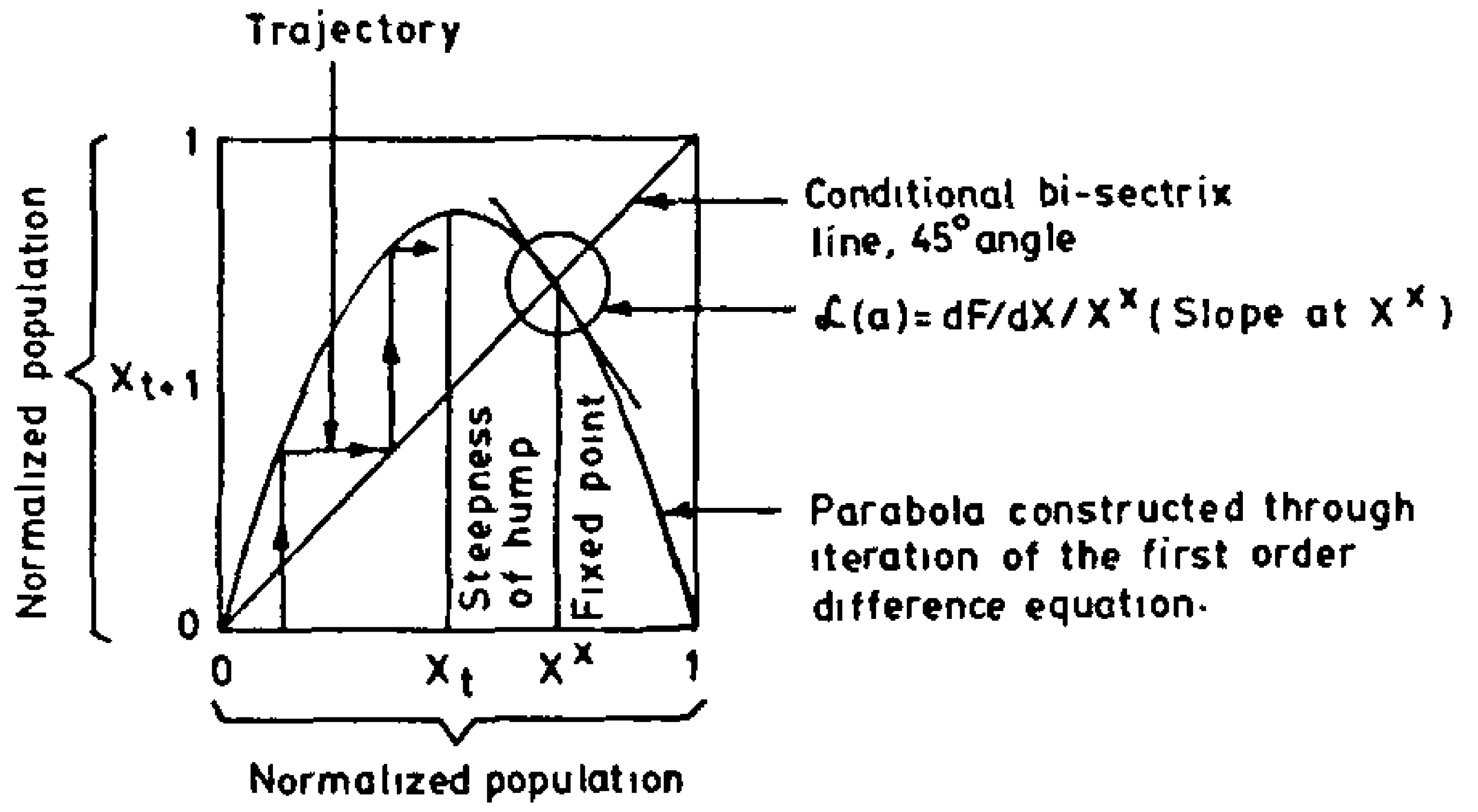

Figure 1. Logistic map and its essential parameters

parameters. The fixed point is the eigenvector (slope), $\lambda(a) = (dF/dX)/X'$, of the bisectrix point of 45° line at X' and determines the local stability of the fixed point $F(X) = X$; fixed point $\lambda(a) < 1$ (Figure 1). If the slope of F at X' lies between 45° and −45°, then $\lambda(a) < 1$, and the fixed point is locally attracting. As the slope steepens beyond −45°, $\lambda(a) > 1$, and the fixed point becomes repelling.

