Fall of liquid drops through a viscous medium: A fresh expression for drag force by the method of dimensions

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An attempt has been made afresh by the method of dimensions to get a fresh expression for the drag force F arising from the motion of a liquid drop not exhibiting oscillation through a viscous medium, assuming that the drag force F is a function of D, u, σ, η and ρ alone (D, diameter of the drop; u, terminal velocity acquired by the drop; σ, density of the liquid in the column; η, viscosity of the column liquid; ρ, density of the liquid drop). This paper also relates to the experimental verification of the fresh drag force expression obtained in the case of a liquid drop with fairly high Reynolds number (20–305), Eotvos number (2×10^{-2} to 105×10^{-2}) and Morton number (4.75×10^{-12} to 3.75×10^{-7}) falling without oscillation through an immiscible liquid medium. Furthermore, the experimental value of the constant in the fresh drag force expression presented in this paper is found to be 3.6094.

SEVERAL authors have conducted studies on the fall of solid spheres[1-5] and liquid drops[6-30] through a viscous medium. So far, to the authors' knowledge, none has dealt with the problem of fall of liquid drops with only six variables, F, D, u, σ, η, ρ, in the dimensional analysis of drag force F. Therefore, an attempt has been made afresh to get a fresh expression for the drag force when the drops fall freely without oscillation with these six variables by dimensional analysis. Furthermore, an attempt has also been made with falling drops of fairly high Reynolds, Eotvos and Morton numbers to determine the value which the fresh drag force expression presented in this paper predicts for the constant K by experiment.

In general, there are at least six dimensionless groups that govern the motion of a drop, namely, $\pi_1 = F/D^2u^2\sigma$, $\pi_2 = \eta/\sigma uD$, $\pi_3 = \rho/\sigma$, $\pi_4 = \eta/\eta_0$, $\pi_5 = \gamma/Du^2\sigma$, $\pi_6 = u^2/gD$, where γ is the interfacial tension, D is the equivalent diameter of the drop and g is the acceleration due to gravity. η_0 is the viscosity of the drop. In general, the shape of the drops need not be spherical. The experimental work shows that for sufficiently small drops which appear to be ellipsoidal in shape over the range of Reynolds number $Re = 20$–305 ($Re = \sigma uD/\eta$), Eotvos number $Et = 2 \times 10^{-2}$–105×10^{-2} ($Et = g(\rho - \sigma)D^2/\gamma$), and Morton number $Mo = 4.75 \times 10^{-12}$ to 3.75×10^{-7} ($Mo = g\eta^4(\rho - \sigma)/\sigma^2\gamma^3$) the following simplification holds good:

$$\pi_1 = \phi(\pi_2, \pi_3) = K(\pi_2, \pi_3)^{1/2}, \quad (1)$$

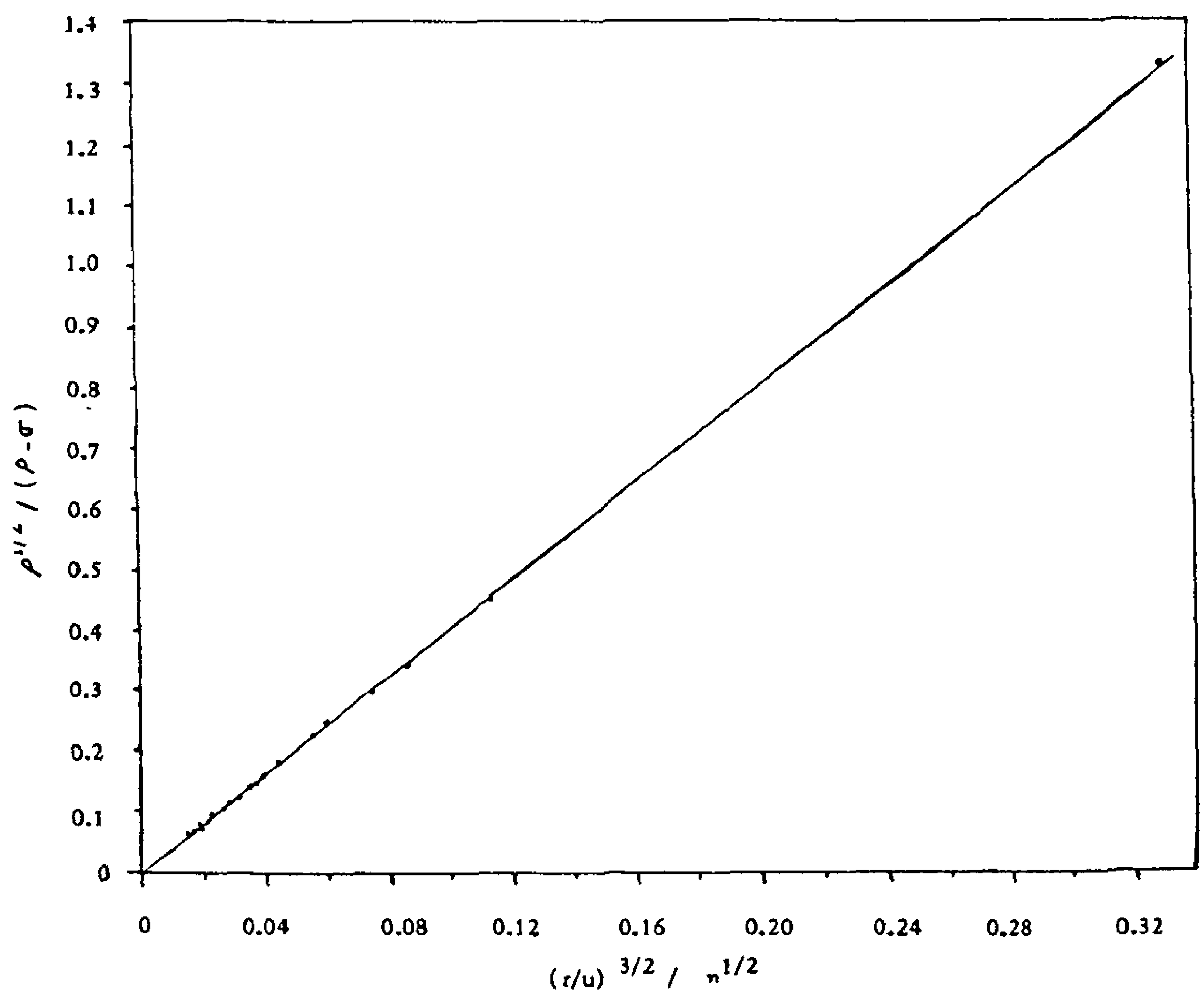

Figure 1. Plot of data given in Table 3

Table 1 The systems and their density, ratio of density, viscosity, interfacial tension and ratio of radius to terminal velocity values

Liquid drop	Column liquid	ρ (kg m^{-3})	σ (kg m^{-3})	$\rho-\sigma$ (kg m^{-3})	ρ/σ	Column liquid η (N s m^{-2}) viscosity	Interfacial tension γ ($\times10^{-3}$ N m^{-1})	r/u (s)
Eugenol	Water	1072 81	1000 00	72 81	1 07281	0 00100	12 82	0 0235
Chlorobenzene	Water	1097 99	1000 00	97 99	1 09799	0 00100	46 03	0 0195
Benzoyl benzoate	Water	1112 97	1000 00	112 97	1 11297	0 00100	34.59	0 0178
1-2-Dichloroethane	Water	1242 91	1000 00	242 91	1 24291	0 00100	31 78	0 0111
1-2-Dichlorobenzene	Water	1291 54	1000 00	291 54	1 29154	0 00100	84 77	0 0100
Trichloroethylene	Water	1444 05	1000 00	444 05	1.44405	0 00100	43 42	0 0077
Chloroform	Water	1477.40	1000 00	477 40	1.47740	0 00100	41 70	0 0073
Bromobenzene	Water	1492 21	1000 00	492 21	1 49221	0 00100	70.20	0 0071
Carbon tetrachloride	Water	1579 63	1000 00	579 63	1.57963	0 00100	39 06	0 0065
Nitrobenzene	Water	1193.92	1000 00	193 92	1.19392	0 00100	46 01	0 0125
Carbon disulphide	Water	1251 70	1000 00	251 70	1 25170	0 00100	31 36	0 0107
Dichloromethane	Water	1314 32	1000 00	314 32	1 31432	0 00100	36.71	0 0094
Water	Kerosene	1000 00	797 34	202 66	1 25417	0 00129	43 78	0 0125
Alcohol	Kerosene	818 89	797 34	21 55	1 02703	0 00129	1 35	0 0522
Trifluoroethanol	Kerosene	1380.57	797 34	583.23	1.73147	0 00129	8 32	0 0069
Ethylene glycol	Kerosene	1100.01	797 34	302 67	1 37960	0 00129	18 30	0 0099
Dimethylformamide	Hexane	947 15	665.37	281 78	1 42349	0 00032	4 69	0 0061
Acetonitrile	Hexane	778.26	665 37	112 89	1 16966	0.00032	5.36	0 0105
Ethylene glycol	Hexane	1100.01	665 37	434 64	1 65323	0.00032	25 55	0 0048
Water	Hexane	1000 00	665 37	334 63	1 50292	0 00032	17 44	0 0056
Water	Xylene	1000 00	857 95	142 05	1 16557	0 00060	12 48	0 0123

r = radius of the liquid drop; u = terminal velocity of the drop; ρ = relative density of the liquid drop, σ = relative density of the liquid in the column; η = viscosity of the liquid in the column; γ = interfacial tension between the liquid drop and the liquid in the column.

