Introduction to Subfactors. V. Jones and V. S. Sunder. London Mathematical Society, Lecture Notes Series 234, Cambridge University Press. 1997. 162 + xii pp. ISBN 0-521-58420-5, paperback.

V. Jones was awarded the Fields Medal in 1990 for a series of astonishing discoveries in the field of operator algebras. He pioneered the theory of subfactors and demonstrated that this theory has significant implications for many other parts of mathematics. For example, based on his theory he came up with new polynomial invariants for knots. The book under review is a friendly invitation to subfactor theory by two well-known masters of this flourishing field.

Von Neumann algebras are weakly closed *-subalgebras of the algebra of all bounded operators on a Hilbert space. One may also think of them as commutants of ranges of unitary representations of groups. Among von Neumann algebras those with trivial centres are known as *factors*. In view of a well-established decomposition theory, factors are to be thought of as building blocks for other von Neumann algebras.

Murray and von Neumann broadly classified factors into three classes known as types I, II and III, by looking at projections contained in them. A type I factor is isomorphic to the algebra of all bounded operators on a Hilbert space. The dimension of such a Hilbert space becomes an invariant for the factor. Surprisingly this notion of dimension can be extended to type II factors as well, but now the dimension is not necessarily an integer and in fact it can be any positive real number (or infinity). This possibility of 'continuous dimension' interested von Neumann. Jones went a step further. He tried to classify inclusions of factors and discovered that there is a strange mixture of the discrete and the continuous.

Suppose N, M are two type II factors such that N is contained in M, then N is said to be a *subfactor* of M. One usually assumes that N, M are of type II_1, i.e. they have dimension 1 (this is just a kind of normalization). Given any such pair, there is a numerical invariant for them known as the (Jones) *index*. This is in fact a generalization of the notion of index for subgroups. The first striking result of Jones is that the index is contained in

$$\{4 \cos^2 \pi/n:: n=3,4,\ldots\} \cup [4,\infty],$$

and that each of these values are realizable. The proof comes about from analysing a 'tower' of type II_1 factors:

$$N \subset M \subset M_1 \subset M_2 \subset \ldots,$$

built out of the given pair $N \subset M$ through a repeated application of a method known as the 'basic construction'. This book describes these concepts in detail and the approach is through the notion of bimodules.

In the tower mentioned above, if one looks at relative commutants $\{M_i' \cap M_j : -1 \le i \le j\}$ $(M_{-1} = N, M_0 = M)$, usually one obtains a grid of finite dimensional von Neumann algebras. But finite dimensional von Neumann algebras are nothing but direct sums of full matrix algebras and their inclusions can be described through certain graphs known as Brattelli diagrams. These graphs become valuable invariants for the factor–subfactor pair one started with. This way, subfactor theory gets transformed into a study of combinatorial and graph theoretic structures. Then the connections with Kronecker's theorem (on norms of matrices with integer entries) and well-known classification of Coxeter graphs and Dynkin diagrams surface naturally.

One can also reverse the process; that is, one can start with special types of combinatorial structures and build new subfactors and analyse them. For instance, one such method known as 'spin models' makes use of complex Hadamard matrices. The last few chapters of this book are devoted to this kind of endeavour. The analysis flavour of von Neumann algebra theory (for example, see classics like Dixmier[1]) is completely missing here and it is mostly combinatorics and graph theory. The treatment is quite detailed, but one must admit that the details are quite technical and may enthuse only a specialist.

It is clear that the authors have assumed that the reader is familiar with basics of operator algebra theory on Hilbert spaces; for instance, the introduction to von Neumann algebras given is a little too brief. A good number of clearly stated, interesting examples of factors and subfactors have been provided before building up the whole theory. Missing in the book are the connections with other areas of mathematics, such as low dimensional topology, algebraic quantum field theory alluded to in the backcover. Except for a short sketch about the relationship with knot theory in the Appendix, no indications are given as to how these other fields come into picture. Perhaps, to learn more about such aspects one should refer to Goodman *et al.*[2], and Jones[3] (surprisingly no reference to this well-written Lecture Notes is to be found in the present book). This book is clearly indispensable for anybody who wants to do research on subfactors. But more importantly this could be very useful to any student or mathematician who wants to get introduced to the subject.

1. Dixmier, J., *von Neumann Algebras*, North Holland, Amsterdam, 1981.
2. Goodman, F., de la Harpe, P., Jones, V., *Coxeter Graphs and Towers of Algebras*, MSRI Publications, Springer, 1989, vol. 14.
3. Jones, V. F. R., *Subfactors and Knots*, CBMS Regional Conference Series in Mathematics, Number 80, American Mathematical Society, 1991.

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The World Wide Web for Scientists & Engineers – A Complete Reference for Navigating, Researching and Publishing Online. Brian J. Thomas. SPIE – The International Society of Optical Engineering, Bellingham, Wash, USA. 1998. ISBN: 0-8194-2775-6. pp. xv + 375. Price $34.00.

Within a few years of its emergence, the Internet has become an essential part of life for most scientists and engineers. It owes a large part of its phenomenal success and growing popularity to the World Wide Web, the penultimate example of distributed computing, and the data ubiquity the Web has made possible. Thanks to the Web scholars and scientists have evolved new, cost-effective and time saving ways of publishing, disseminating and searching information. Major publishers and information providers have recognized the tremendous potential of the Web – witness *Science Direct* of Elsevier, *Web of Science* of the Institute