Wave–particle duality and Bohr's complementarity principle in quantum mechanics

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Interest in Bohr's complementarity principle has recently been revived particularly because of several thought experiments and some actually performed experiments to test the validity of mutual exclusiveness of wave and particle properties. A critical review of the situation is undertaken and it is pointed out that the problem with mutual exclusiveness arises because of some vagueness in the conventional formulation. An attempt is made to remove this vagueness by connecting the origin of mutual exclusiveness to some principles of quantum mechanics. Accordingly, it becomes obvious that to contradict complementarity principle without contradicting quantum mechanics would be impossible. Some of the recent experiments are critically analysed.

IN classical physics energy is transported either by particles or by waves, and particle and wave are the sole ultimate categories of physical entities. Particles are described by Newton's equation of motion and a wave by a space–time field function which satisfies a differential equation – the so-called wave equation (or field equation), and there is no mix-up. But the classical 'particle' is described in quantum mechanics (QM) by a field function (ψ) satisfying Schrödinger's equation which has a wave-equation-like flavour. Similarly, classical 'waves' are described by quantum field equations in QM which may be recast in a Schrödinger equation[1] for the wave function of the field quanta. The basic difference in the mathematical description of the two distinct classical entities disappears and we are in for the typical wave–particle duality in the world of quantum mechanics.

The wave–particle duality according to Feynman[2] 'contains the only mystery of the theory', while Bohm[3] considered it as one of the crucial 'new features of primary significance' introduced by the quantum theory. Eddington has coined the name 'wavicle' for microparticles and the term 'wave function' has been chosen to emphasize the wave–particle duality, considered generally so fundamental in quantum mechanics. Wave–particle duality is pointed out even in a quark model based on finite-dimensional quantum mechanics[4]. On the other hand, Popper[5] writes '. . . I nevertheless believe that the quantum theory is in a very definite sense a particle theory (here I disagree with Schrödinger) and in a sense which excludes a duality, analogy or complementarity between particles and waves'.

Notwithstanding such comments, the notion of duality in the physical description which grew in the years following Einstein's work in 1905 continues to spur fresh debate in the wake of some recent experiments on matter–wave interferometry and quantum optics.

Almost 20 years after the seminal work of Einstein, de Broglie extended the wave–particle dualism to material particle in 1924 and signalled the formulation of wave mechanics. Acknowledging the significance of the work, Einstein[6] immediately wrote back that 'de Broglie has lifted a corner of the great veil'. However, in the following years multiplicity of dualist interpretation led Bohr[7] to 'harmonize the different views, apparently so divergent' and to offer a general description of 'atomic phenomena' in the framework of the complementarity principle.

The recent glut of critical comments regarding the validity of Bohr's complementarity principle (BCP) or, more precisely, the conceptual content of the wave–particle duality in the light of new experiments, however, makes it clear that much of the controversy arises because of the prolixity of the formulation, which contains some amount of vagueness (see Box 1). In January 1949 Einstein[8] complained that 'despite much effort which I have expended on it, I have been unable to achieve the *sharp formulation* of Bohr's principle of complementarity' (italics ours). In our view two major shortcomings of the conventional formulation of BCP are: (1) There is no guideline to decide whether a suggested pair of variables/properties should be considered complementary. (2) No precise quantum-mechanical principle is adduced from which the incompatibility can be derived.

When Bohr remarked '. . . any given application of classical concepts precludes the simultaneous use of other classical concepts which in a different connection are equally necessary for the elucidation of the phenomena'[9], he kept one important question unanswered. If an experiment is performed to measure some properties and use classical concepts for the elucidation of the phenomenon, how do we ascertain which concepts are to be used and which to be precluded? This difficulty arises mainly because Bohr formulated the complementarity principle without addressing the principles of QM. We can find a parallel in the principle of uncertainty. Heisenberg formulated it without using QM. But later it was demonstrated that the uncertainty principle is implicit in the formalism of QM. Some physicists seem to accept BCP as independent of QM, and its relation to QM remains illdefined. Any attempt to show that mutual exclusiveness (ME) in BCP can be violated without violating QM presumes the view that BCP is independent of QM. It is surprising that no serious attempt so far has been made to see if BCP can be formulated in a way so that it becomes completely consistent with the principles of QM.

