Fathoming the unfathomable

Infinity and the Mind. Rudy Rucker. Princeton Univ Press, 41 William Street, Princeton, New Jersey 08540, USA. 1995. Price: $ 12.95. 342 pp.

The concept of *infinity* is at once romantic, intriguing and challenging to the pure thought process. Like its 'reciprocal' partner *zero*, infinity has exerted the minds of great thinkers almost since the dawn of civilization. The earliest evidence from recorded history comes from Greek philosophers like Pythagoras, Plato and Aristotle from a rather 'negative' angle: its very name *apeiron* (unbounded, infinite, indefinite, or undefined) bearing eloquent testimony to the *unease* (born out of a lack of comprehension?) with which they looked upon it. All these thinkers firmly believed that any given aspect of the world could be described by a *finite* arrangement of 'natural' numbers. The encounters with 'infinities in the small' were no less vexing, as Pythagoras and his colleagues had a taste of this 'issue' when they could not represent the 'square-root of two' by a *finite* ('rational') arrangement of 'natural' numbers. The nearest move for coming to terms with *apeiron* was made by Aristotle who went as far as inventing the notion of the *potential infinity* which could be visualized in terms of natural numbers, versus the *actual infinity* which could not be so described. The latter came nearest to the definition of God at the hands of philosophers (like Plotinus) and missionaries (like St. Augustine), as 'Absolute One' who stands outside of number, 'and so is under no limit w.r.t. anything external or internal' (as any such determination would bring something of the *dual* into it). Further, He could think 'infinite thoughts' simultaneously. This was the forerunner of an already modern position, which now goes by the name of the Platonic philosophy (see below). The next landmark in the encounter with *infinity*, which heralded the modern attitude towards the 'actual infinite' in mathematics, came through a novel idea introduced by Galileo based on a *repudiation* of the usual counting method applicable to *finite-sets*, through his insistence that *infinite sets* obey a different 'arithmetic' from finite sets. The precise modification of this arithmetic was to take shape some 250 years later (1880s) at the hands of George Cantor, the Father of modern notion of infinity. Galileo's attitude towards '∞' in turn had been conditioned by his desire to treat space and time as *continuous variables* which was at the root of the development of the calculus of infinitesimals by the standard 'limit' process.

This brings us to the modern era in which the concept of infinity exists in all its forms, potential and actual, spatial and temporal, as well as large and small; where the principal players have been Cantor, Hilbert, Goedel and Von Neumann among others. Cantor's theory of *transfinite numbers* was the first to offer a glimpse of the vastness of *infinity* through a *new* form of counting in which an *ordinal number* is defined as the *set* of all *natural numbers preceding* it. This produces a well-defined hierarchy of 'infinities' which play the role of 'landmarks' in the journey towards bigger and bigger infinities. The first such landmark is a mere 'ω' = lim(n). The next stages are obtained by continuing the process through $\omega^2, \omega^3, \ldots, \omega^\omega$ (exponentiation); $\omega^{\omega^\omega} \equiv {}^{\omega}\omega$ (tetration); and so on and on. When this process of counting is exhausted, one arrives at the next benchmark: aleph one—a different 'kind of infinity' from the ω-series. The 'complete' hierarchy all the way to God (Ω) where the buck stops (?) is[1]

$$0, \ldots 6, \ldots W, \ldots \omega;\ \omega + \omega;\ \text{aleph-1};$$
$$\text{aleph-}\omega;\ \theta;\ \rho;\ \lambda;\ \alpha;\ \ldots \text{'}\Omega\text{'}.$$

