Different Theories of the Spiral Nebulae.*

By A. C. Banerji and Nizamuddin.
(*Allahabad University.*)

HUBBLE found that about 97 per cent. of extra-galactic nabulæ fall into two classes which are more or less regular in shape, *viz.*, (*a*) those which have no spiral arms and are "elliptical" in shape, (*b*) those which possess "true spiral" forms each form consisting of a central region which is rather vaguely defined and from which two spiral arms emerge.

The elliptical nebulæ are classified into eight types, namely, E_0, E_1, E_2, E_3, E_4, E_5, E_6 and E_7, the numerical integer being nearest to $10 \cdot \frac{a-b}{a}$, where a and b are the greatest and the least diameters of the projections of the nebulæ on the sky. We may notice here that E_0 nebulæ are almost circular in shape as in this case $b > 0 \cdot 95\, a$.

Among the spiral nebulæ a much larger number consists of a circular nucleus from which two or more spiral arms emerge, whereas in the second type of spiral nebulæ whose number is much fewer the arms appear to emerge from the ends of a bar-shaped mass.

The remaining 3 per cent. of the nebulæ including the two Magellanic clouds are of irregular shape.

The spiral shapes of the nebulæ raise great difficulties and several theories have been suggested to explain them. We shall discuss briefly the more important theories in this paper.

1. *Jeans' Theory.*—Jeans worked out the case of a rotating compressible mass and obtained a series of configurations with increasing rates of rotation. The surfaces of equi-density will clearly coincide with equi-potential surfaces. Here the minor axis OZ is the axis of rotation. Over the boundary we get $\nu + \frac{1}{2}\omega^2(x^2+y^2) =$ constant. Where ω is the angular velocity. If ν_1 be the potential at any point of the equator and v_2 the potential at one of the poles, $\nu_2 = \nu_1 + \frac{1}{2}\omega^2 a^2$ (where a is the equatorial radius). If $\bar{\rho}$ is the mean density and b the polar radius, then $M = \frac{4}{3}\pi\bar{\rho}\, a^2 b$ and $\nu = \frac{\gamma M}{a}$ and $\nu_2 = \frac{\gamma M}{b}$ approximately.

Jeans thus finds that $\frac{4e}{3} = \frac{\omega^2}{2\pi\gamma\bar{\rho}}$ where ellipticity $e = \frac{a-b}{a}$. From observed ellipticity of nebular forms we can find the values of $\frac{\omega^2}{2\pi\gamma\bar{\rho}}$, and a method can be found out for calculating $\bar{\rho}$.

Bok has applied Jeans' method to Roche's compressible model of a massive point nucleus surrounded by an atmosphere of negligible total mass. He has further assumed that this configuration is rotating with uniform angular velocity around the Z axis. As before the surfaces of equi-density would coincide with equi-potential surfaces.

Let us consider the equatorial section of the configuration. We have at the equator if Ω is the total potential $\Omega = \frac{\gamma M}{a} + \frac{1}{2}\omega^2 a^2$.

Also if p be the pressure then, $\frac{dp}{da} = \rho \cdot \frac{d\Omega}{da}$.

Therefore $\frac{dp}{da} = \rho\left(-\frac{\gamma M}{a^2} + \omega^2 a\right)$.

If a is small then $\frac{\gamma M}{a^2} >> \omega^2 a$ and $\frac{dp}{da}$ is negative, and so this is a stable configuration since p decreases as a increases.

The pressure gradient vanishes for the value a_0, where $a_0{}^3 = \frac{\gamma M}{\omega^2}$, we then get $\Omega_0 = \frac{3}{2}\omega^2 a_0{}^2 = \frac{3}{2}(\omega\gamma M)^{\frac{2}{3}}$

So the equation for the limiting equipotential surface becomes $\frac{1}{2}\omega^2(x^2+y^2) + \frac{\gamma M}{r} = \frac{3}{2}(\omega\gamma M)^{\frac{2}{3}}$.