In a recent communication[10] we made such an attempt and pointed out that both the uncertainty and the complementarity principles are implied within the formalism of QM. Also it must be clearly understood that they add nothing to the formalism. Instead, they bring out explicitly certain features of QM not quite obvious from the formalism itself. In addition, they focus our attention on some sharply contrasting situations in classical and quantum mechanics and thereby help us realize their fundamental differences. According to Scully *et al.*[11], 'complementarity distinguishes the world of quantum phenomena from the realm of classical physics'.

In the quantum-mechanical description of a physical entity which is classically described either as a particle or a wave, we find that pairs of variables/properties exist which are mutually exclusive in the sense that a *sharp simultaneous knowledge of both is impossible.* Actually these are termed as complementary pairs.

In the above statement we have made ME a *defining property* of complementary pairs. On going through the writings of Bohr on BCP, it is evidently clear that ME is intended to be the essence of complementarity and we conclude that if for a system mutually exclusive variables do not exist, the system is a purely classical

Box 1.

N. Bohr on complementarity principle

'An adequate tool for a complementary way of description is offered by the quantum-mechanical formalism... It must be remembered that even in the indeterminacy relation we are dealing with an implication of the formalism which defies unambiguous expression in words suited to describe classical physical pictures. Thus, a sentence like "we cannot know both the position and the momentum of an atomic object" raises at once questions as to the physical reality of two such attributes of the object which can be answered only by referring to the conditions for the unambiguous use of space–time concepts on the one hand and the dynamic conservation laws on the other hand. While the combination of these concepts into a single picture of a causal chain of events is the essence of classical mechanics, room for regularities beyond the grasp of such a description is just afforded by the circumstances that the study of the complementary phenomena demands mutually exclusive experimental arrangements.' – Bohr, N., in *Atomic Physics and Human Knowledge*, Science Editions, New York, 1961, p. 41.

Box 2.

Pros and cons

1. '... the development of modern physics has enriched our thinking by a new principle of fundamental importance, the idea of complementarity.' – Born, M., *Physics in my Generation*, Springer, New York, 1969, p. 171.
2. 'Complementarity denotes the logical relation, of quite a new type, between concepts which are mutually exclusive, and which, therefore, cannot be considered at the same time – that would lead to logical mistakes – but which nevertheless must both be used in order to give a complete description of the situation.' – Rosenfeld, N., *Nature*, 1961, **190**, 384–388.
3. 'I have, for a long time, adopted the idea of complementarity in the realm of quantum physics, whilst at the same time realizing that it was inadequate. In recent years, I have been led to regard the concept of complementarity with increasing suspicion.' – de Broglie, L., in *The Current Interpretation of Wave Mechanics*, Elsevier, Amsterdam, 1964, p. 7.
4. 'This illogical conflation of concepts was the great quantum muddle which led to the emergence of the notorious wave–particle duality and hence the complementarity interpretation.' – K. R. Popper's view on the Copenhagen interpretation as presented by Jammaer, M., in *The Philosophy of Quantum Mechanics*, Wiley, New York, 1974, p. 448.

one. In every complementary pair he has discussed, Bohr stressed the ME aspect. But a gap was created when he said that wave properties are complementary to particle properties – it is absolutely necessary to know in a given experimental set-up which particle properties are to be regarded as complementary to which wave properties. Classical waves and particles have many common properties. As Bacry[12] puts it, '..., there are situations where the two aspects compete to interpret a phenomenon'. But in this respect Bohr's specific statement is significant: 'It is evident, however, that any concrete wave picture is as unable to account for basic experience regarding the individuality of the electron as a corpuscular picture for the superposition properties of radiation fields'[13]. This statement very clearly indicates that the complementarity Bohr meant was between properties of superposed and unsuperposed states and, as we shall see presently, these properties definitely satisfy ME.

In ref. 10 we made a precise statement of BCP in the spirit of the statements made by Bohr and in this paper we intend to examine critically in the light of this precise statement the thought experiments or the actual experiments which claim to violate the Bohrian notion of ME.