This is only one aspect of the mind-boggling nature of infinity—its 'unfathomability' which the author of the book (who is a distinguished set theorist and science fiction writer) narrates in a most entertaining fashion. He then goes on to discuss some *logical paradoxes* named after Berry, Richard and 'The Liar' which point to the existence of mental concepts that defy any exact formalization, and are in some sense 'infinite', insofar as the human mind 'understands' them. The exposition is non-technical, but the author nevertheless maintains a strictly logical rigour in a delightful narration of 'The Unnameable'. In the modern age of ever expanding computer logic and storage capacity, these purely logical problems which refer respectively to 'limitations of language', 'randomness' (in the sense of having no finite description), and 'the meaning of truth' (?) (*not* mathematically definable) reveal the chinks in this formidable armour in a human reservoir to tackle these basically 'semantics' aspects of such paradoxes. The last one exposes rather dramatically the finite capacity of the most powerful computer logic in its pathetic helplessness before the celebrated Goedel Incompleteness Theorems of mathematical logic. This result which has a deep philosophical significance, brings out the intrinsic limitations of any basically *finite* (man-made) system, howsoever big, to capture *all* the aspects of the *infinite* (a higher reality—God?); see below.

The last but not least issue of this logical nature concerns the *One—many problem*: Can 'everything' be regarded as a 'unity', i.e. a *single* definite thing? In other words, is the world a one or many? For an analysis of this problem, Rucker employs the modern language of set theory in his logical efforts to reduce 'diversity to unity'. According to Cantor, the founder of set theory: A set is a 'many' which 'allows' itself to be thought of as 'one'. Cantor believed that a pure set has an 'ontological' existence irrespective of notice by anyone, and by the *thought* $\Leftrightarrow$ set correspondence the same is true of pure *thought*. Are 'sets' members of themselves? A logical pursuit of this question again gives rise to an infinite regression just as in the Berry, Richard and Truth paradoxes, which can be resolved only by demanding that a *class* of all *sets* is *not* a set.

This last point brings one perilously close to the situation relative to the metaphysical or theological *absolute* which is *not rationally knowable* (all thinkers agree on this). At least the logic of set theory (Ernst Zermelo) leads to the 'reflection principle' which paraphrased in simple terms reads something like this; the mind does not attain God, but to what is beneath Him. (This is not just pious phrase-mongering but rigorous logic.) In this scenario there are two classes of mathematicians: i) The Platonist who believes in the ontological existence of infinite sets; ii) the Formalist who believes that there are only finite descriptions of mathematical theories. In this dual scenario, the role of Intuitionism is to synthesize the two viewpoints:

Actual Infinity $\Leftrightarrow$ ONE;
Potential Infinity $\Leftrightarrow$ MANY.

The 'Book' which made its first appearance earlier[1] under a different publisher, explains the *logic* of infinity and related concepts in a non-technical fashion and is therefore of value to the non-specialist in mathematics who is interested in its logical basis without bothering about the technicalities. More such books are appearing in the market, covering very similar themes[2]. The 'pioneer' in this regard was perhaps George Gamow's entertainer of half a century earlier[3], which provides the 'appetizer' for them. The present book[1] however also deals with some technical aspects like the formal proofs of the Goedel Theorems and a fuller exposition of the 'transfinite cardinals' of Cantor theory, topics which are not easily found in most textbooks, and should thus be useful to the more serious student of mathematics, who is further helped to develop his logical thinking capacity without external aid, through a collection of puzzles/paradoxes whose answers are given at the end of the book. Another item of delight is the story of Rucker's personal encounters with the 'Mathematical Genius' (who very rarely granted interviews). The serious student of mathematics may profitably combine books of this kind with more 'substantial' classics like Courant's 'What is Mathematics?'[4] to grasp the abstract ideas and methods of mathematics in a more palatable and wholesome fashion.

To summarize, the book whose style will please both the uninitiated and the more serious thinker in mathematics, ventures to convey to both the abstract concept of *infinity* in all its forms, both 'potential' and 'actual', spatial and temporal, large and small. The main lesson seems to be that even mathematics as an 'exact science' has its limitations (thanks to the Goedel Theorems and the like) which have led to at least *two schools* of thought: Platonistic versus Formalistic, and encouraged the (more holistic) intuitionists to lean towards Mysticism[1], a subject which is beyond the scope of this review.