Using these return maps, a qualitative understanding of the dynamics of the logistic maps without performing any calculations can be done[1,5]. It is intended to use these logistic maps to predict the successive values of the areal extent in lakes (population of pixels in discrete space). These logistic maps can be constructed using the first-order difference equations. The water body dynamics can be explained through a single logistic map rather than through conventional algebraic analysis. By tracing the trajectories, it is easy to know how the water body areal extent fluctuates with time.

As lakes exhibit fluctuations, there is a need to classify them according to their dynamical behaviour from stable to unstable to apparently random fluctuations. The magnitude of variation between fixed time intervals is an important parameter for studying the dynamics of a particular system. The water body dynamics vary with time and physiography. Conceptually, the water bodies which exhibit fluctuations within seasons may also show variations at annual intervals. This may be confirmed by approximating lake behaviour within the seasons and at annual intervals by return maps. The logistic model approaches the problem of quantifying lake behaviour from two directions: (i) assessment of external disturbances and (ii) the rate of change in areal extent.

Therefore, it is important to develop a mathematically derived model for a better understanding of the role of the significant parameters and their quantification in terms of decrease in areal extent of lakes. Thus, some water bodies may behave chaotically; some others are attracted either to initial conditions or to a fixed point (equilibrium stage). The decreasing areal extents of water bodies when represented in return or logistic maps attract all the trajectories towards initial status.

The prime reason for retrogression of a natural system towards initial conditions, say behind X_t, can be quantified through logistic maps. This retrogression of a natural system towards its initial status as exemplified by population (natural system) decrease due to deceases (factor)[1], can be represented by two aspects in two logistic maps, one showing population (natural system) decrease and the other intensity of decrease or the factor. Similarly, the areal extents of many lakes and water spread decrease periodically due to meteoro–geophysiographic conditions.

Fluctuations in areal extents of some water bodies are very high and hence unpredictable, and stability is a very rare phenomenon. These fluctuations in water body areal extent can be quantified by means of logistic equations and return maps.

This method of deducing the system's behaviour needs the following parameters: areal extent at specified time intervals and the strength of nonlinearity (λ). To fit a logistic map to a set of observations $X_t, X_{t+1}, \ldots, \lambda$ would be estimated by plotting X_{t+1} versus X_t in order to get the curve $X_{t+1} = \lambda X_t(1 - X_t)$ (personal communication, May and Lloyd, 1994).

In a return map representing the behaviour of fluctuations of areal extents in lakes which are being eutrophied with time, trajectories are attracted towards initial conditions (areal extents will decrease with time). By contrast, in lakes where no fluctuations are recorded, trajectories will be attracted to a fixed point (stable in nature).

A natural system like water bodies behave differently in different seasons. Water bodies represented seasonally in logistic maps show their pixel population progressing towards equilibrium and hence trajectories are also attracted towards equilibrium point during monsoon, whereas during summer the pixel population is attracted towards initial status. Thus, trajectories may be attracted towards equilibrium point or they may repel. These fluctuations can be identified as different levels of the strength of nonlinearity.

In logistic maps the magnitude of variation or the strength of nonlinearity (λ) between initial and final values can be represented in terms of X_t and X_{t+1}, areal extents of water body at time t and $t+1$. Generally, the water bodies tend to show changes in areal extents at annual intervals. If X_t is the initial value, the changes during successive years, $X_{t+1}, X_{t+2}, \ldots, X_{t+n}$, can be determined by temporal evaluation. A similar methodology can be applied for measuring the areal extent during any specific season (summer, rainy or winter seasons).

As the strength of nonlinearity of a system increases, the behaviour becomes more and more unpredictable.