The critical value of the mean density is obtained from $\frac{\omega^2}{2\pi\gamma\rho} = 0 \cdot 36$.

Bok suggests that for $\bar{\rho} > \rho_0$, the surplus matter would probably stream out in equatorial plane.

* From a lecture delivered at the Mathematical Conference, Lucknow, March 16, 1938.

Jeans supposes that as soon as the ellipsoidal nebulæ becomes unstable, gas will be ejected out in the equatorial plane along the spiral arms and ultimately stars would be formed by condensations along these arms.

The main difficulty in Jeans' theory is that he supposes that ellipsoidal nebulæ by their very nature are gaseous in composition and this is not corroborated by observation. His theory rules out the possibility of star clouds existing in ellipsoidal nebulæ. This is also not verified by observation. Our own galaxy is probably a highly flattened system of ellipsoidal form and is known to contain many star clouds.

Lindblad has also calculated the mass of the Andromeda nebula and he has found that the luminous part of the nucleus is composed mainly of stars like our Sun.

The observed spectra of ellipsoidal nebulæ cannot be explained by Jeans' theory about their composition.

Jeans' theory would give a long-time scale for the age of stars which are believed to be formed in the spiral arms of the nebulæ but Bok has pointed out that long-time-scale presents many difficulties which would disappear if you accept the short-time-scale.

2. *Brown's Theory.*—Brown assumes that originally every spiral nebulæ was a highly flattened homogeneous ellipsoid of revolution inside which the gravitational force of attraction is of the form $-Ax$, $-Ay$, $-Cz$. Later on, minor variations in the uniform density are assumed to be due to perturbations caused by rather close encounters with passing galaxies. These perturbations also lead to the formation of the spiral arms of the nebulæ. Brown further concludes that after the encounter the spiral arms gradually coil up and the nebulæ ultimately reverts to its original ellipsoidal shape. According to him the spiral form is not a permanent structure and its formation is being repeated more or less periodically by encounters.

Inside the homogeneous ellipsoid of revolution all stars will have the same angular velocity. Brown superposes on the uniform density, the additional small density

$$\delta_1\sigma = \lambda \cos\left(2\phi - 2q \log \frac{r}{a} - 2\alpha\right) \sin^2\theta,$$ where

$\lambda << $ constant density. Here r and ϕ are the co-ordinates in the equatorial plane and θ is the angle which the radius vector makes with the polar axis. We now see that $\phi - q \log \frac{r}{a} = \alpha$ is equation to an equiangular spiral.

We may also notice here that the factor $\sin^2 \theta$ in the superposed density leads to rapid density decrease perpendicular to the equatorial plane. Here q is the tangent of the angle which the radius vector makes with the tangent at any point of the spiral. As the superposed density is a periodic term, it may cause the resonance trouble. So in Brown's theory it is necessary to add another term $\delta_2\sigma = -\mu \log \frac{r}{a}$, where μ is of the same order of magnitude as λ. This extra term leads to a gradual density decrease as r increases.

We also find that on account of superposed density the rate of angular motion is slower in the front regions than in the back regions. Hence it leads to the gradual coiling up of the spiral like a watch spring.

Assuming the density of galaxies in intergalactic space to be 10^{-72}, Brown calculates that in every 10^{12} years there will be an encounter which would lead to spiral formation. There is one great drawback in Brown's theory—apart from the small superpositions he assumes uniform density throughout the galaxy even perpendicular to the equatorial plane; but this is not borne out by observation. Moreover there is also no evidence yet to show that spiral formation is a periodic phenomenon.