1. Rucker, R., *Infinity and the Mind*, Birkhauser, Boston, 1982.
2. Barrow, J. D., *Pi in the Sky*, Little-Brown, Boston, 1993.
3. Gamow, G., *One, Two, Three, Infinity*, 1950.
4. Courant, R. and Robbins, H., *What is Mathematics*, Oxford Univ. Press, NY, 1978.

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Conversations on Mind, Matter and Mathematics, by Jean-Pierre Changeux and Alain Connes, edited and translated by M. B. DeBevoise. Princeton University Press, 41, William Street, Princeton, New Jersey, USA. 1995. Price: $24.95; £19.95. 260 pp.

Many thinkers over millennia have pondered over the problem of the relationship between mind and matter. In the secular part of the Western tradition, avoiding reference to individual development and destiny, the origins of such enquiry, as in so much else, go back to the Greeks. Here one encounters the contrasting views of Plato on the one hand, and of Democritus, Epicurus and Lucretius on the other. For Plato, all objects and experiences in the world were shadows or pale imitations of a fixed sphere of ideas, beyond space, time and sensation, where alone perfection and the highest truths were attained. Democritus and his followers adopted in contrast a more pragmatic materialist point of view, giving primacy to experience and knowledge of the world directly accessible to us. Passing over many centuries, we come to Descartes and his 'cut': a sharp distinction between mind and matter, a dualism, with the former not reducible to the latter. In our own times, with stupendous advances in both the physical and the life sciences, gifted scientists have devoted themselves to these questions. Thus one recalls Schrödinger's *Mind and Matter*, subtly influenced both by his wave mechanics and his leanings towards Vedanta; and more recently Max Delbruck's *Mind from Matter? An Essay on Evolutionary Epistemology*, where a deep understanding of Darwinian evolution and Lorenz's ideas on phylogenetic and ontogenetic 'learning' were brought to bear on the problem.

Questions in this realm are easily posed; not so easy however to find convincing answers, or even agreement on the characteristics of an acceptable answer. Some perennial problems that suggest themselves: is the statue already present in the uncut marble before the sculptor goes to work, and does she merely uncover what was pre-existing? Is a new mathematical concept an invention or a discovery, an exploration of a fixed continent of mathematical truths?

Conversations on Mind, Matter and Mathematics is an exhilarating dialogue between a biologist and a mathematician, faithfully recorded and beautifully translated, on these and related problems, with special reference to mathematics. Changeux is an outstanding neurobiologist, from the Lwoff-Monod-Jacob school; while Connes is an equally gifted mathematician with a deep feel for theoretical physics. Much of their conversation is an argument that is never resolved: through his experience in mathematics Connes firmly believes that mathematics exists 'out there' as a territory independent of us, available for exploration. He feels so because of the frequent occurrence of different individuals finding the same answer to a given problem; by the power of axioms to hold within them many consequences which with effort we unravel; and by the apparent independence of many mathematical creations from sensory experience. Equally passionately Changeux—surely influenced by neurobiology— insists on the constructivist viewpoint: mathematics is a product of the human brain, taking inputs from the physical world; all mathematical steps and arguments are specific achievements within the brain, using its material basis and organization, its evolved capacity for logical reasoning. The relative complexity of certain parts of mathematics is no proof of its independent existence or reality. Mathematics as a language to describe nature, so spectacular in physics, is not independent of the brain; even those results which today appear as finished products evolved in societies and cultures, and once did not exist. To Connes' challenge: 'Will we ever "see" a brain conceiving a mathematical idea, a concept, a step or an argument?', Changeux answers confidently—'Yes, one day we will!'.

Such is the heady stuff recorded in these conversations. That such a gifted pair should have come together for this exploration—though they agree to disagree—is remarkable indeed. The range is wide, the pace breathtaking. Perhaps Connes' clearest statement of his position is this: 'It's humility, finally, that forces me to admit that the mathematical world exists independently of the manner in which we apprehend it, that it isn't localized in time and space'. Yet he admits that the tools used to explore this world are products of human culture. In answer, Changeux traces the origins of speech and logic to homo erectus 400,000