The Pancharatnam phase: New applications in stellar interferometry

P. Hariharan and W. J. Tango

School of Physics, University of Sydney, NSW 2006, Australia

The Pancharatnam phase has been regarded for many years as a phenomenon of purely academic interest; however, recent work has shown that it has many practical applications including its use in a long-base stellar interferometer to permit measurements of faint stars.

The Pancharatnam phase

In what has become a seminal paper, Pancharatnam[1] was able to show that if a beam of light was taken from one polarization state, without introducing any phase changes, through two other polarization states and back to its original state, it could exhibit a phase shift. He also showed that the magnitude of this phase shift (the Pancharatnam phase) was equal to half the solid angle subtended at the center of the Poincaré sphere by the circuit traversed by the point representing the state of polarization of the beam.

The Poincaré sphere

The Poincaré sphere is a convenient way of representing the state of polarization of a beam of light which also makes it very easy to visualize the effects of retarders[2,3].

As shown in Figure 1, right-circular and left-circular polarized states are represented by the North and South poles of the sphere, while linearly polarized states lie on the equator, with the plane of polarization rotating by 180° for a change in longitude of 360°. The effect of passage through a birefringent plate with a retardation δ, whose fast axis is at an angle θ with the vertical, is represented by a rotation of the sphere by an angle δ about a diameter running through a point on the equator at a longitude 2θ.

A system that can be used to demonstrate the Pancharatnam phase with a linearly polarized beam consists of a $\lambda/2$ plate mounted between two $\lambda/4$ plates[1,4]. The two $\lambda/4$ plates have their optic axes fixed at an azimuth of 45°, while the $\lambda/2$ plate can be rotated by known amounts. In this arrangement (known as a QHQ phase-shifter), rotation of the $\lambda/2$ plate through an angle θ transports the input vertically polarized state represented by the point A_1 around the loop $A_1SA_2NA_1$ on the Poincaré sphere, and shifts the phase of the vertically polarized output beam by 2θ.

Achromatic phase-shifting

Because the Pancharatnam phase is a geometric one, and, therefore, a topological phenomenon, it is, in principle, achromatic. Even with a simple QHQ phase-shifter, the variation of the phase shift with the wavelength is quite small over a wide range of wavelengths[5]. With such a phase-shifter in one arm of an interferometer, the total phase difference between the beams is

$$\phi_{\text{total}} = (2\pi/\lambda)\, d + \phi, \tag{1}$$

where d is the optical path difference and ϕ is the Pancharatnam phase. Accordingly, the effect of varying the Pancharatnam phase is also to move the interference fringes across the field of view. However, unlike the dynamic phase introduced by a change in the optical path, which depends on the wavelength, the Pancharatnam phase is the same for all wavelengths. The effect of a change in the Pancharatnam phase of $2m\pi$, where m is an integer, is therefore to move the fringes formed at all wavelengths by an integral number of fringe spacings, so that the interference pattern formed with white light returns to its original configuration. As shown in Figure 2,

