

Ramanujan: Letters and Commentary. Bruce C. Berndt and Robert A. Rankin. American Mathematical Society, London Mathematical Society, Providence, 1995.

This is a wonderfully welcome book. The worldwide public has been fortunate to have a variety of biographies of the Indian mathematical genius, Ramanujan. Most recently, Robert Kanigel published *The Man Who Knew Infinity* for a broad audience. Before this, S. R. Ranganathan had published *Ramanujan, The Man and the Mathematician*. And, of course, there are the wonderful accounts of Ramanujan's life in G. H. Hardy's *Ramanujan*, and in Ramanujan's *Collected Papers*.

However each biography always carries with it some of the views of the author. This is perhaps most strikingly in evidence in the following account that Hardy (taken from page 4 of *Ramanujan*) provides of Ramanujan's religious views:

'Now the two memoirs of Ramanujan printed in the *Papers* (and both written by men who, in their different ways, knew him very well) contradict one another flatly about his religion. Seshu Aiyar and Ramanchandra Rao say: "Ramanujan had definite religious views. He had a special veneration for the Namakkal goddess... He believed in the existence of a Supreme Being and in the attainment of Godhead by men... He had settled convictions about the problem of life and after..."; while I say... his religion was a matter of observance and not of intellectual conviction, and I remember well his telling me (much to my surprise) that all religions seemed to him more or less equally true...'

'Which of us is right? For my part I have no doubt at all; I am quite certain that I am.'

'Classical scholars have, I believe, a general principle, *difficilior lectio potior* – the more difficult reading is to be preferred – in textual criticism. If the Archbishop of Canterbury tells one man that he (the Archbishop) believes in God, and another that he does not, then it is probably the second assertion which is true, since otherwise it is very difficult to understand why he should have made

it, while there are many excellent reasons for his making the first whether it be true or false. Similarly, if a strict Brahmin like Ramanujan told me, as he certainly did, that he had no definite beliefs, then it is 100 to 1 that he meant what he said.'

'This was no sufficient reason why Ramanujan should outrage the feelings of his parents or his Indian friends. He was not a reasoned infidel, but an "agnostic" in its strict sense, who saw no particular good, and no particular harm, in Hinduism or in any other religion. Hinduism is, far more, for example, than Christianity, a religion of observance, in which belief counts for extremely little in any case, and if Ramanujan's friends assumed that he accepted the conventional doctrines of such a religion, and he did not disillusion them, he was practising a quite harmless, and probably necessary, economy of truth.'

To paraphrase what I said in *The Hindu* (21 December 1987, page 8), the day before the Ramanujan centenary:

Hardy believed that Ramanujan was more or less a Western European agnostic. I doubt it. In my dealings with academic Indians, I have found them quite polite. If you contradict their beliefs, I have found few, if any, who would bluntly remark: 'That's the stupidest remark I've heard!' They are more likely to smile and keep their own opinions. I believe that is related to Ramanujan's response to Hardy. Hardy was Ramanujan's great benefactor and was also a man who referred to God as his personal enemy. If you were Ramanujan and you were speaking about religion with Hardy, what could you say that both would be an honest statement consistent with your religious beliefs and would not antagonize your great friend? What could be a better statement than '... all religions seem... more or less equally true?'

This statement reflects the great tolerance of Hinduism, and (to paraphrase again):

... if Ramanujan's friend (Hardy) assumed that he accepted the conventional doctrines of (agnosticism), and he did not disillusion (him), he was practising a quite harmless, and probably necessary, economy of truth.

A welcome function of this magnificent collection of letters is that it serves to bring to life the many names that occur in the biographies. In addition, a mathematician finds many mathematical gems buried in its pages. For example, we find (pp. 147–151) Hardy's letter to Ramanujan with a thorough accounting of his subsequently famous extensions of their joint method to sums of an odd number of squares. Also, we have permanently recorded his last letter (pp. 220–223), which indirectly hints at the existence of the Lost Notebook. Incidentally, the editors provide quite a plausible history of the Lost Notebook on page 267.

One of the great mysteries concerning Ramanujan is: how did he think? What thought processes led him to his wild and strange theorems? In reading this book, I had hoped we might gain at a minimum a few more ideas. However, as before, we are left in mystified amazement at the surprising mathematical fertility of Ramanujan's brain, even during his years of severe illness. This is especially evident in his last letter (mentioned above), and in the four mathematical letters on pages 175 to 191. It is with interest that we learn on page 191 that most of the unproved formulae on these pages presented in the pure, succinct style of Ramanujan have actually been proved in a Ph D thesis by P. Bialek. Unfortunately Bialek's work has apparently not been published elsewhere.