The amount of nonlinearity is said to be high when the ratio between X_{t+1} and X_t is high. As the variation in water bodies is a natural phenomenon, occurring through ages, it is essential to assess its magnitude in temporal sequence. The highest magnitude of variation can possibly occur in water bodies (due to many factors) at specified time intervals t and $t+1$, provided the areal extent at time t is near zero and that at $t+1$ is at its equilibrium state. The reverse phenomenon, i.e. where t has a definite value at $t+1$ becoming zero may also occur. It is essential to fix the parameter representing the amount of nonlinearity λ to between 1 and 4 to construct the one-dimensional return map and to quantify the dynamical behaviour of a nonlinear lake system. For values of $\lambda < 1$ the areal extent always decreases to 0 (as shown for $\lambda = 0.92$ in Figure 5 *b*). The intersection of the parabola with the 45° line at $X_t = 0$ represents a stable fixed point on the maps. However, for $\lambda > 1$ (as shown in Figures 4 *a*, 4 *b*, 6 *a* and 6 *c*) this fixed point becomes unstable. Instead, the parabola now intersects at $X = (\lambda - 1)/\lambda$, which corresponds to a new fixed point.

The strength of nonlinearity should always lie between 1 and 4 for representation in logistic maps, all the natural systems, including water bodies, reaching equilibrium. Alternatively, two cases exist: (a) if $0 < \lambda < 1$, the water body is attracted towards initial conditions and (b) hypothetically, if $\lambda > 4$, the water body reaches extinction. The former can also be represented logistically. Figure 2 shows the strength of nonlinearity (λ) of three different hypothetical lakes, 1, 2, 3, where the strengths of nonlinearity $\lambda_1 > \lambda_2 > \lambda_3$. Thus, the logistic maps aid in the comparison of lake behaviour in temporal sequence.

Temporal satellite data are of significant use in studies on natural systems as water bodies as they show fluctuations conforming to nonlinear mode. They offer synoptic coverage revealing the quantitative involvement of relevant factors and are also available at short time intervals. The simplified mathematical models based on the first-order difference equations can be adopted with remotely sensed data to quantify the lake behaviour.

Different possibilities have been discussed with reference to hypothetical lakes and their possible behaviours are also approximated in terms of return maps.

Figure 3 represents different possibilities of water bodies in different seasons. The magnitude of variation in Figure 3 *a* is computed considering the water body