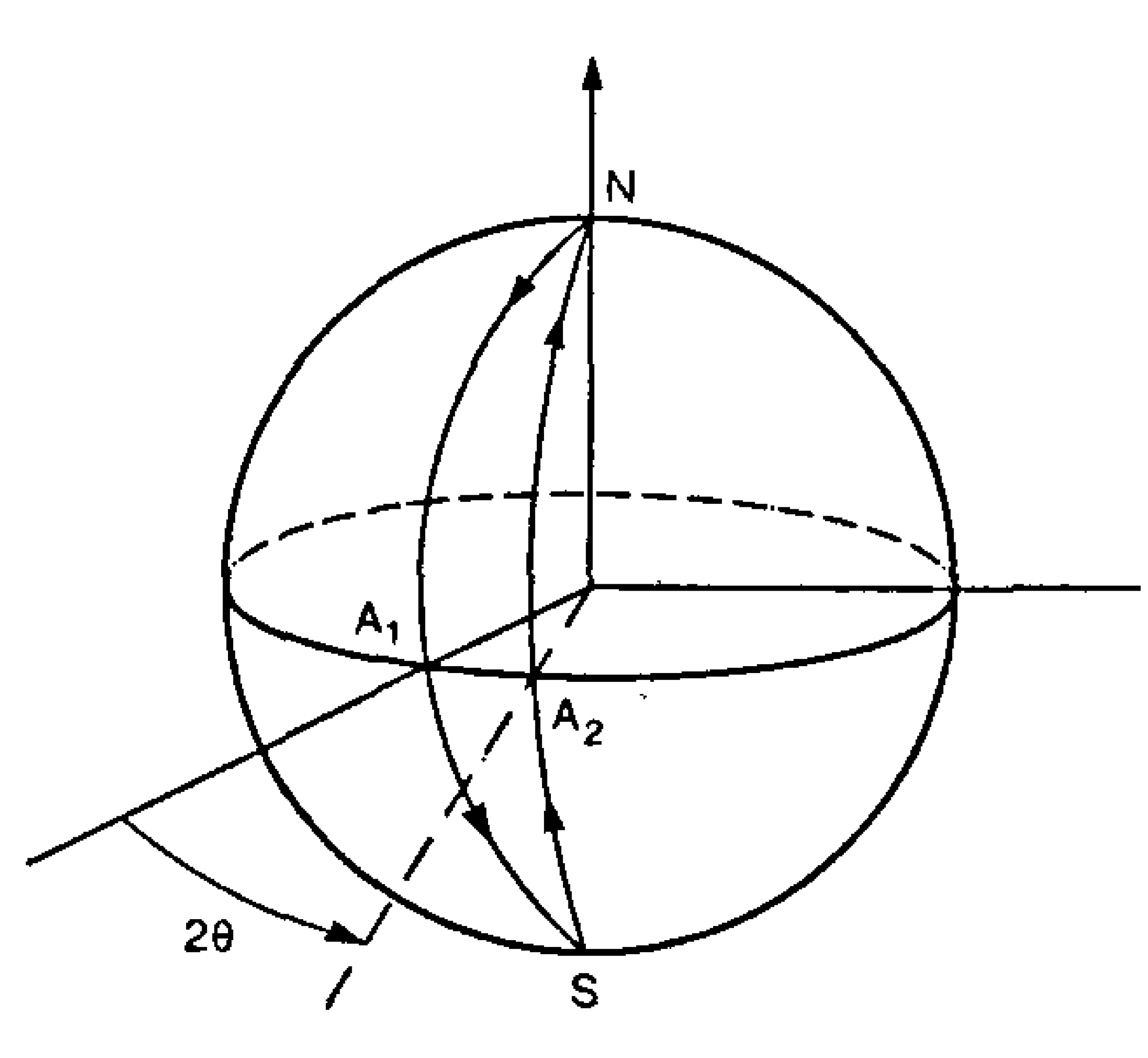

Figure 1. The Poincaré sphere: effect of a QHQ phase shifter on the state of polarization of a beam of light.

the intensity at any point in the interference pattern varies between its maximum and minimum values, but the position of the fringe envelope in the field of view remains unchanged[6].

If, then, four measurements are made of the intensity in the fringe pattern corresponding to additional phase differences of 0°, 90°, 180° and 270°, the visibility of the interference fringes can be calculated using the relation

$$V_d = \frac{2[(I_0 - I_{180})^2 + (I_{90} - I_{270})^2]^{1/2}}{I_0 + I_{90} + I_{180} + I_{270}}, \qquad (2)$$

which, normally, is valid only for monochromatic light.

The Pancharatnam phase has been regarded for many years as a phenomenon of purely academic interest; however, the fact that it is intrinsically achromatic has led to the development of new techniques in surface profiling[7] and interference microscopy[8]; a recent application is in stellar interferometry where it makes possible measurements on faint stars.

Stellar interferometry

Since the dimensions of a star are very small compared with its distance from the earth, it follows, from the van Cittert–Zernike theorem, that the complex degree of coherence between the light vibrations from the star reaching two points on the earth's surface is given by the normalized Fourier transform of the intensity distribution over the stellar disc. The angular diameter of the star can, therefore, be obtained from a series of measurements of the complex degree of coherence at pairs of points separated by different distances. This can be done by observations of the visibility of the fringes in an interferometer which samples the wave field from the star at two such points.