That complementary pairs of variables exist for quantum physical entities, no one doubts, and when we measure the precise position of an electron, we know that the concept of momentum has to be excluded. This knowledge comes from the principles of QM. Similarly, if we are doing an interference experiment, say double slit it is the which-state information that is complementary to the appearance of the interference pattern. We want to apply the same criterion in the so-called wave–particle complementarity, identifying the quantum principle responsible for ME in this case.

We can distinguish between two classes of complementarity on the basis of the origin of mutual exclusiveness. These are:

Class I: Pair of variables obtained from a Fourier transform of the state vector and any pair of dynamical variables for which the corresponding operators satisfy a noncommutation relation. These include pairs of canonically conjugate variables.

In this class we have the complementary pairs $\mathbf{r}, \mathbf{p}$ and t, $\mathcal{E}$ derived from $\Psi(r, t)$ and its Fourier transform $\Phi(p, \mathcal{E})$. Also, all pairs of noncommuting variables such as components of angular momentum $J_x, J_y; J_x, J_z$, etc., belong to this class. Similarly, the radiation field variables E and H, which satisfy field commutation relation, may serve as the complementary pair in the case of a light beam. Existence of more esoteric complementary pair of variables has also been reported from experiments on elementary particles[14]. ME follows quite clearly from the general quantum-mechanical uncertainty principles for this class of complementary pairs[15].

Class II: A property which depends on interference terms arising from the superposition of a number of states and the property associated with the which-state information form a pair of complementary properties.

In the simplest example of the widely discussed double-slit experiment, one may observe class II type of complementarity by measurement of a single variable. Let the state of a beam of electrons of a definite momentum be given by $\exp(ipz/\hbar)$, the momentum p being along the z direction. At $z = z_1$, a screen is placed with two slits whose centres are at $y = \pm a$ (we take y along the vertical direction). On the right-hand side of the screen the state vector is modified to $\psi = \psi_1 + \psi_2$, where ψ_1 is the wave vector originating from the upper slit and ψ_2 that from the lower slit. If we place a screen in this region at $z = z_2$, we shall observe the interference

Box 3.

Precise statement of BCP and wave–particle duality according to formal quantum mechanics

Existence of complementary pair of variables/properties distinguishes the world of quantum phenomena from the realm of classical physics. Mutual exclusiveness is the *defining property* of complementary pairs.

On the basis of the origin of mutual exclusiveness we can distinguish between two classes of complementary properties. These are:

Class I: Pair of variables obtained from a Fourier transform of the state vector and any pair of dynamical variables for which the corresponding operators satisfy a noncommutation relation. These include pairs of canonically conjugate variables. For this class of complementary pairs, ME follows quite clearly from the general quantum-mechanical indeterminacy principles.

Class II: A property which depends on interference terms arising from the superposition of a number of states and the property associated with the which-state information form a pair of complementary properties. Superposition of states is interpreted with respect to the properties which are measured to show the interference effect and give the which-state information. *This should be considered as the precise expression for the wave–particle complementarity.* The collapse hypothesis of QM ensures mutual exclusiveness in this type of complementarity.

effect. For simplicity we consider ψ_1, ψ_2 as functions of y and z alone. Thus, the probability density function for y as seen on the screen at z_2 is

$$\rho(y, z_2) = |\psi_1(y, z_2)|^2 + |\psi_2(y, z_2)|^2 + \psi_1^*(y, z_2)\,\psi_2(y, z_2) + \psi_2^*(y, z_2)\,\psi_1(y, z_2)$$

The last two terms here give the interference effect. If at $z = z_1$ we make a measurement of y, the result will give us the which-state information and the state vector will collapse to either ψ_1 or ψ_2 and the interference effect in $\rho(y, z_2)$ will disappear. Another interesting feature here is that ψ_1 or ψ_2 need not be eigenfunctions of y; only the respective dispersion in y has to be small.