3. *The Theory of Vogt and Lambrecht.*—In accordance with this theory most of the mass of the spiral nebulæ is supposed to be concentrated in the nucleus. So that everywhere outside the nucleus the gravitational force may be taken to vary as $\frac{1}{r^2}$, r being the distance from the centre of the nucleus. In addition to the force of attraction they assume that there is also a force of repulsion proportional to the distance from the centre of nucleus. If h is the areal constant, $u = \frac{1}{r}$ and θ is the position angle, we get our equation $\frac{d^2u}{d\theta^2} + u = \frac{\gamma M}{h^2} - \frac{a^2}{h^2u^3}$. From this

we get,

$$\theta + \text{const.} = \int \frac{dr}{r^2 \sqrt{\frac{2\gamma M}{rh^2} + \frac{a^2 r^2}{h^2} - \frac{1}{r^2} + c}}.$$

If a^2 has a small value between 0 and $\frac{\gamma M}{r^3}$ then spiral orbits will be formed. For large values of a^2 convex hyperbolas will result.

Vogt finds that for Andromeda nebula a^2 is small and equal to $\frac{1}{50}\frac{\gamma M}{r^3}$.

Criticism of Vogt's Theory.—The main objection to Vogt and Lambrecht's theory is the assumption of the force of repulsion. They did not give any explanation of its cause. It is analagous to cosmic force of repulsion in Einstein's theory of Relativity which is believed to expalin the so-called phenomenon of recession of g^laxies. The theory does not satisfactorily explain why should there be two arms in the spiral nebulæ. Lambrecht tries to explain it by ascribing it to encounters. Moreover there is no justification in assuming that the mass is concentrated in the nucleus. Hacker has also criticised Vogt's theory. He has pointed out that the spiral orbits would also have a point of inflexion. Moreover the form of the spiral orbit as given by Vogt's theory does not very well agree with the observed spiral arms of the nebulæ. The orbit according to Vogt's theory proceeds rather steeply outwards after reaching the point of inflexion.

4. *Lindblad's Theory.*—Lindblad assumes that there is a small condensed nucleus which is surrounded by a spheroidal galaxy of stars of uniform density from which spiral arms emanate. He takes the mass of the nucleus to be λM and the total mass to be $(\lambda + 1)$ M. For orbits in equatorial plane Lindblad gets the equation

$$\left(\frac{du}{d\theta}\right)^2 = -u^2 + \frac{2}{h^2}\int \frac{f}{u^2}\,du,$$

where $f =$

$$\frac{\lambda\gamma M}{r^2} + \frac{3}{2}\frac{r\gamma M}{a^3 e^3}\left[-\frac{ae}{r}\sqrt{\frac{1-a^2e^2}{r^2}} + \text{arc sin}\,\frac{ae}{r}\right]$$

where e is the eccentricity of the meridional section, a is the semi-major axis and θ is the longitude. f is measured positively towards the centre. The first term in f arises from the nucleus and the remaining terms from the outer ellipsoid of revolution.

After substituion and integration we get

$$\left(\frac{du}{d\theta}\right)^2 = -u^2 + \frac{3}{2}\cdot\frac{\gamma M}{h^2 a^3 e}\left[\left(2a^2 - \frac{1}{e^2u^2}\right)\text{arc sin}\,aeu + \frac{a\sqrt{1-a^2e^2u^2}}{eu} + \frac{4}{3}\lambda a^3 eu\right] + c.$$

Lindblad has proved that tidal action will cause a slight perturbation which will not change h but will increase c by a small amount δc. Lindblad has put symbolically the above equation as $\left(\frac{du}{d\theta}\right)^2 = \phi(u) + c$.

If $\frac{1}{u_0}$ be the semi-major axis of undisturbed elliptic orbit just before perturbation, then the spiral form would be possible provided $\phi''(u_0)$ is positive. If $\phi''(u_0)$ is negative no spiral form is possible. So when $\phi''(u_0) = 0$, we get the transitional case. If there be little mass in the nucleus, that is if λ is small, there is great possibility for the formation of the spiral arms.

Lindblad's Recent Investigations.—In a recent paper Lindblad has assumed that the stellar system may be divided into a number of sub-systems of approximately the same extension in the galactic plane but with different degrees of flattening towards this plane and different speed of rotation at the same distance from the axis.