In Michelson's stellar interferometer[9], as shown schematically in Figure 3, four mirrors M_1, M_2, M_3, M_4 were mounted on a 6 m long support on the 2.5 m (100 inch) telescope at Mt Wilson. The light from the star received by the two mirrors M_1, M_2, whose spacing could be varied, was reflected by the fixed mirrors M_3, M_4, to the main telescope mirror, which brought the two beams to a focus at O.

In this arrangement, when the two images of the star are superimposed, and the two optical paths are equalized, the visibility of the interference fringes is

$$V = |\mu_{12}|, \qquad (3)$$

where μ_{12} is the complex degree of coherence of the wave fields at M_1 and M_2. When M_1, M_2 are close together, V is nearly equal to unity, but as D, the separation of M_1 and M_2, is increased, V decreases until, eventually, the fringes vanish.

If we assume the stellar disc to be a uniform circular source with an angular diameter 2α, the visibility of the fringes is, from eq. (3)

$$V = 2J_1(u)/u, \qquad (4)$$

where $u = 2\pi\alpha D/\lambda$. The fringe visibility then varies with the separation of the mirrors, as shown in Figure 4, dropping to zero when

$$D = 1.22\lambda/2\alpha\,. \qquad (5)$$

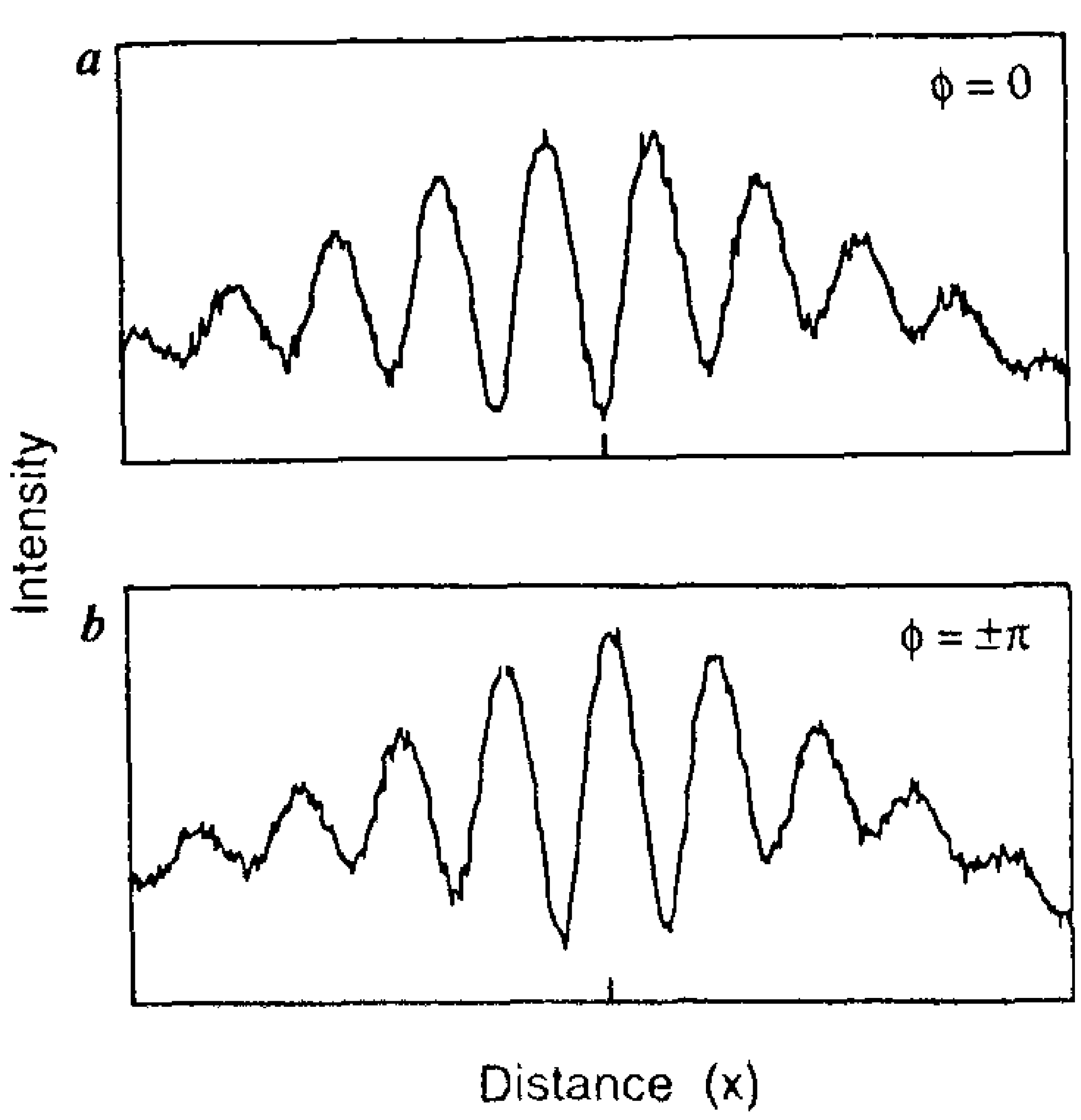

Figure 2. Effect of changes in the Pancharatnam phase on the intensity in the interference pattern formed with white light: (*a*), $\phi = 0°$; (*b*), $\phi = 180°$.