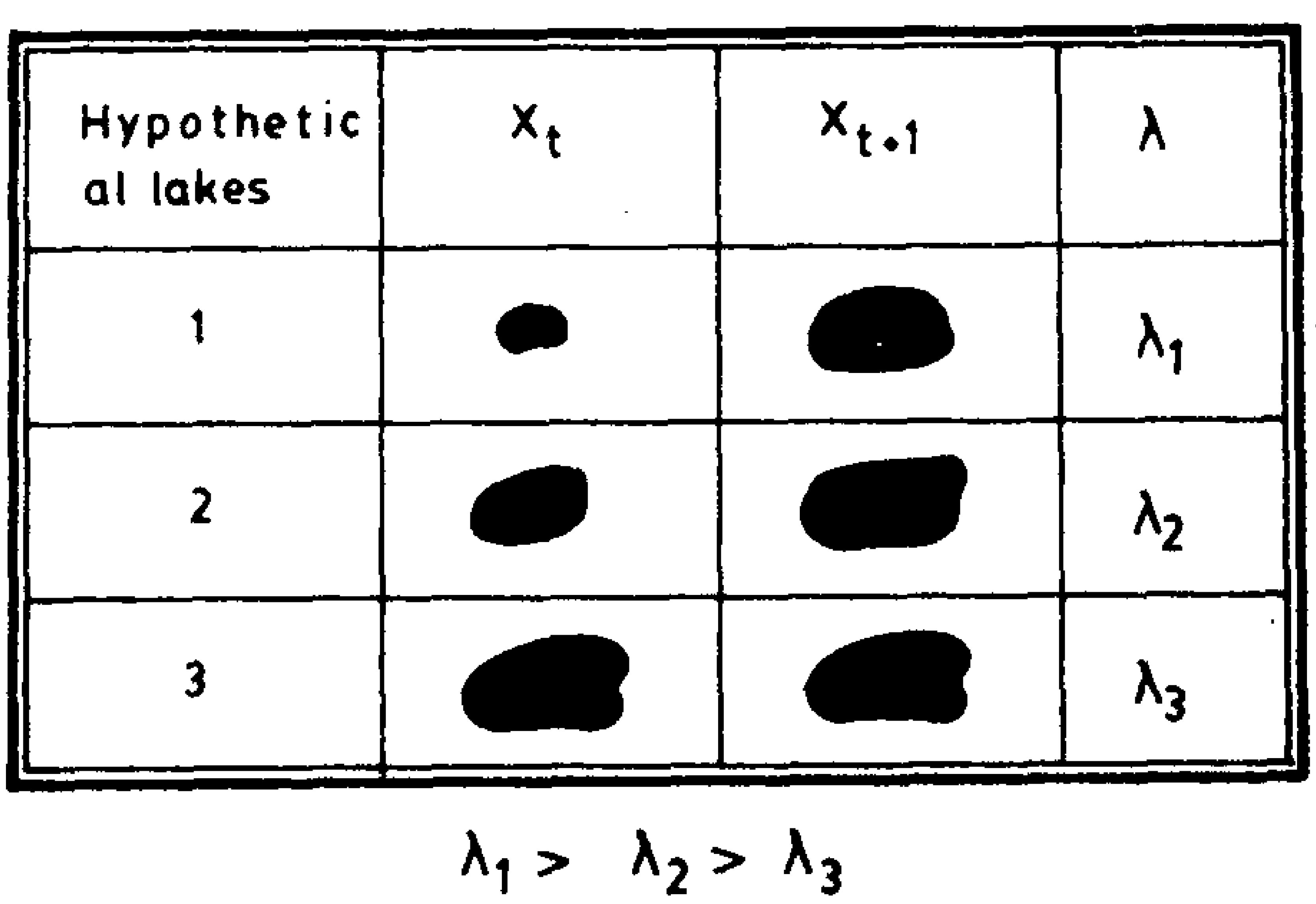

Figure 2. Involvement of the population of water body pixels at different time periods and its strength of nonlinearity.