Experiments based on matter–wave interferometry[2,11,16] belong to this type. A straight edge or a single-slit diffraction experiment may also be included as examples manifesting this type of complementarity. But they seem to have never been discussed in connection with complementarity. The difficulty lies in the fact that the initial state vectors in this case are superposition of a continuous infinity of position eigenfunctions. For example, in the straight-edge case, if we put a series of detectors in the open part near the edge and consider those spots on the screen produced only when the detectors click, the totality of all these spots gives no interference effect. To obliterate the entire pattern, we have to put detectors over a large region near the edge.

Both the interference terms and the which-state information are also obtained from measurement of some suitable pair of variables. Suppose the relevant observables for demonstrating the interference effect is $\hat{\alpha}$ and $\hat{\beta}$. To produce interference effect, the state vector in question must be a superposition of at least two eigenvectors belonging to two distinct eigenvalues of $\hat{\alpha}$ (say). Let us, therefore, take the state vector as $\psi = a_1 |\alpha_1\rangle + a_2 |\alpha_2\rangle$. If we measure the expectation value of $\hat{\beta}$ in this state, we get

$$\langle\hat{\beta}\rangle = |a_1|^2 \langle\alpha_1|\hat{\beta}|\alpha_1\rangle + |a_2|^2 \langle\alpha_2|\hat{\beta}|\alpha_2\rangle + a_1^* a_2 \langle\alpha_1|\hat{\beta}|\alpha_2\rangle + a_2^* a_1 \langle\alpha_2|\hat{\beta}|\alpha_1\rangle.$$

The last two terms give the interference effect. If $\hat{\beta}$ commutes with $\hat{\alpha}$ then the nondiagonal matrix elements vanish and there will be no interference effect. For this type of experiments it is necessary that $\hat{\beta}$ and $\hat{\alpha}$ correspond to noncommuting variables. Instead of the expectation value, a measurement of the probability distribution of the results of measurement of $\hat{\beta}$ will also reveal the interference effect. If before measuring $\hat{\beta}$ we make a measurement of $\hat{\alpha}$ and the outcome is α_1 (or α_2), the state vector collapses to $|\alpha_1\rangle$ (or $|\alpha_2\rangle$). Subsequent measurements of $\langle\hat{\beta}\rangle$ will give only the first two terms and the interference effect will disappear, showing the complementarity between the interference term in $\langle\hat{\beta}\rangle$ and the which-state information about the state vector. The ME arises here because of the collapse of the state vector and not because of noncommutation between $\hat{\alpha}$ and $\hat{\beta}$.

The simplest example of complementarity of this type is realized in spin interference experiments with neutrons. Let the spin wave function of the neutron beam be given by

$$X = \chi_+ \cos(\theta/2)\, e^{-i\phi} + \chi_- \sin(\theta/2),$$

where $X_\pm$ are the spin eigenstates of $\hat{\sigma}_z$ and the spin of the beam in the state X is polarized along $\hat{\theta}, \phi$ direction. If we measure the expectation value of $\hat{\sigma}_x$, we get

$$\langle\hat{\sigma}_x\rangle = \sin\theta \cos\phi.$$

This is indeed an interference effect. If we measure $\hat{\sigma}_z$ to extract the which-state information, then $\langle\hat{\sigma}_x\rangle$ becomes zero, i.e. the interference effect disappears.

Other examples of class II type of complementarity can be cited from different areas of physics. In class I type of complementarity, the state vector is unimportant. It is the specific pair of variables which satisfy ME irrespective of the state of the system. But in class II type it is the structure of the state vector which is important. Depending upon a specific state vector, one can find a variable (or a pair of variables) and a measurement which will show interference and another measurement which will cause disappearance of interference. The origin of ME for class II type of complementarity is always the quantum-mechanical principle of collapse of a state vector.

So far we have discussed only the extreme situations. Either there is no which-state knowledge and there is complete interference effect or just the opposite. The question arises whether we can quantify the concepts of a degree of interference and a degree of distinguishability of the states. Indeed, using projection operators it is possible to define two operators $\hat{I}$ and $\hat{D}$ which have these properties and whose expectation values satisfy a complementarity relation.