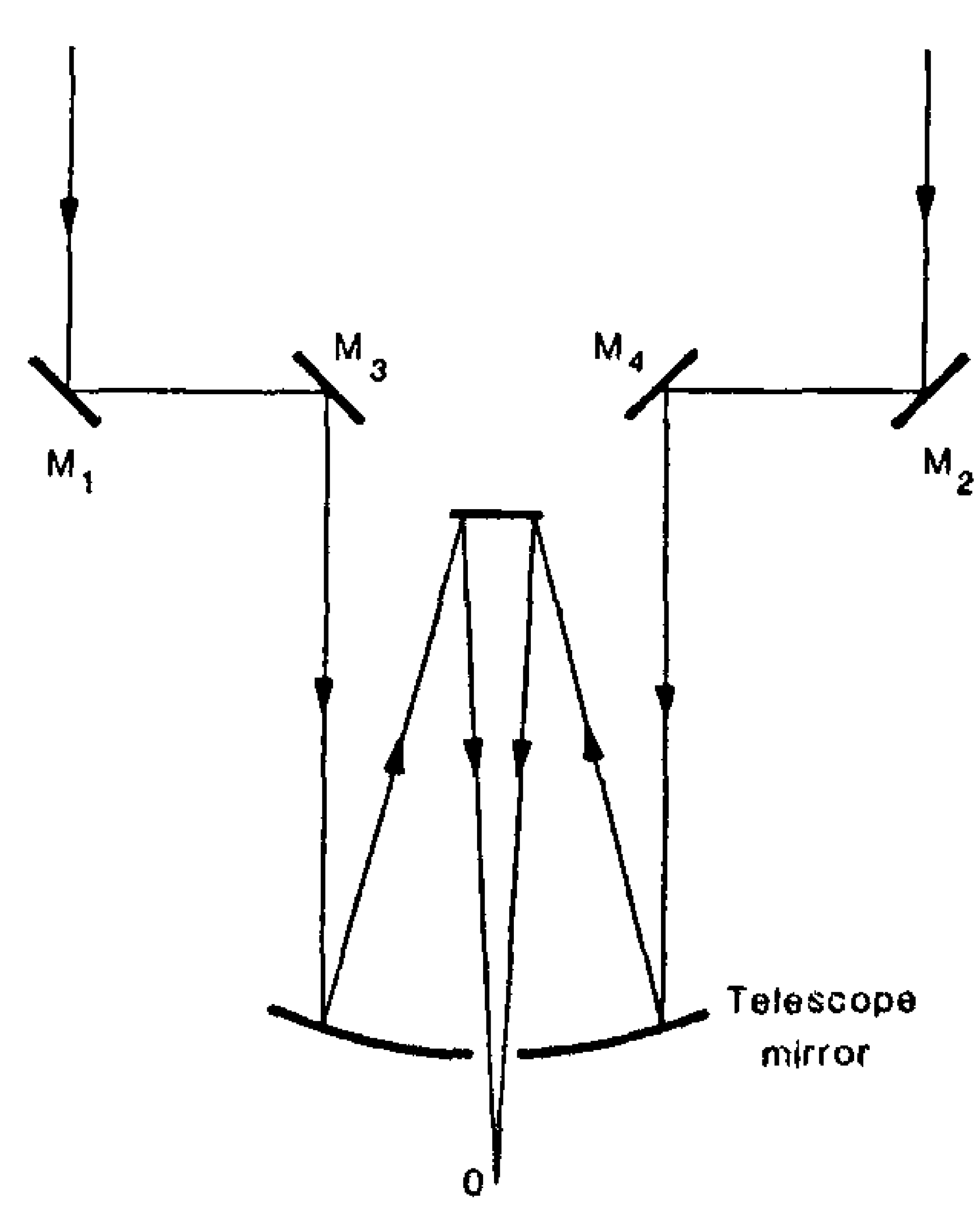

Figure 3. Michelson's stellar interferometer.

Measurements with Michelson's stellar interferometer present serious difficulties because of two very stringent requirements that must be met to be sure that the observed disappearance of the fringes is actually due to the finite diameter of the star. One is that the optical path difference between the two beams must be small compared to the coherence length of the light. The other is that the optical path difference must be stable to a fraction of a wavelength. The latter condition is very difficult to satisfy with longer baselines, since atmospheric turbulence produces rapid random changes in the two optical paths.

Long-baseline stellar interferometers

With the development of modern detection, control and data-handling techniques it became possible to overcome the difficulties encountered with Michelson's stellar interferometer[10], and instruments designed to make measurements over baselines up to 640 m, have been constructed.

In the Sydney University Stellar Interferometer (SUSI)[11], two siderostats at the ends of a North–South baseline direct light from the star, via two beam-reducing telescopes and an optical path-length compensator, to a beam combiner. Error signals from two quadrant detectors in each channel, viewing the image of the star, are used to control the siderostats and two piezoelectric-actuated tilting mirrors to keep these images exactly on the axis. Interference therefore takes place between two pairs of nominally parallel wave fronts leaving the beam splitter.

Two photon counting detectors measure the total flux in a narrow spectral band in the two complementary interference patterns. The signals from the two detectors are then proportional to $(1+|\mu_{12}|\cos\phi)$ and $(1-|\mu_{12}|\cos\phi)$, where ϕ, the phase difference between the beams, varies randomly with time because of changes in the lengths of the optical paths through the atmosphere.