where K is a constant and π_2 is the reciprocal of the Reynolds number Re and $\pi_3=\rho/\sigma$, the ratio of the densities. Subsituting the values of π_1, π_2, and π_3 in eq. (1), the expression for drag force is

$$F = KD^2u^2\sigma(\eta/\sigma uD)^{1/2}(\rho/\sigma)^{1/2},$$

or

$$F = KD^{3/2}u^{3/2}\eta^{1/2}\rho^{1/2}. \quad (2)$$

Since at the terminal velocity

$$F = 4\pi r^3(\rho-\sigma)g/3, \quad (3)$$

where r is the radius of the drop and is equal to $D/2$, we get the constant K as

$$K = \sqrt{2}\pi g(r/u)^{3/2}(\rho-\sigma)/3\eta^{1/2}\rho^{1/2}, \quad (4)$$

or the terminal velocity u as

$$u = (2\pi^2g^2r^3(\rho-\sigma)^2/9\eta K^2\rho)^{1/3}. \quad (5)$$

Rewriting eq. (4), we get

$$[(r/u)^{3/2}(\rho-\sigma)]/[\eta^{1/2}\rho^{1/2}] = 3K/\sqrt{2}\pi g, \quad (6)$$

i.e. $\sqrt{2}\pi g\, S/3K = 1$, (7)

where

$$S = [(r/u)^{3/2}(\rho-\sigma)]/[\eta^{1/2}\rho^{1/2}]. \quad (8)$$

Equation (8) is of vital importance, in the sense that it can be used to determine the density (ρ) of liquids available in microquantities when the conventional methods fail. S has been found to be approximately a constant for all systems (see Table 3), the mean value of which is 0.248319 m^{-1} s^2.

Equation (8) may be written as a quadratic equation which gives the value for ρ as

$$\rho = \{2\sigma + \lambda + [\lambda(\lambda + 4\sigma)]^{1/2}\}/2, \quad (9)$$

where

$$\lambda = S^2\eta/(r/u)^3. \quad (10)$$

Equation (9) may be employed to determine the density of liquids available in microquantities.

A graduated cylinder (diameter 5 cm) filled with the experimental liquid was used. Liquid drops of known volume were gently injected into the liquid column using a graduated Hamilton precision microsyringe. The drops and the liquid were immiscible. The terminal

Table 2. Experimental data for liquid drop pairs

Liquid drop pair	Volume V of the drop (l)	Radius r of the drop (10^{-4} m)	Distance S travelled (10^{-2} m)	Time t taken (s)	Observed terminal velocity u (10^{-2} m s^{-1})	r/u (s)	Reynolds number	Eotvos number ($\times 10^{-2}$)
1-2-Dichlorobenzene in water	0.5	4.9237	80	16.4	4.8780	0.0101	48.04	3.280
Morton no = 4.7403 × 10^{-12}	1.0	6.2035	50	8.1	6.1728	0.0100	76.59	5.207
	2.0	7.8159	50	6.4	7.8125	0.0100	122.12	8.265
1-2-Dichloroethane in water	0.5	4.9237	40	9.1	4.3956	0.0112	43.29	7.263
Morton no = 7.4139 × 10^{-11}	1.0	6.2035	40	7.2	5.5556	0.0112	68.93	11.529
	2.0	7.8159	40	5.7	7.0175	0.0111	109.70	18.301
Water in kerosene	0.5	4.9237	40	10.2	3.9216	0.0126	23.87	4.399
Morton no = 1.0307 × 10^{-10}	1.0	6.2035	40	8.1	4.9383	0.0126	37.87	6.983
	2.0	7.8159	40	6.4	6.2500	0.0125	60.39	11.084
Ethylene glycol in kerosene	0.5	4.9237	40	8.0	5.0000	0.0098	30.43	15.779
Morton no = 2.1089 × 10^{-9}	1.0	6.2035	40	6.4	6.2500	0.0099	47.93	24.953
	2.0	7.8159	40	5.1	7.8431	0.0100	75.78	39.610
Dimethylformamide in hexane	0.2	3.6278	40	6.8	5.8824	0.0062	88.74	30.988
	0.3	4.1528	40	5.8	6.8966	0.0060	119.10	40.606
Morton no. = 6.3348 × 10^{-10}	0.4	4.5708	40	5.3	7.5472	0.0061	143.46	49.191
Acetonitrile in hexane	0.2	3.6278	40	11.5	3.4783	0.0104	52.47	10.857
Morton no = 1.6974 × 10^{-10}	0.3	4.1528	40	10.1	3.9604	0.0105	68.39	14.227
	0.4	4.5708	40	9.2	4.3478	0.0105	82.64	17.235
Water in xylene	0.5	4.9237	40	10.0	4.0000	0.0123	56.32	10.818
Morton no = 1.2613 × 10^{-10}	1.0	6.2035	40	7.9	5.0633	0.0123	89.83	17.172
	2.0	7.8159	40	6.3	6.3492	0.0123	141.92	27.259