Origin of ME in class II type of complementarity

In a recent paper Uffink and Hilgevoord[17] have made a quantitative analysis of class II type of complementarity. They introduced a measure for the indistinguishability U $(0 \le U \le 1)$ of two quantum states in a given measurement and the amount of interference I observable in the same measurement and established an inequality $U \ge I$, which they regarded as a 'quantitative expression of Bohr's claim that one cannot distinguish between two possible paths of a particle while maintaining an interference phenomenon'.

In a similar way, but in still simpler terms, we can define I and a distinguishability parameter D (see also ref. 10) with the following considerations: Consider two orthogonal quantum states ψ_1 and ψ_2 expressed in the basis $|i\rangle$ (set of eigenvectors of some quantum-mechanical observable $\hat{\Omega}$) where some eigenvectors are common to both ψ_1 and ψ_2 and others are all different:

$$\psi_1 = \sum_{1}^{n} \alpha_i\,|i\rangle + \sum_{j\neq i} \alpha_j\,|j\rangle$$

and

$$\psi_2 = \sum_{1}^{n} \beta_i\,|i\rangle + \sum_{l\neq i} \beta_l\,|l\rangle,$$

where $|j\rangle \neq |l\rangle$ for all j's and l's. In a given measurement, we can define two operators $\hat{D}$ and $\hat{I}$ corresponding to the degree of distinguishability and the amount of interference, observable in the same measurement on a superposed state function ψ as

$$\hat{D} = |\psi\rangle\,\langle\psi| - \sum_{1}^{n} |i\rangle\,\langle i|, \quad \hat{I} = \sum_{1}^{n} |i\rangle\,|i\rangle.$$

The distinguishability parameter and the amount of interference observable in the same measurement can be obtained from the expectation values of $\hat{D}$ and $\hat{I}$ on the normalized superposed state:

$$\psi = [\psi_1 + \psi_2],$$

where

$$\Sigma\,|\alpha_i|^2 + \Sigma\,|\alpha_j|^2 + \Sigma\,|\beta_i|^2 + \Sigma\,|\beta_l|^2 = 1;$$

hence,

$$D = \langle\hat{D}\rangle = 1 - \sum_{1}^{n} (|\alpha_i|^2) + |\beta_i|^2).$$

So, D lies between 0 and 1. The zero value indicates complete lack of distinguishability and 1 complete distinguishability. A value of 0.6 for D signifies 60% distinguishability, implying that if $\hat{\Omega}$ is measured N times in 60% cases, we shall be able to distinguish the state and in 40% cases we fail,

Similarly,

$$I = \langle\hat{I}\rangle = \sum_{1}^{n} (|\alpha_i|^2 + |\beta_i|^2)$$

and, hence, $D + I = 1$.

When $D = 1$, $I = 0$, and vice versa; thus, D and I are complementary aspects. As an illustration consider the case of scattering of two identical particles (i e. indistinguishable and $D = 0$); here interference of amplitude in the differential scattering cross-section is observed ($I = 1$). But for two distinguishable particles, no interference effect is observed. However, in the case of a double-slit experiment $D = 1$ does not necessarily mean that we have the which-slit information unless we make a measurement. $D = 1$ means that we have, in principle, a 100% efficient which-slit detector for the particular experiment.

We also note that the commutation relation satisfied by the two operators D and I is

$$[\hat{D}, \hat{I}] = [\,|\psi\rangle\,\langle\psi| \sum_{1}^{n} |i\rangle\,\langle i| - \sum_{1}^{n} |i\rangle\,\langle i|\,|\psi\rangle\,\langle\psi|\,]$$

$$= \sum_{1}^{n} [(\alpha_i^* + \beta_i^*)\,|\psi\rangle\,\langle i| - (\alpha_i + \beta_i)\,|i\rangle\,\langle\psi|\,]$$

$$\Rightarrow \langle\psi|\,[\hat{D}, \hat{I}]\,|\psi\rangle = 0.$$

So the commutation relation does not lead to any uncertainty type of relation between ΔD and ΔI, representing the corresponding dispersions.