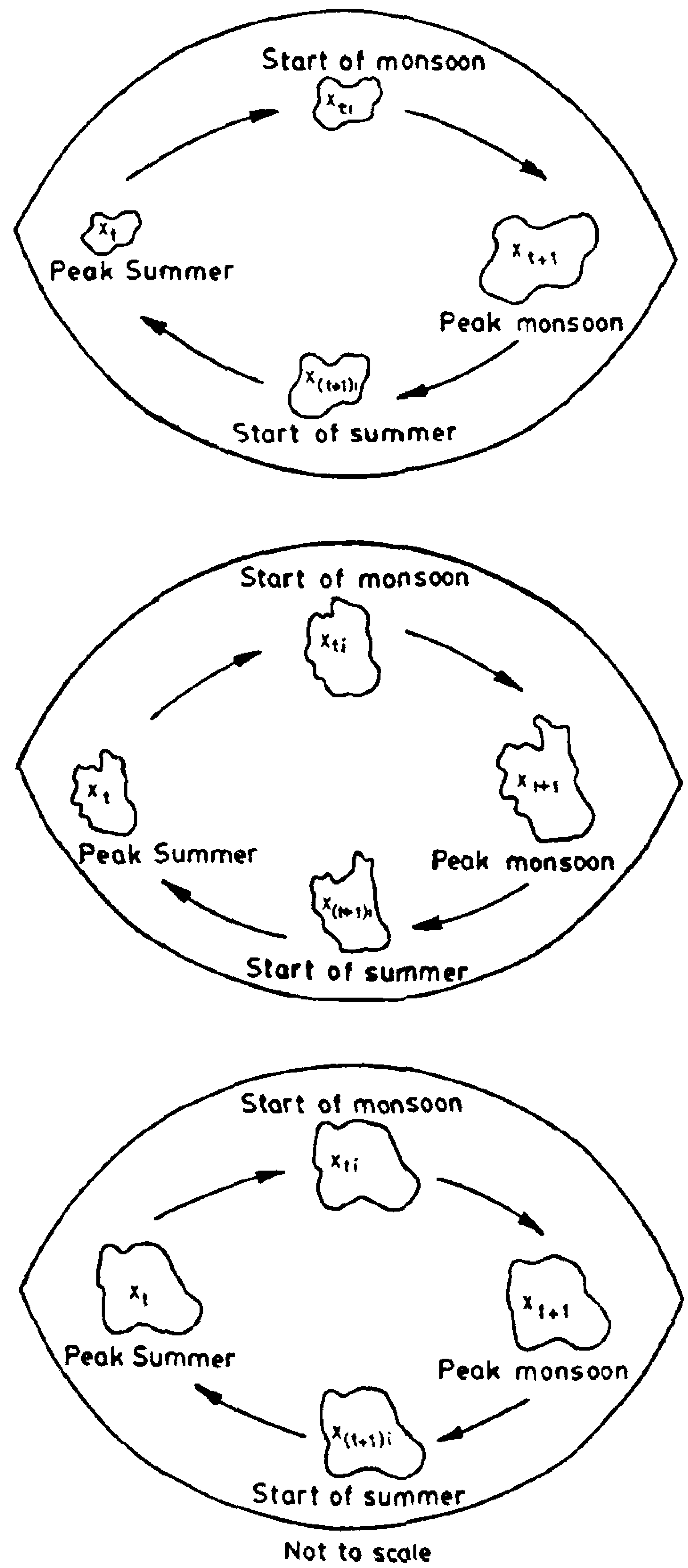

Figure 3. Conceptual cycle of water body behaviour of different regions.

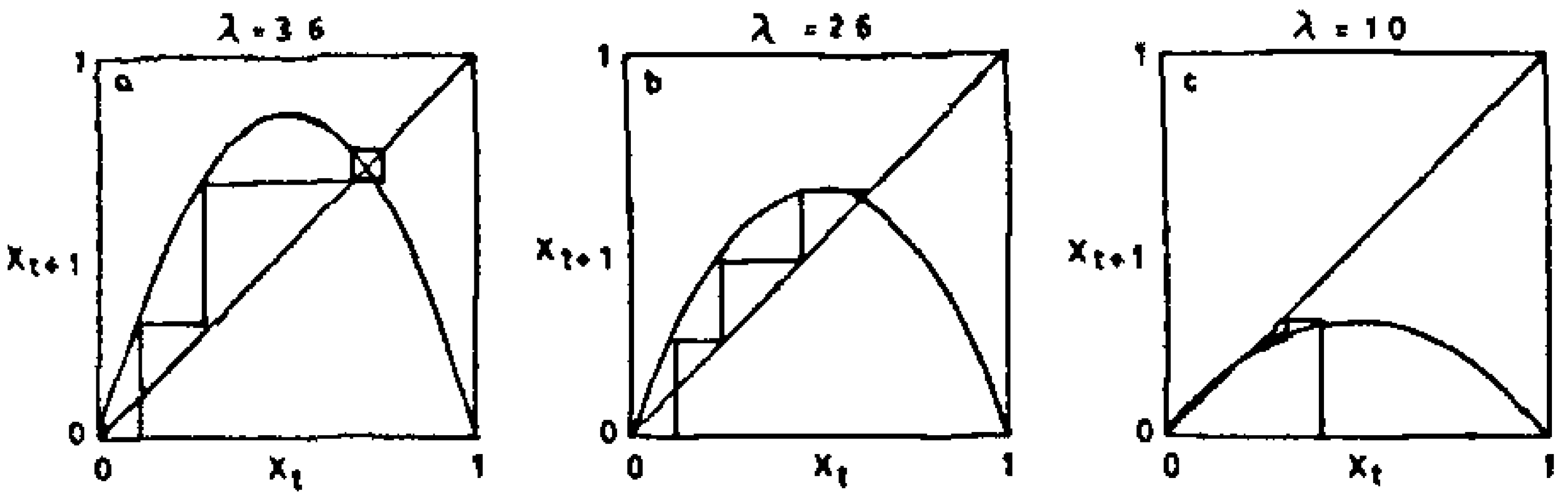

Figure 4. The return maps constructed by taking the areal extents from the possibilities given in Figure 3 and the computed strength of nonlinearity into account. X_t and X_{t+1} are populations of the water bodies at peak summer and peak rainy season, respectively

