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RESEARCH COMMUNICATIONS

Strain determination from three known stretches – A trigonometric solution

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The determination of strain from three coplanar, non-parallel stretches has been a classical problem in structural geology. A simple trigonometric solution to this problem is presented here.

THE problem of determination of the shape of strain ellipse from three known extensions was first recognized by Ramsay[1]. He proposed a Mohr circle solution which was also adopted by Ramsay and Huber[2]. Another graphical method using an alternate Mohr circle construction was proposed by Lisle and Ragan[3]. The graphical Mohr circle method tends to be rather lengthy and less accurate than an equivalent mathematical method. A few workers have therefore attempted alternate mathematical solutions to this problem. Ragan[4] has used matrix inversion and eigenvector determination while Sanderson[5] and De Paor[6] have used other algebraic methods. The simplest of them is by De Paor[6]. In the present paper, a simple equation is derived for the direction of the longest principal strain axis, which has further been used for determination of magnitudes of both the principal stretches. Whereas the '... amount of calculation involved is time-consuming without the aid of a computer...'[6], the present simple expression can easily be evaluated using a hand calculator.

The derivation is based on the appreciation of the fact that to define a strain ellipse, three variables are required, viz. λ_1 (long axis), λ_2 (short axis) and θ (direction of λ_1 with respect to any direction A along which the strain is known). If we know three coplanar non-parallel strain vectors (A, B, and C) with quadratic elongations λ_A, λ_B and λ_C respectively and their angular interrelationship (Figure 1), the shape and orientation of the strain ellipse can be determined.

The inverse quadratic strain along any direction A (λ_1^A $= \theta$) is:

$$\lambda'_A = \lambda'_1 \cos^2\theta + \lambda'_2 \sin^2\theta \text{ [From eqs. (3)–(31) of ref. 1]}$$

Therefore inverse quadratic strain along A, B and C (Figure 1) can be represented by:

$$\lambda'_A = \lambda'_1 \cos^2\theta + \lambda'_2 \sin^2\theta, \quad (1)$$

$$\lambda'_B = \lambda'_1 \cos^2(\theta+\theta_1) + \lambda'_2 \sin^2(\theta+\theta_1), \quad (2)$$

$$\lambda'_C = \lambda'_1 \cos^2(\theta+\theta_2) + \lambda'_2 \sin^2(\theta+\theta_2), \quad (3)$$

where θ_1 and θ_2 are known.

By means of trigonometric relationships:

$$2\cos^2\alpha = 1 + \cos 2\alpha,$$

$$2\sin^2\alpha = 1 - \cos 2\alpha,$$

we get:

$$2\lambda'_A = \lambda'_1 + \lambda'_2 + (\lambda'_1 - \lambda'_2)\cos 2\theta, \quad (4)$$

$$2\lambda'_B = \lambda'_1 + \lambda'_2 + (\lambda'_1 - \lambda'_2)\cos 2(\theta+\theta_1), \quad (5)$$

$$2\lambda'_C = \lambda'_1 + \lambda'_2 + (\lambda'_1 - \lambda'_2)\cos 2(\theta+\theta_2). \quad (6)$$