In all so-called wave–particle complementary experiments involving one particle at a time, time is an important parameter. We can introduce time in the above analysis and discuss the possibility whether we can have at $t = 0$, $D = 1$ and $I = 0$ and at a subsequent instant in the interference zone $D = 0$ and $I \neq 0$. A double-slit experiment closely resembles this situation. The essence of wave–particle complementarity problem is whether we can make a which-state measurement at $t = 0$ (or in the noninterfering region in the vicinity of the slits) and still get interference at a subsequent instant of time t? This is ruled out in quantum mechanics because of the collapse hypothesis.

Here we note that the operators $\hat{D}$ and $\hat{I}$ are state-dependent and the quantification of the qualitative properties of distinguishability and interference phenomenon is brought by $D + I = 1$. In complementarity phenomena the extremes, i.e. $D = 1$, $I = 0$ or $D = 0$, $I = 1$ are considered. But here the whole range is included. From the foregoing discussions it is clear that class II complementarity is consistent with the principles of QM. ME follows from collapse hypothesis of QM and in that sense we may say that class II BCP is derived from QM.

With this formulation we conclude that any experiment which claims to violate ME in wave–particle complementarity will imply some limitations of the quantum-mechanical formalism itself. This we believe is in true spirit of Bohr's original ideas.

In the following section we discuss some significant experiments on wave–particle duality in the light of the present formulation. All these experiments may be broadly classified into three classes. The first group of experiments[2,11,18,19] are claimed to provide experimental evidence in favour of ME in wave–particle duality. In a rather novel thought experiment suggested by Scully *et al.*[11], the incompatibility between 'welcher weg' (which path) knowledge and the appearance of interference cannot be traced to any uncertainty relation. On the other hand, certain 'welcher weg' experiments[20–22] which made joint unsharp measurements of complementary observables have been used to argue that they yield 'partial particle' and 'partial wave' information and are, therefore, in conflict with the Bohrian notion of ME.

Recently, Petroni and Vigier[16] and Ghose *et al.*[23] have proposed experiments to demonstrate that a quantum-mechanical entity may display its particle and wave aspects at the same time, thereby contradicting the principle of complementarity. In what follows the significance and import of these experiments are analysed in the light of the precise statement of BCP.

Discussion of some experiments

We now take a brief look at the well-known thought experiments on wave–particle duality. In the archetypal example of the recoiling-slit arrangement Einstein hoped to devise a gedanken experiment to provide the which-path information without destroying the interference pattern of the photons; Bohr[18] pointed out that we must also treat the recoiling slits by the laws of QM. Following the arguments of Feynman[2], we note that in order to observe the interference pattern, the slits must be localized with an accuracy of the order of the slit width and the corresponding uncertainty in the slit momentum is always much greater than the momentum transfer in the scattering process. Consequently, the which-path information is not available when the photons do produce interference pattern. The entire analysis here envisages a classical particle-like passage of the photons (even when they do interfere!) through the double slit, while a correct quantum-mechanical explanation should be obtained in terms of the quantum-mechanical field function satisfying appropriate boundary condition at the slits – the concept of momentum transfer being out of place[15]. However, in the similar example of double-slit interference experiment with electrons, when Feynman intends to 'watch' the electron using scattered photon to do the spying, the interference pattern gets washed out, and Feynman's analysis invoking the uncertainty principle for the scattering process in this case may be regarded as an intuitive description not quite consistent with the formalism of QM[15].

In the interference region, the wave function describing the interfering beams may be written as the sum of two terms referring to the two slits:

$$\Psi(r) = [\Psi_1(r) + \Psi_2(r)]$$

The corresponding probability density at $r = R$ on the screen, denoted by $P(R)$, will be given by the squared modulus of $\Psi(R)$ and contains the interference term represented by $\Psi_1^*\Psi_2 + \Psi_2^*\Psi_1$. Now, if we consider two detectors 1 and 2 in the vicinity of the two slits, either making transition from ground state g to an excited state e induced by the passage of the interfering quantum object, the state of the beam plus the detector system (after entanglement) may be given by