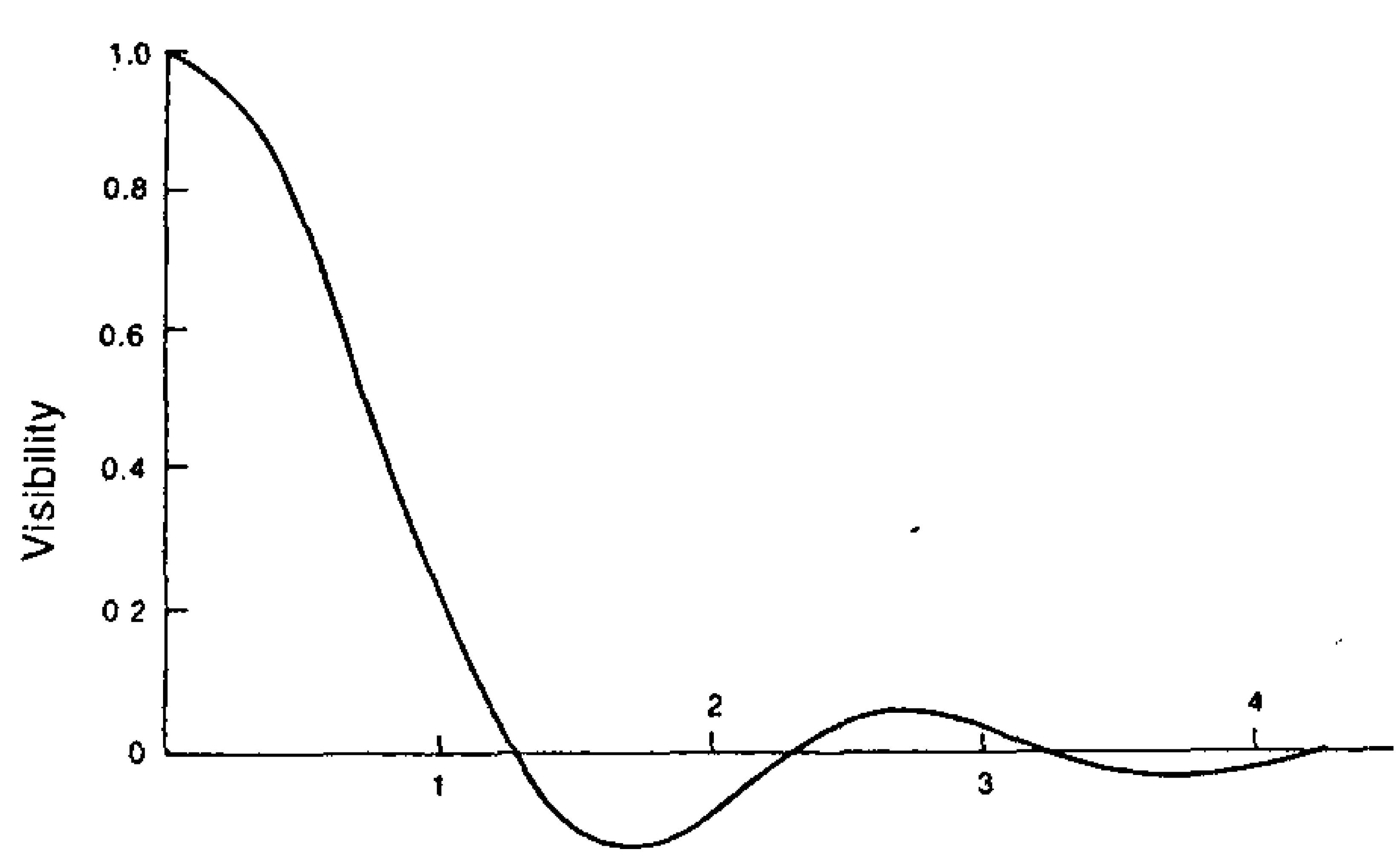

Figure 4. Variation with the length of the baseline, for a uniform circular source, of the visibility of the fringes in Michelson's stellar interferometer.

If the variations in the lengths of the optical paths are large enough that ϕ has a uniform circular distribution, the average value of $\cos^2\phi = 1/2$; the average of the square of the difference between the two signals is then $2|\mu_{12}|^2$ or, in other words, twice the square of the visibility.

To ensure that the optical path difference is small compared to the coherence length of the radiation, photon-counting array detectors (PCADs) are used to observe the channeled spectrum produced by the combined beams. The spacing of the fringes then gives a direct indication of the optical path difference. Data from the PCADs are accumulated in a switchable memory, which allows one data set to be processed to obtain a control signal while the next is being stored[12].

Achromatic phase-shifting for stellar interferometry

A drawback of this technique is that it assumes that the visibility of the fringes is constant over the range of optical path differences introduced by the random variations in the lengths of the optical paths. To satisfy this condition, the spectral bandwidth has to be limited by a narrow-band filter, resulting in a serious loss of light. This problem can be overcome by holding the optical path difference at a value close to zero and using a phase shifter operating on the Pancharatnam phase to introduce phase shifts of $\pm 90°$ that are almost independent of the wavelength[13].

Unfortunately, it is not possible to use QHQ phase shifter in a stellar interferometer, since measurements have to be made within a time interval that is much shorter than the period of the atmospheric fluctuations, to avoid errors due to variations in the optical paths caused by atmospheric turbulence. However, very rapid achromatic phase shifts can be obtained with a switchable phase-shifter in which the $\lambda/2$ plate is replaced by two ferroelectric liquid crystal cells[14].

As shown in Figure 5, the two linearly polarized beams incident on the beam combiner (BC) in SUCI are brought into orthogonal states of polarization by the half-wave retarders H_A and H_B, so that differential measurements on the two outputs can be used to eliminate scintillation noise. It is, therefore, possible to use a switchable phase-shifter incorporating two FLC devices in each of the two outputs, with a minimum of modifications to the system.