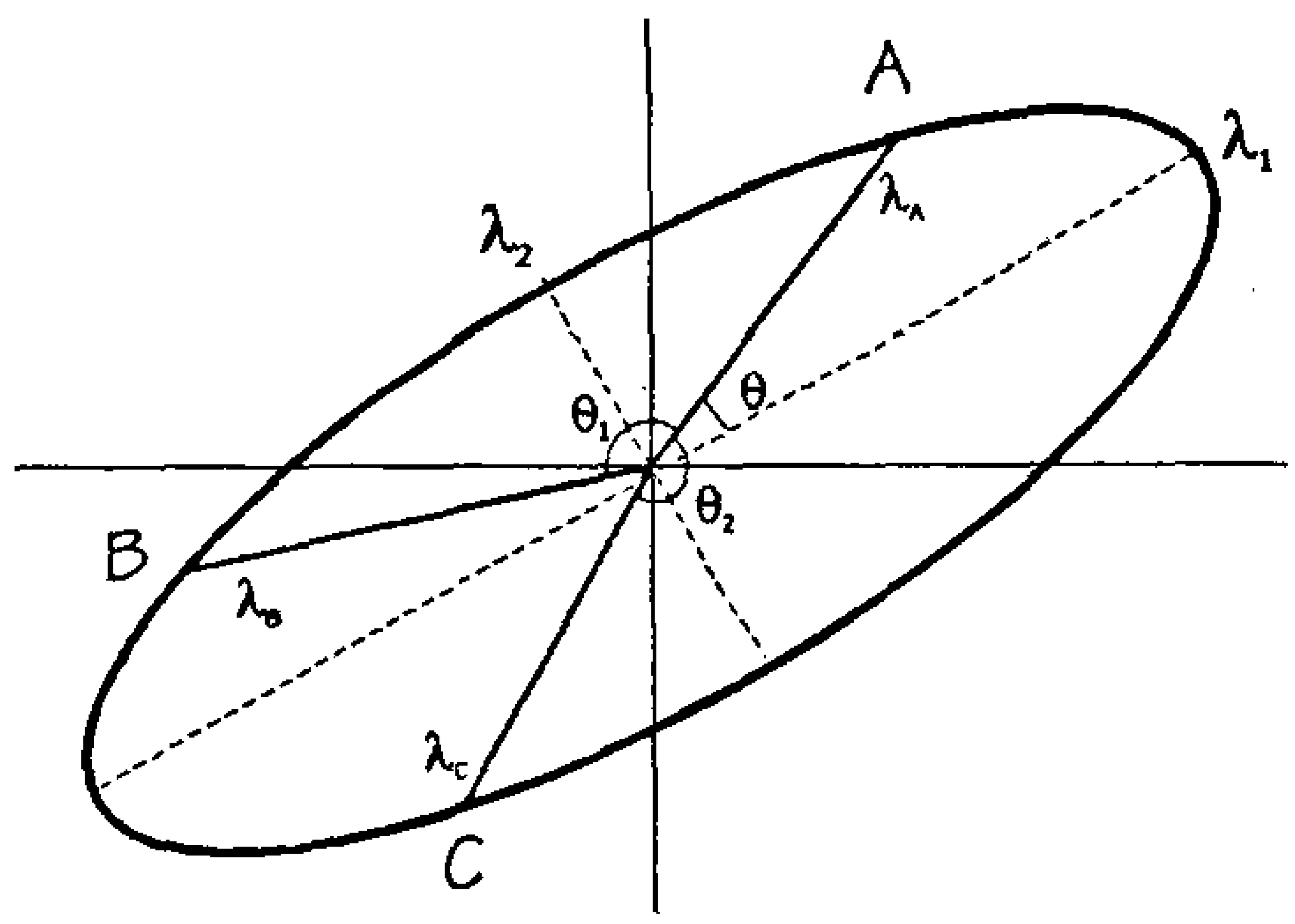
Subtracting eq. (5) from (4) and eq. (6) from (4) we get:

$$2(\lambda'_A - \lambda'_B) = (\lambda'_1 - \lambda'_2)[\cos 2\theta - \cos(2\theta + 2\theta_1)], \quad (7)$$

$$2(\lambda'_A - \lambda'_C) = (\lambda'_1 - \lambda'_2)[\cos 2\theta - \cos(2\theta + 2\theta_2)]. \quad (8)$$

Because for any value of α and β:

$$\cos\alpha - \cos\beta = 2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2},$$



igure 1. Strain vectors A, B and C of magnitudes λ_A, λ_B, λ_C in a rain ellipse.