$$\Psi(r) = [\Psi_1(r)\ |1_e 2_g\rangle + \Psi_2(r)\ |1_g 2_e\rangle].$$

For nonorthogonal detector states, partial or complete interference phenomenon is observed and no which-state information is available. In some special cases, as for detectors having coherent-state-like energy spectrum, one can have the so-called 'ein weg' (one way) knowledge, as claimed by Petroni and Vigier[16]. But if the detector states are orthogonal, the entanglement with the detector states results in a loss of coherence in interferometer and we get no interference. However, from an appropriate choice of the detector basis, as in the case of Scully *et al.*'s[11] micromaser detector, possibility of quantum erasers appears naturally[24] and the interference pattern can be retrieved. On the other hand, from a subsequent measurement on the detector state (with the aid of an atomic reader as in Scully's example) the superposed state gets collapsed and we get the which-state information.

The experiment of Petroni and Vigier

In the proposed neutron interferometry experiment[16,25] a pulsed neutron beam polarized vertically with respect to the apparatus is incident on a perfect crystal interferometer. At any instant, there is at most one neutron inside the interferometer. The beams from the beam splitter together with the coil states, described by the wave function

$$\Psi(r) = [\Psi_1(r) + \Psi_2(r)]\ |\uparrow\rangle \circ |C_1 C_2\rangle,$$

are then made to pass through two radio-frequency (rf) spin flip coils, one in each arm, and are superposed beyond the final set of crystal planes.

The wave function of the emerging beams from the coils is given by[26]

$$\Psi(r) = [\Psi_1(r)\ |C_1' C_2\rangle + \Psi_2(r)\ |C_1 C_2'\rangle]\ |\downarrow\rangle$$

and retains the coherent superposition as the coil states (denoted by the C's) are nonorthogonal. The final beam thus produces an interference in the measured intensity.

There are no resonant frequencies provided by the rf field which correspond to half the energy of spin flip. If we assume that energy and momentum are conserved in each energy exchange, it implies that a spin flip occurs in either of the coils and not in both. It is also possible to obtain interference in the intensity of the detected neutrons in the same experimental set-up. However, as pointed out by Scully and Walther[27] and also by Unnerstall[26], the rf coils generate coherent states of photons and the passage of neutron through the coil cannot leave the which-path information in the coil since the coherent photon distribution remains essentially unchanged by the addition of a single photon associated with spin flip. After the passage of a large number of neutrons, the energy transferred to each coil is summed up to an amount that is detectable, and one can tell that out of N neutrons involved in the experiment, n (say) neutrons have exchanged energy with coil I and the rest $N-n$ with coil II. This is precisely what Petroni and Vigier[16] claimed as 'ein weg' information. It seems legitimate to realist physicists to infer that each neutron followed a particular trajectory or the other through the rf coils, leaving the so-called 'ein weg' information in the process. But this extrapolation is, however, not endorsed by quantum orthodoxy[26]. 'Ein weg' experiment does not contradict BCP. The only problem it creates is how one would accommodate the 'ein weg' information within the framework of the orthodox interpretation of QM.

Experiment of Ghose et al.[23]

Ghose *et al.*[23] have proposed an experiment in which single-photon states are incident on a combination of two prisms placed opposite to each other. Mizobuchi and Ohtake[28] have performed the actual experiment and verified the quantum-optical predictions outlined in the former paper. When the gaps between the prisms is larger than the wavelength, the incident 'photon states' suffer total internal reflection inside the first prism (registered by counter 1). When the gap is shorter than the wavelength, there is a possibility of their 'tunnelling' across the gap (registered by counter 2).

Since they consider the presence of 'tunnelling' to be accounted for by assigning a wave aspect to the photon, the perfect anticoincidence in the clicking of the two counters led them to claim that the 'experimental result contradicts the tenet of mutual exclusiveness of the classical wave and particle pictures assumed in Bohr's complementarity principle'.

We[23] have adopted the viewpoint of Grangier *et al.*[19] and consider anticoincidence as a signature of particle property. However, the anticoincidence observed in two detectors in a single-photon field has a totally different explanation in quantum field theory[25], devoid of the photon particle picture, a photon being just an excitation of a mode.