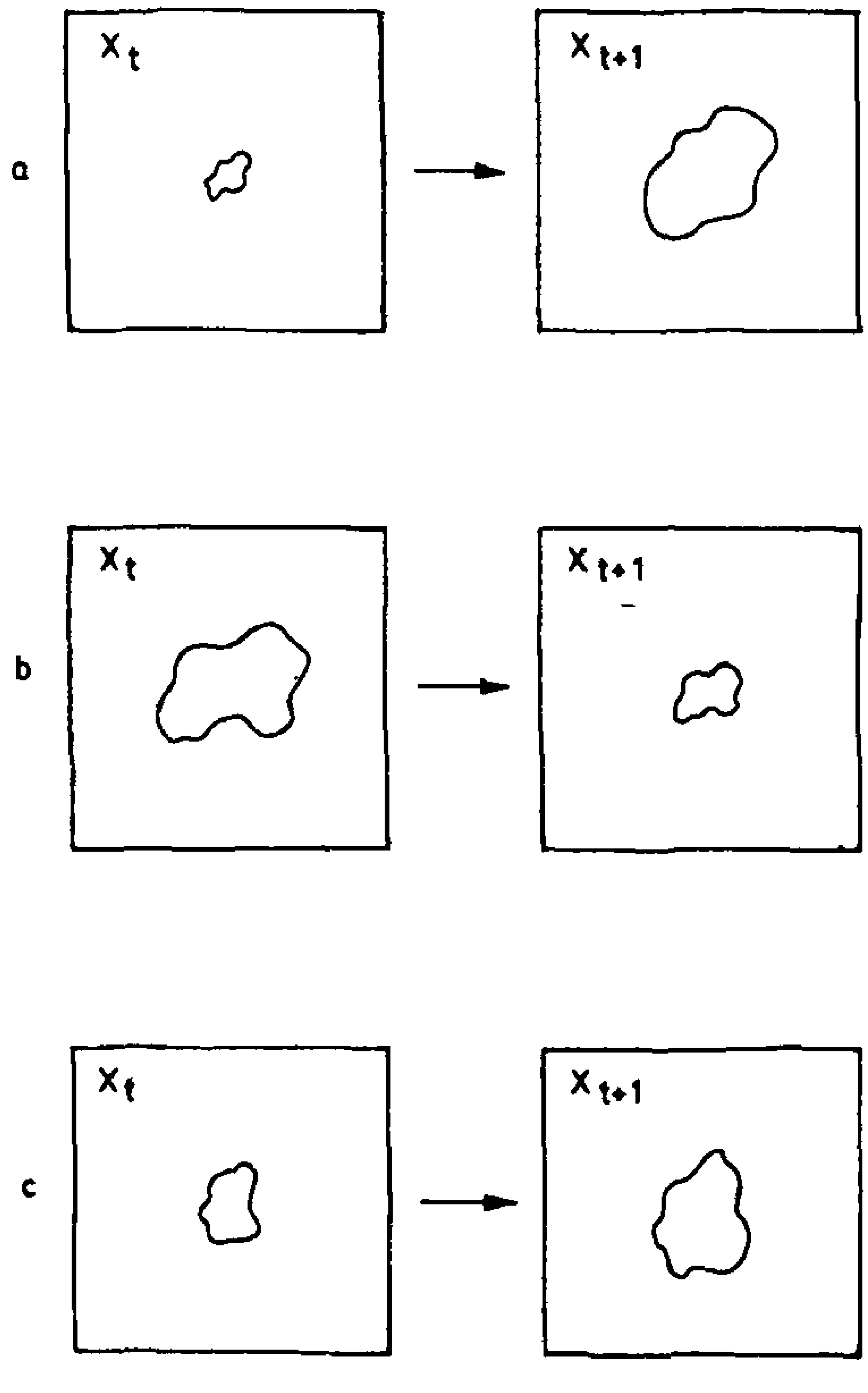

Figure 5. The hypothetical water bodies at time t and $t+1$ of two different years *a*, the magnitude of variation is > 3.8, and *b*, magnitude of variation is < 1; and *c*, the amount of nonlinearity is exactly 2.