The sub-system of greatest flattening towards the galactic plane is represented by the Milky Way clouds and one of the smallest flattening and smallest velocity of rotation is represented by the distant globular culsters. He also assumes that no given natural system of objects would belong to a single sub-system, but would spread over a number of such sub-systems. Spectro-graphical determinations of the rotational motions of the nebulæ show a fairly uniform angular speed of rotation in the central parts of the systems. It is extremely probable that in the outer less dense regions, the angular velocity is far less than in the central parts and that in these regions it decreases rapidly.

Perhaps the transition between these two states of motion is fairly rapid. Lindblad has shown that asymptotic spiral orbits will naturally occur in such cases. Moreover due to the escape of high velocity objects and formations of condensations all rotating systems will become flattened with age and the condition that is necessary for formation of asymptotic spiral orbits will automatically be realised after some time.

Lindblad has shown that in the outer regions of the central system there would be a marked tendency towards a formation of local condensation of matter. He suggests that as the result of an encounter between two such condensations near the edge of the spheroidal system, one condensation may move in an asymptotic orbit and cause the "initial disturbance" to produce the spiral form. So according to him tidal ejections due to outside cause such as encounter with passing galaxies, are not essential for the formation of two or more spiral arms.

Lindblad has also suggested, contrary to Vogt, that the points of ejections should have a tendency to recede relatively to the matter at the edge of the central system in opposite direction to the rotation. It is quite possible that these points of ejections may be fixed in space. According to Lindblad when the spiral arms are fairly thin, the decrease of size and mass of the central system may be neglected and spiral arm may without much error be supposed to indicate the real orbit of a single particle of the arm (M. 81). In the case of nebulæ of heavier arms continuous decrease of the central body by the formation of arms must be taken account of and the arm no longer represents exactly the orbit of one of its particles (M. 51).

Lindblad's theory seems to be much more tenable than any other theory so far put forward as he does not make any untenable assumption concerning the structure of ellipsoidal and spiral nebulæ.

Wellman's Theory.—Wellman has assumed that a slow expansion of the system due to a secular decrease of its mass would make all elliptical orbits of the system take the shape of very close spirals. He assumes that there is a difference in the rate of expansion between outer and inner orbits and that the spiral arms are the loci of the ejected matter that comes out as a steady outflow from diametrically opposite points of the system. Although these assumptions are very interesting they can hardly be applied to the actual system.

Jehle has tried to explain the spiral arm by generalised theory of wave-mechanics.

Narlikar and Moghe have suggested that the two-dimensional geodesics of an expanding spherical universe have spiral arms, but their theory has been criticised by McCrea. Narlikar has also replied to the criticisms of McCrea.

In conclusion, it may be mentioned that no satisfactory theory about the formation of spiral arms can be established until and unless we can have a more thorough knowledge of the constitution of the galaxy. The recent investigations by Plaskett and Pearce tend to show that inter-stellar matter extends throughout the local cluster and that there is a gaseous substratum involving the local system and perhaps extending beyond it "a continuum rather than a cloud". They have also found that the inter-stellar diffused matter partakes in the rotation of the galaxy and they believe that the whole galactic system is immersed in a gaseous substratum consisting of atoms of various elements, the density being of the order of 10^{-26}. "The separate atoms while obeying the ordinary gas laws partake in a rotational movement around a distant central mass in galactic longitude 325°. The observed rotational accelerations seem to be the same as for the stars so that the atoms are not subjected to any appreciable radiation pressure from the central mass."

From observational matter now available we may accept the view that the space in our stellar system, at least to the distance observed, is pervaded by very diffuse matter in the gaseous form of a composition similar to stellar matter and ionised by general radiation of stars.

The authors are studying the problem of the spiral arms by assuming a central rotating homogeneous mass of finite dimensions (not small) surrounded by a rotating gaseous and ionised matter of low density, so that apart from the gravitational forces, electrical forces are also to be taken into account.

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