On the other hand, the so-called 'tunnelling' cannot be accepted as a hallmark of wave property. In that case we should consider the tunnelling of α-particle through atomic nucleus as a violation of ME in wave–particle dualism. Furthermore, while Ghose *et al.* consider the existence of evanescent waves leading to transmission of light at the critical angle as a specific wave property, Chiao *et al.*[29] described the phenomenon of 'frustrated total internal reflection' of the purely nonclassical single-photon state exploiting an analogy with particle (quantum) tunnelling through the classically forbidden region. They have also proposed an experiment to measure the photon tunnelling time using the 'particle aspect' of photon tunnelling.

According to Chiao *et al.*[29], there exists a classical limit associated with coherent states of the electromagnetic field in which the tunnelling phenomenon can be understood entirely as a classical wave phenomenon. The two arguments are, therefore, irreducible. However, it seems that tunnelling can be interpreted both as a particle and a wave property.

If we accept the interpretations of Grangier *et al.*[19] and Ghose *et al.*[23] of wave–particle duality in BCP then Mizobuchi and Ohtake's experiment[28] definitely contradicts ME. Similarly, a measurement of photon velocity on refraction in a denser medium coupled with anticoincidence will also violate ME. Again, since the probability of joint detection of more than one photon is exactly zero for an 'ideal single-photon' state, should we claim that both 'particle' and 'wave' appear in the single run of an interference experiment with 'ideal single-photon' states? One can multiply this sort of examples indefinitely[30]. On this point it is also important to note that photon enters the theory as a secondary quantity – an elementary excitation of the field. The concept of photon does not presuppose that the quantum of field energy is concentrated at a point in space, i.e. that the photon is a localized entity analogous to a very small bullet. The photon is only approximately localizable and the wave function in coordinate space is nonlocal[1]. We may also recall that Einstein always stressed the provisional nature of the light quantum postulate. In December 1951, he wrote to his friend Besso: 'All these fifty years of pondering have not brought me any closer to answering the question "What are light quanta?". Nowadays, every Tom, Dick and Harry thinks he knows it, but he is mistaken'[31] Pushing the classical analogy too far in the photon picture may, therefore, lead to contradictions.

Some physicists are, in effect, asserting that any property for which a standard classical corpuscular theory does not exist is a wave property and vice versa. This rather simplistic interpretation of Bohr's idea is untenable, for a 'particle' is not always a classical particle (when it tunnels!) and similarly a 'wave/field' may have nonclassical features. *In fact, the use of the term wave–particle duality is at the root of much confusion and QM does not guarantee the claim of complementarity between arbitrarily defined wave and particle properties.*

In this experiment the pair of properties involves no superposed state and hence no interference effect is there. Therefore, it belongs neither to class I nor to class II types of BCP. We conclude that this experiment is not suited to test BCP in its precise version.

The unsharp joint measurements[20–22] which claim to observe both interference and the path of the particle do not really fall within the compass of BCP. Bohr himself in the Como lecture discussed the wave packet description of a particle (where approximate knowledge of both momentum and position exists) as an illustration of the complementarity principle, implying that unsharp knowledge of conjugate variables does not violate BCP. Both class I and class II types of BCP imply sharp measurement.

Finally, let us summarize the situation. If we interpret the wave–particle duality aspect of BCP as a principle independent of the formalism of QM, then mutual exclusiveness becomes a doubtful proposition. In the quantum region we have neither a true classical particle nor a true classical wave. In some experiments, for some limiting value of a parameter, results can be interpreted as those of a classical particle and in the other limit as those of a classical wave. Naturally, for any intermediate value of the parameter, this type of exclusive interpretation would be impossible. Moreover, to try to designate an entity as either a particle or a wave by measuring a single property is not a very satisfactory and unambiguous procedure. A classical particle or a wave is defined by a totality of many properties.

On the other hand, we can look at BCP as focusing our attention on an important and characteristic feature of QM that every physical entity has a number of pairs of mutually exclusive variables/properties which is unknown in classical physics. Then ME becomes a defining feature of BCP. Selection of pairs of such variables/properties has to be made consistently with QM. This is really possible, as has been shown in this paper. With this modified formulation, BCP becomes consistent with QM and violation of BCP without violating QM becomes impossible.

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