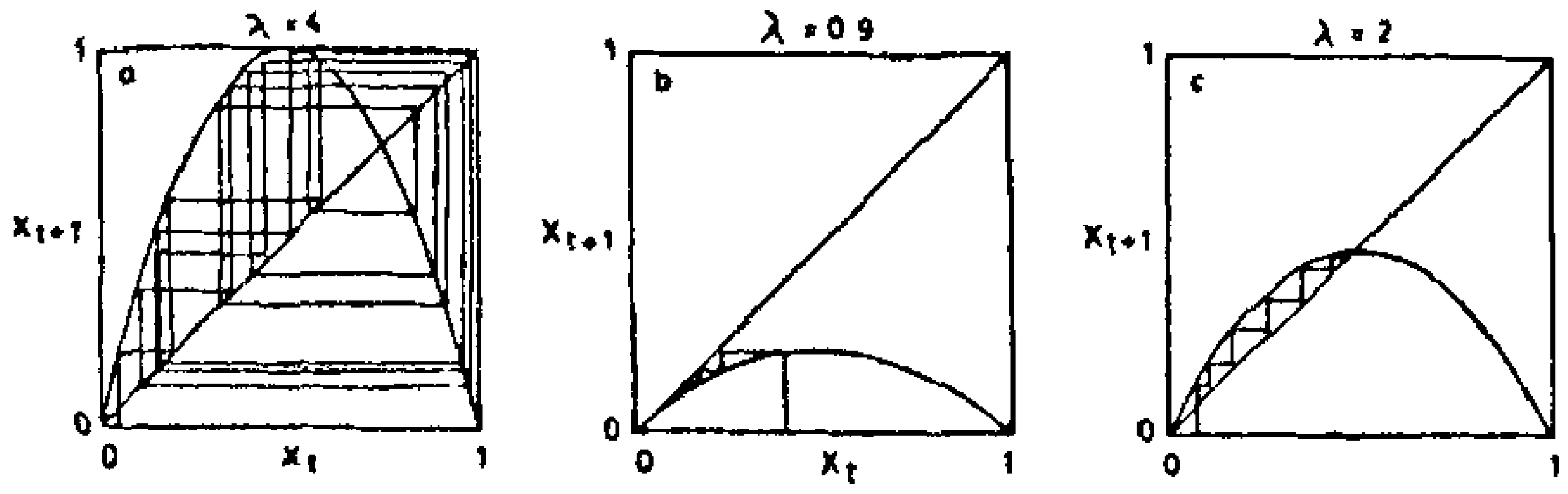

Figure 6. Return maps plotted

areal extents at peak summer (X_t) changing via X_{t+1} during the advent of monsoon to X_{t+1} at the end of monsoon. The trajectory during this course is attracted to a fixed point (Figure 4 *a*). Reverse course is possible because of seasonal cycle. The areal extent of the water body at peak rainy season is X_t and at peak summer is X_{t+1} with intermediate X_n during start of summer; the trajectories in the return map will be attracted to initial conditions.

In Figure 3 *b*, the magnitude of variation is less than in the first case, as the trajectory is attracted to a fixed point (X') (stable) (Figure 4 *b*). In Figure 3 *c* the amount of nonlinearity is exactly 1. This indicates that all the trajectories are attracted to a fixed point (Figure 4 *c*). Such types of water bodies are very stable in nature.

Figure 4 shows the return maps constructed by taking the areal extents from the possibilities given in Figure 3 and the computed strength of nonlinearity into account. X_t and X_{t+1} are the populations of the water bodies at peak summer and peak rainy season, respectively.

Similar studies can also be carried out at annual intervals with the help of quadratic maps, to find out the lake's behaviour. Figure 5 shows the hypothetical water bodies at times t and $t+1$ of two successive years. In Figure 5 *a*, *b* and *c* the magnitude of variations are assumed as > 3.8, < 1 and 2, respectively.

Generally, in eutrophied lakes, areal extent reduces with time as shown in lakes of Figure 5 *b*. In Figure 5 *a*, the increase in areal extent is represented hypothetically. Figure 6 shows the quadratic maps of the areal extent fluctuations in the model lakes of Figure 5 *a*, *b* and *c* due to the given possibilities.

The magnitude of variation in water bodies is very high (Figure 5 *a*). Trajectories behaviour, being random, is unpredictable as shown in the logistic map with $\lambda = 4$ (Figure 6 *a*), whereas in the other logistic maps, the trajectories are attracted towards initial conditions (Figure 6 *b*) and to equilibrium point (Figure 6 *c*), respectively.