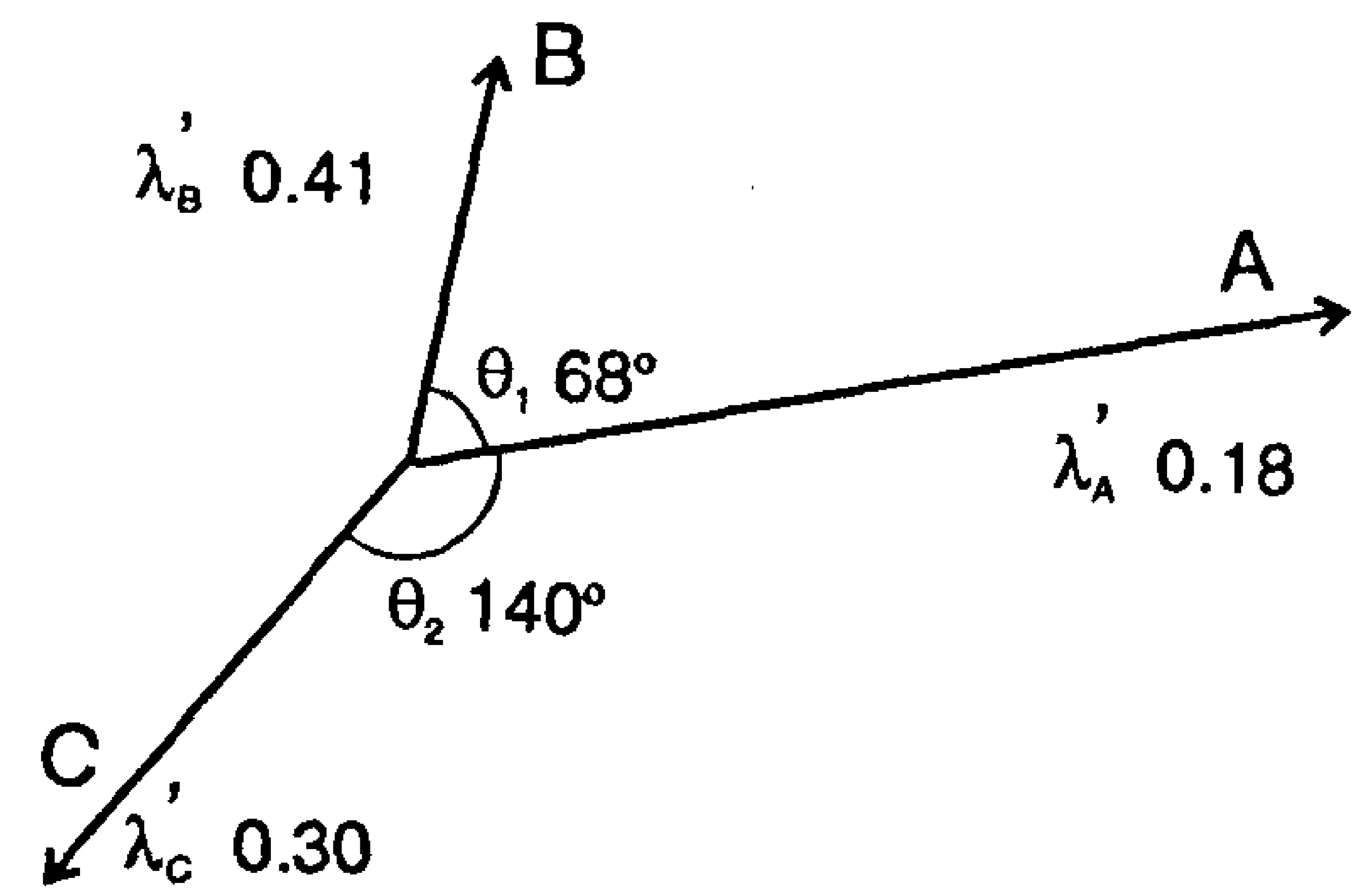


Figure 2. The modified three belemnite problem[2].

qs (7) and (8) become:

$$2(\lambda'_A - \lambda'_B) = (\lambda'_1 - \lambda'_2)\left[2\sin\frac{4\theta + 2\theta_1}{2}\sin\frac{2\theta_1}{2}\right], \quad (9)$$

$$2(\lambda'_A - \lambda'_C) = (\lambda'_1 - \lambda'_2)\left[2\sin\frac{4\theta + 2\theta_2}{2}\sin\frac{2\theta_2}{2}\right]. \quad (10)$$

lence

$$\lambda'_1 - \lambda'_2 = \frac{\lambda'_A - \lambda'_B}{\sin(2\theta + \theta_1)\sin\theta_1}, \quad (11)$$

$$\lambda'_1 - \lambda'_2 = \frac{\lambda'_A - \lambda'_C}{\sin(2\theta + \theta_2)\sin\theta_2}. \quad (12)$$

:ombining eqs (11) and (12) we get:

$$\frac{(\lambda'_A - \lambda'_B)/\sin\theta_1}{\sin(2\theta + \theta_1)} = \frac{(\lambda'_A - \lambda'_C)/\sin\theta_2}{\sin(2\theta + \theta_2)}. \quad (13)$$

.et

$$a = \frac{\lambda'_A - \lambda'_C}{\sin\theta_2}, \quad (14)$$

$$b = \frac{\lambda'_A - \lambda'_B}{\sin\theta_1}. \quad (15)$$

;ubstituting a and b in eq. (13) and multiplying with the lenominators we get:

$$a \sin(2\theta + \theta_1) = b \sin(2\theta + \theta_2). \quad (16)$$

3ecause for all α and β

$$\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta,$$

Therefore eq. (16) yields

$$a \sin 2\theta \cos\theta_1 + a \cos 2\theta \sin\theta_1 = b \sin 2\theta \cos\theta_2 + b \cos 2\theta \sin\theta_2, \quad (17)$$

or

$$\sin 2\theta\,(a \cos\theta_1 - b \cos\theta_2) = \cos 2\theta\,(b \sin\theta_2 - a \sin\theta_1) \quad (18)$$

Hence,

$$\tan 2\theta = -\frac{a\sin\theta_1 - b\sin\theta_2}{a\cos\theta_1 - b\cos\theta_2}. \quad (19)$$

Eq. (19) is the expression for the direction of A with respect to the longest principal axis of the strain ellipse θ. A choice is needed between two values for $\tan 2\theta$ depending on the quadrant in which θ lies. However, if the longest-known stretch is taken as A as a convention, the θ would generally lie in the first quadrant. Eqs (1)–(3) can now be solved.

The problem of three boudinaged belemnites described by Ramsay and Huber[2], is chosen as an example, the same that was also selected by others[3,4,6] for validation of their methods. In keeping with the convention proposed above, the belemnites A, B and C of Ramsay and Huber[2] have been renamed as B, A and C respectively (Figure 2).

The calculations show that the direction of A from the largest principal extension is –3.8° (or the direction of largest principal extension with respect to A is 3.8° positive clockwise) and λ'_1 and λ'_2 are 0.179 and 0.436 respectively which are in agreement with the values 4°, 0.18 and 0.43 of graphical method of Ramsay and Huber[2]; 4.3°, 0.18 and 0.43 from calculation by equation of Ragan[4]; and 4.32°, 0.175 and 0.43 using equations of De Paor[6].

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