However, a problem which arises in using such a switchable phase shifter with two orthogonally polarized beams in each channel is that the two beams experience phase shifts of $+2\theta$ and -2θ, respectively, when the axis of the FLC device switches through an angle θ. With normal FLC devices, in which the optic axis switches through an angle of 45°C this would result in

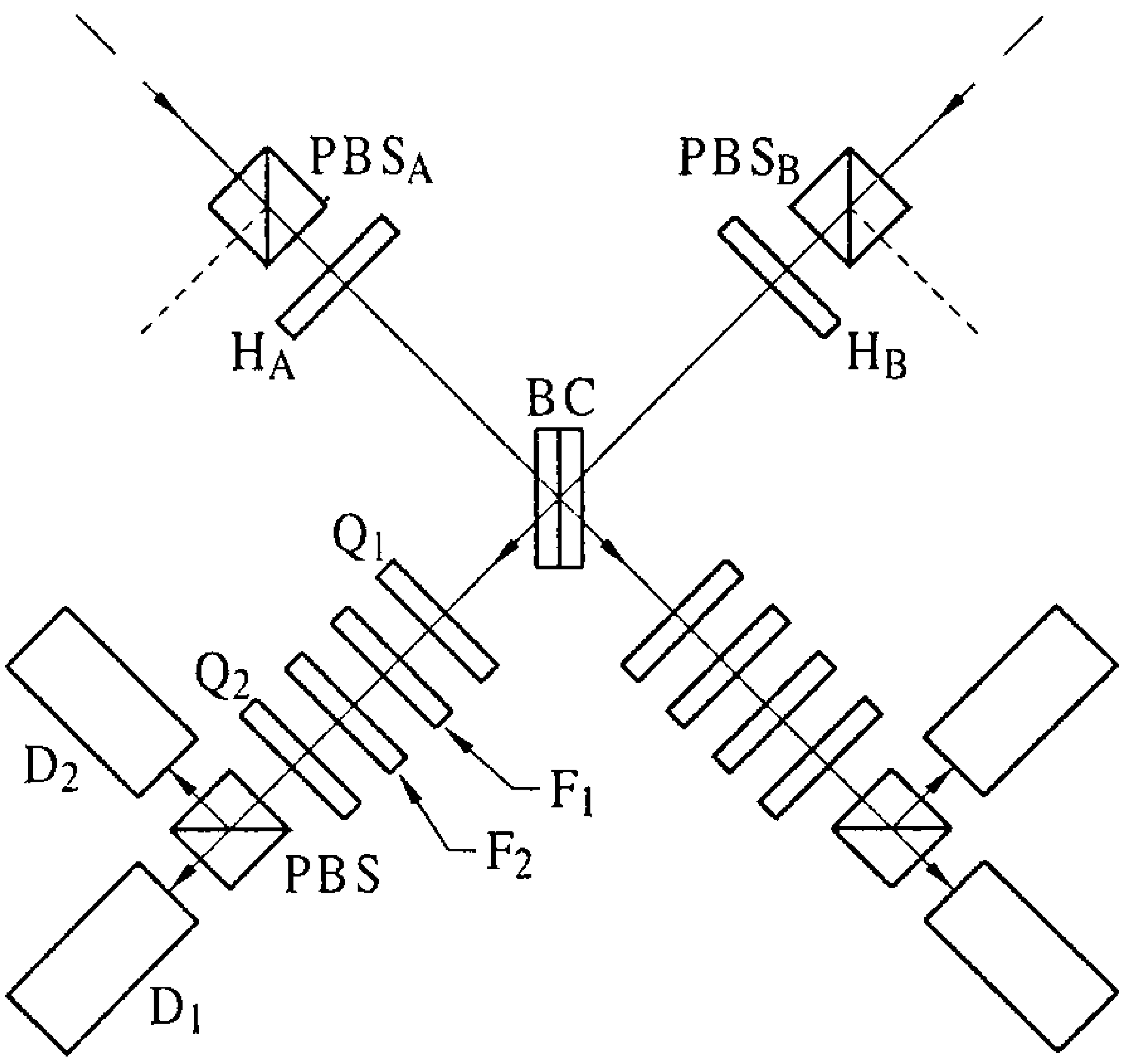

Figure 5. Optical system of a stellar interferometer using switchable phase-shifters operating on the Pancharatnam phase.

phase shifts of $\pm 180°$, which are not useful. A solution would be to use an FLC material with a switching angle of 22.5°, but such a material was not available. Fortunately, it has been shown that it is possible to use materials with other switching angles, provided they differ significantly different from 45° (ref. 15).