If more data over a longer time period, sampled at more regular intervals, are obtained, the experimentally constructed return map can be examined by plotting the data as X_{t+1} versus X_t. This resembles the logistic map, in which case such maps can be fitted and used to make predictions. On the other hand, such a simple map fails to capture the complexity of the system.

Simple mathematical models like the first-order difference equations (logistic equations) are helpful in quantification of seasonal and temporal behaviour of water bodies. Certain possibilities with reference to lake behaviour are illustrated hypothetically in this short note. It would be very interesting if the data on changing areal extents are obtained over a longer time period, sampled at more regular intervals to observe the validity whether or not the logistic model conforms to the behaviour of natural lakes. This logistic model approach will be of use in segregating lakes according to their behaviour. The accuracy and validity of the logistic model approach to predict the dynamical behaviour depends mainly on the computation of the strength of nonlinearity.

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Petrogenesis and tectonic setting of Malani rhyolites: Evidence by trace elements and oxygen isotope composition

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The trace element and oxygen isotope studies of Malani rhyolites from Gurapratap Singh and Diri area, district Pali, Rajasthan, indicate rift-related, within-plate nature of these rhyolites. The rhyolites have probably been derived by partial melting of crustal rocks.

THE Malani volcanism marks a late proterozoic tectonomagmatic event over an area of 50,000 km^2 in western and southwestern Rajasthan[1]. The rocks are exposed in isolated hills and ridges and are characterized by a preponderance of rhyolitic rocks over the intermediate and basic rocks. The present communication describes in brief the results of trace element and oxygen isotope studies of Malani rhyolites from Gurapratap Singh and Diri area, district Pali, Rajasthan (latitude 25°35′–25°40′N and longitude 73–73°10′E). The study volcanics are confined to hill ranges running E–W to NE–SW in semiarcuate fashion and have been extruded on a basement of argillaceous rocks belonging to the Aravalli or Delhi Supergroup.

The rhyolites from Gurapratap Singh and Diri area are fine-grained/glassy and sparsely phyric[1]. The phenocrysts are mainly of plagioclase feldspar and quartz. The presence of devitrified glass shards, angular phenocrysts of feldspar and collapsed pumice fragments indicates tuffaceous nature of these volcanics[2], though a few true rhyolite flows have also been encountered in the area.

Table 1. Major (wt %) and trace element (ppm) analyses, CIPW norms and oxygen isotope composition of Malani rhyolites from Gurapratap Singh and Diri, Pali district, Rajasthan. Major element data are from Srivastava *et al.*[1] (WR – whole rock, Qtz – quartz mineral separate)

	D41	G151	G25	G24
SiO_2	73 08	75.11	75 11	76 56
TiO_2	0.2	0.22	0 17	0 03
Al_2O_3	14.85	12.91	14 30	13 44
Fe_2O_3	1.14	1 63	1 64	0 57
FeO	0.72	0 2	0 12	0 4
MnO	0 05	0.03	0 01	0 01
MgO	0 08	0.2	0 19	0.19
CaO	0.8	1.3	0 85	0.32
Na_2O	3.05	2 7	2.35	0 55
K_2O	5.0	4 4	3 65	7.0
P_2O_5	0 09	0 03	0.02	0 0
LOI	0.8	1.2	0.6	0.0
Total	99 86	99 93	99 01	99 07
Rb	224	220	136	332
Ba	501	402	949	582
Sr	85	63	41	14
Zr	242	196	158	65
Th	28	26	17	18
Y	79	110	65	111
Nb	19	19	15	17
La	56	68	32	29
Ce	139	138	65	47
V	5	15	7	6
$\delta^{18}O$	2.61WR		9.19Qtz	11.94WR
CIPW norms				
q	34 80	39 66	44 10	45 54
or	29.47	26 13	22.24	41 14
ab	25.68	22.53	20.96	4 72
an	3.06	6.39	4 17	1 67
c	3 26	1.33	4 59	4.39
hy	0 46	0.50	0 50	0 76
mt	1.61	–	–	0 70
hm	–	1.60	1 60	–
il	0.30	0 50	0 30	–
ap	0 23	–	–	–