Table 3 The value of $(r/u)^{3/2}/\eta^{1/2}$ and $\rho^{1/2}/(\rho-\sigma)$ and the constant K obtained by using eq (6)

Liquid drop	Column liquid	$(r/u)^{3/2}/\eta^{1/2}$	$\rho^{1/2}/(\rho-\sigma)$	$S^{\dagger}$	K^{*}
Eugenol	Water	0.11392	0.44985	0.253239	3.6754
Chlorobenzene	Water	0.08611	0.33816	0.254644	3.6958
Benzoyl benzoate	Water	0.07510	0.29531	0.254302	3.6908
1-2-Dichloroethane	Water	0.03698	0.14514	0.254806	3.6981
1-2-Dichlorobenzene	Water	0.03162	0.12327	0.256533	3.7232
Trichloroethylene	Water	0.02137	0.08558	0.249676	3.6237
Chloroform	Water	0.01972	0.08051	0.244972	3.5554
Bromobenzene	Water	0.01892	0.07848	0.241058	3.4986
Carbon tetrachloride	Water	0.01657	0.06857	0.241681	3.5076
Nitrobenzene	Water	0.04419	0.17818	0.247730	3.5997
Carbon disulphide	Water	0.03500	0.14056	0.249707	3.6139
Dichloromethane	Water	0.02882	0.11534	0.248279	3.6265
Water	Kerosene	0.03891	0.15604	0.250319	3.6192
Alcohol	Kerosene	0.33206	1.32790	0.249674	3.6292
Trifluoroethanol	Kerosene	0.01582	0.06371	0.248203	3.6039
Ethylene glycol	Kerosene	0.02743	0.10958	0.249257	3.6324
Dimethylformamide	Hexane	0.02663	0.10922	0.241214	3.5391
Acetonitrile	Hexane	0.06015	0.24712	0.242377	3.5324
Ethylene glycol	Hexane	0.01859	0.07631	0.242101	3.5358
Water	Hexane	0.02343	0.09450	0.244846	3.5978
Water	Xylene	0.05569	0.22262	0.250073	3.6307
			Mean	0.248319	3.6109

*The constant of eq (6)

†S of eq (8)

velocity u was determined by noting the time of transit t required by the liquid drop of radius r to cover a distance S between two graduations on the column. The drops studied here while falling are ellipsoidal in shape. If V is the volume of the drop of equivalent radius[7] r, then $r = (3V/4\pi)^{1/3}$.

The densities of the liquids were determined accurately by the specific gravity bottle method (Table 1). Viscosity values were determined by using the Ostwald viscometer (Table 1) and interfacial tension values have been determined by the method of drops (Table 1).

Twenty-one liquid systems were studied (Table 1). Only seven liquid drop pairs having three data points each are presented to show that r/u is approximately constant (Table 2). The other fourteen liquid drop pairs for which experimental results have been obtained satisfy the same. The right-hand side of eq. (6) is a constant. A graph is drawn with $(r/u)^{3/2}/\eta^{1/2}$ along the X-axis and $\rho^{1/2}/(\rho - \sigma)$ on the Y-axis (Figure 1). The slope of the line in the graph gives the value of $\sqrt{2\pi g/3K}$.

Substituting the experimental value (0.248319) obtained by eq. (8) (Table 3) on the left-hand side of eq. (6), the value of K can be determined. The value of K thus determined (Table 3) is 3.6109, and that determined from the graph (Figure 1) is 3.6079, the mean being 3.6094. Therefore, for the motion of a liquid drop falling freely and vertically, and not exhibiting oscillation, through an immiscible liquid medium of fairly high Reynolds, Eotvos and Morton numbers, eq.(2) is the fresh drag force expression, in which the experimental mean value of the constant K is 3.6094.

Equation (5) with $K = 3.6094$ may be used to predict the terminal velocity of the drop falling without oscillation. Using eq. (8) (with mean experimental value 0.248319, Table 3), if the terminal velocity and radius are known, one can determine the density of a micro-quantity liquid drop for which the density cannot be determined by the capillary tube method (where weighing is a problem) or by any other conventional methods.

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