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RESEARCH COMMUNICATIONS

Some FRW models with constant active gravitational mass

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The consequences of taking the active gravitational mass of the universe constant in the background of FRW models are investigated. It is found that the dependence of the nature of expansion on the curvature parameter k may be altered.

In the standard big bang cosmology, wherein the universe is assumed to be filled with a distribution of matter represented by the energy momentum tensor of a perfect fluid

$$T_{ij} = (\rho + p)v_i v_j + p g_{ij}, \quad \text{(in the units with } c = 1) \tag{1}$$

and the geometry of the universe is described by the Robertson–Walker line element

$$ds^2 = -dt^2 + R^2(t)\{(dr^2/1 - kr^2) + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\}, \quad k = \pm 1, 0, \tag{2}$$

the Einstein field equations

$$R_{ij} - \tfrac{1}{2} R^h_h\, g_{ij} = -8\pi G T_{ij} \tag{3}$$

obtain two independent equations

$$-\ddot{R}/R = (4\pi G/3)(\rho + 3p), \tag{4}$$

and

$$\dot{R}^2/R^2 + k/R^2 = (8\pi G/3)\rho. \tag{5}$$

The expression for the scale factor R is obtained by solving the differential equation resulting from equations (4) and (5) by assuming an equation of state. Out of the three cases of the model obtained for different values of the curvature parameter k, one is closed for $k = +1$ and the remaining two are ever expanding for $k = 0$ and -1. In this communication, motivation is given for taking the active gravitational mass of the universe as constant and it is found that the dependence of the nature of expansion of the resulting models on the curvature parameter k may be different from that of the standard models.

The field equation (4) may be deemed as an analogue of the Newtonian force law and suggests that the force per unit mass at each space-time point is determined by the active gravitational mass density $(\rho + 3p)$. Here we note that the gravitational pull is exerted not only by ρ as in the Newtonian theory but rather by $(\rho + 3p)$ which exhibits a relativistic effect. It is this additional pressure and internal energy contribution to the gravitational force which is the major cause of the problem of gravitational collapse in general relativity[1].

One notes that equations (4) and (5) obtain $\rho R^3 =$ constant (ref. 2) in the present pressureless phase of evolution, which can be interpreted as the conservation of the total active gravitational mass of the comoving sphere of radius R. As there is no justification of conferring a special status upon the present epoch, we speculate that this constancy feature of the active gravitational mass is met not only in the present phase of evolution but in the early phases too. We thus assume that in the presence of pressure, the active gravitational mass is constant.

$$(\rho + 3p)R^3 = \text{constant} = A \quad \text{(say)}. \tag{6}$$

Equation (6), taken together with the energy momentum conservation equation

$$\dot{\rho} + 3(\rho + p)\ \dot{R}/R = 0\,, \tag{7}$$

which may be obtained from equations (4) and (5), leads to

$$p^3 = K(\rho + 3p)^2, \quad K = \text{constant} \geq 0\,, \tag{8}$$

which is the equation of state for the perfect fluid constituting the matter content of the universe. It is a physically reasonable equation of state since

$$(\mathrm{d}p/\mathrm{d}\rho) = [2/3 \cdot (p/(\rho + p))]\,, \tag{9}$$

indicating that $(\mathrm{d}p/\mathrm{d}\rho) < 1/3$ for $\rho > p$.

Equations (4) and (6) obtain

$$\ddot{R} = -(4\pi GA/3R^2)\,, \tag{10}$$

which, by integrating, gives

$$\dot{R}^2 = (8\pi GA/3R) + B\,, \tag{11}$$

where B is some constant of integration. Equations (5), (6) and (11) may be used to obtain

$$\rho = A/R^3 + 3(B+k)/8\pi GR^2 \tag{12}$$

and

$$p = -(B+k)/8\pi GR^2\,, \tag{13}$$

which are in agreement with equation (8) and suggest that $K = -(B+k)^3/(8\pi G)^3 A^2$ for the self-consistency of the system.

Equation (13) indicates that in order to have $p \geq 0$, one must have $(B+k) \leq 0$.

We note from equation (6) that for $\rho > 0$ and $p > 0$, $A > 0$. Equation (10) hence indicates that $\ddot{R} < 0$. This, together with $R > 0$, implies that the curve $R(t)$ must have reached $R(t) = 0$ at some finite time in the past, say at $t = 0$, which is a singularity, where equation (6) holds in the limiting case. The models thus start from a big bang.

Equation (11) may be integrated for different values of k as:

$k = 0$

We thus have $B \leq 0$ for $p \geq 0$. When $B < 0$ say $B = -n^2$, we obtain

$$t = -\frac{(24\pi GAR - 9n^2R^2)^{1/2}}{3n^2} + \frac{4\pi GA}{3n^3} \times \sin^{-1}\left[\frac{3n^2R}{4\pi GA} - 1\right] - \frac{2\pi^2 GA}{n^3}\,. \tag{14}$$

When $B = 0$, we have

$$R = (6\pi GA)^{1/3}\, t^{2/3}\,. \tag{15}$$

Equation (14) indicates that the model reaches its maximum radius $R_{max} = 8\pi GA/3n^2$ at $t = 4\pi^2 GA/3n^3$ and thereafter starts contracting back to origin in a way similar to the $k = 1$ case of the standard model though here $k = 0$. Equation (15) suggests that $R \to \infty$ as $t \to \infty$ with the deceleration parameter $q = ½$ always as in the $k = 0$ case of the standard model.

$k = -1$

In this case $p \geq 0$ demands $B \leq 1$. If $B > 0$, say $B = n^2 \leq 1$, we obtain

$$t = \frac{(24\pi GAR + 9n^2R^2)^{1/2}}{3n^2} - \frac{4\pi GA}{3n^3} \times \cosh^{-1}\left[\frac{3n^2R}{4\pi GA} + 1\right], \tag{16}$$

which indicates that $R \to \infty$ and $\dot{R} \to n$ as $t \to \infty$. The deceleration parameter, which is initially ½ in this case tends to zero as $t \to \infty$. For $B \leq 0$, the evolution is described by equations (14) or (15).

$k = 1$

In this case, $p \geq 0$ demands $B \leq -1$ and the evolution is governed by equation (14) which describes a closed universe as mentioned earlier.

We thus see that the dependence of the nature of expansion on the curvature parameter k may be altered by the incorporation of the conservation law (6) in the standard cosmology.

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