If, then, we consider one of the output channels (say, the left-hand one), the intensities at the photo detectors D_1 and D_2, when a phase shift of 4θ is introduced between the two beams, are

$$I_1(d, 4\theta) = I_0 \,[1 + V_d \cos(2\pi d/\bar{\lambda}) + (\phi_R + 4\theta)]\,, \tag{6}$$

$$I_2(d, 4\theta) = I_0 \,[1 - V_d \cos(2\pi d/\bar{\lambda}) - (\phi_R + 4\theta)]\,, \tag{7}$$

where I_0 is the intensity that would be obtained in the absence of interference, d is the optical path difference between the two interfering beams, V_d is the visibility of the interference fringes and ϕ_R is the phase difference introduced by reflection at BC.

The visibility V_d of the interference fringes can then be calculated from three sets of intensity measurements with phase shifts of 0, $+4\theta$ and -4θ, using the formula

$$V_d = \frac{[(\frac{c-1}{s})^2 (I_{+4\theta} - I_{-4\theta})^2 + (2I_0 - I_{+4\theta} - I_{-4\theta})^2]^{1/2}}{2cI_0 - I_{+4\theta} - I_{-4\theta}}, \tag{8}$$

where $c = \cos 4\theta$ and $s = \sin 4\theta$. With two FLC devices with a switching angle of 60°, yielding phase shifts of 0°, 240° and $-240°$, eq. (8) becomes

$$V_d = \frac{[3(I_{240} - I_{-240})^2 + (2I_0 - I_{240} - I_{-240})^2]^{1/2}}{I_0 + I_{240} + I_{-240}}. \tag{9}$$

Since independent outputs can be obtained from the two channels, and such FLC materials typically have switching times less than 40 μs, the output from one channel can be used to locate and track the zero-order fringe with white light. The use of a fast fringe-tracking system should improve the accuracy of measurements by holding the optical path difference close to zero, while the use of an achromatic phase-shifter should permit the use of a wide spectral bandwidth for measurements, making possible measurements on fainter stars.

1. Pancharatnam, S., *Proc. Indian Acad. Sci.*, 1956, **A44**, 247–262.
2. Ramachandran, G. N. and Ramaseshan, S., in *Handbuch der Physik* (ed. Flügge, S.), Springer, Berlin, 1961, vol. 25, pp. 1–217.
3. Jerrard, H. G., *J. Opt. Soc. Am.*, 1954, **44**, 634–640.
4. Martinelli, M. and Vavassori, P., *Opt. Commun.*, 1990, **80**, 166–176.
5. Hariharan, P. and Ciddor, P. E., *Opt. Commun.*, 1994, **110**, 13–17.
6. Hariharan, P., Larkin, K. G. and Roy, M., *J. Mod. Opt.*, 1994, **41**, 663–667.
7. Hariharan, P. and Roy, M., *Opt. Commun.*, 1996, **126**, 220–222.
8. Hariharan, P. and Roy, M., *J. Mod. Opt.*, 1994, **41**, 2197–2201.
9. Michelson, A. A. and Pease, F. G., *Astrophys. J.*, 1921, **53**, 249–259.
10. Tango, W. J. and Twiss, R. W., in *Progress in Optics* (ed. Wolf, E.), North-Holland, Amsterdam, 1980, vol. 17, pp. 241–276.
11. Davis, J., in Proceedings of the International Symposium on Measurement and Processing for Indirect Imaging, Sydney, 1983 (ed. Roberts, J.), Cambridge University Press, Cambridge, 1984, pp. 125–141.
12. Tango, W. J., Davis, J., Lawson, P. and Booth, A. J., in *High Resolution Imaging by Interferometry* (eds Beckers, J. and Merkk, F.), ESO, Garching, 1991, pp. 1267–1269.
13. Tango, W. J. and Davis, J., *Appl Opt.*, 1996, **35**, 621–623.
14. Hariharan, P. and Ciddor, P. E., *Opt. Commun.*, 1995, **117**, 13–15.
15. Hariharan, P., *J. Mod. Opt.*, 1997, **44**, 857–861.

Received 23 January 1998; accepted 9 February 1998