Reviewing this book allows me to make one addition to the collection of Ramanujan's letters, and to add a further intriguing bit of information to the continuing speculation about Ramanujan's thought. Oliver Atkin, who became famous for fully settling the conjectures of Ramanujan concerning congruences for $p(n)$, once lent me a collection of F. H. Jackson's reprints. F. H. Jackson was a chaplain in the British Navy and an amateur mathematician who contributed much to the study of q -series of the type Ramanujan loved. The letter below, probably written in the 1930s or 1940s, is from Jackson to J. L. Burchall, who was a professor at Durham for many years. The paper and the 'theorem 5' referred to by Jackson appeared in the *Messenger of Mathematics*, 1920, vol. 50, pp. 101–112. The infinite series that he lists

are the celebrated Rogers–Ramanujan identities which are described in detail on page 65 of the book under review:

Dear Burchnall

I sent the theorem 5 (which is the foundation of this paper and is also the basis of Hardy's and Watson's proofs of Ramanujan's theorems giving product expressions for

$$1 + \frac{q}{(1)} + \frac{q^4}{(2)!} + \frac{q^9}{(3)!} + \dots$$

$$1 + \frac{q^2}{(1)!} + \frac{q^6}{(2)!} + \frac{q^{12}}{(3)!} + \dots$$

and other related theorems) to the London Math Socy in 1905. It was returned to me by next post from the Council meeting (never sent to a referee). I gave up membership, kept the paper for 15 years then sent it to Glaisher who put it in the *Messenger of Mathematics*.

In 1920 I wrote to Ramanujan, 3 weeks before his death (I did not know of his illness), pointing out there was some connection with his theorems. He wrote a long letter in reply showing how he came to guess theorems: I am glad that Hardy in his *Ramanujan, Life & Works* (1940) does tardy justice to theorem (5). Last summer during a tedious evacuation into Welsh borderland, I spent some weeks at it & found the transformation into form (1) & (2) & (3). Bailey of Manchester made a flattering note on this transformation.

Yours
F. H. Jackson'

How I would love to have the missing long Ramanujan letter alluded to by Jackson. How wonderful to possess a letter '... showing how he came to guess his theorems'. This tantalizing, missing letter dovetails with other accidents, including a missing page of a Ramanujan letter to Hardy that originally contained the Rogers–Ramanujan identities.

At the end of the day then, we are left with this wonderful book of letters, along with many unanswered questions about the enigmatic genius of Ramanujan. We must emphatically thank the

authors for providing us with this rich historical treasure.

GEORGE E. ANDREWS

*The Pennsylvania State University,
University Park, PA 16802, USA*

Science and Engineering Indicators 1998. National Science Board, National Science Foundation, Arlington, VA, USA, 1998.

This is the thirteenth in the series of biennial science indicators reports, submitted to the President and the Congress of the United States by the National Science Board. Designed to provide a broad base of quantitative information about science, engineering, and technology for use by policymakers in government, industry and academia, this report provides meticulously collected information and analysis that have a bearing on a variety of critical trends and issues, such as (1) increasing globalization of science, technology, and the economy; (2) greater emphasis on science and engineering education and training; (3) structural and priority changes in the science and engineering enterprise; and (4) increasing impact of science and engineering on our daily lives.

The topics covered include: science and mathematics education from the precollege level, through graduate school, and beyond; worldwide increase in S&E educational capabilities; public attitudes and understanding of science and engineering; migration and R&D employment; funding of R&D; and international trends in industrial R&D and competitiveness in the marketplace. In keeping with the needs of the times, the major theme of the report is international comparisons and global trends.

A significant departure from the earlier reports is the inclusion of a whole chapter on the economic and social significance of information technologies, which addresses both positive and negative aspects of the new technologies. This chapter points out that there is significant educational inequity in access to computers and the Internet – between schools attended primarily by minority or economically disadvantaged

students and schools primarily attended by white or nondisadvantaged students. What is more, the poor and minority students cannot compensate for this disparity in their schools in their homes as very few blacks, Hispanics and other poor people have computers at their homes. Indeed, a recent report of the National Telecommunication and Information Administration of the US Department of Commerce points out that between 1994 and 1997 although access to computers, modems and online access has increased countrywide, the disparity between the privileged, largely the whites and non-Hispanic professionals, and the blacks, Hispanics and inner city population has increased considerably. This evidence is often lost on the champions of the digital revolution who harp on the democratizing influence of the information revolution.

Other new features in this year's report include enhanced and new indicators of intersectoral and international collaboration/partnerships, new indicators of Internet and World Wide Web use, coverage of the restructuring of the defence industry and its impact on the nation's S&T enterprise, greater emphasis on indicators of international mobility, new venture capital indicators, and indicators of the impacts of information technologies on education.

According to the report, both science and technology are becoming increasingly global. The American S&T workforce is becoming more global; more and more foreign doctoral recipients, especially those from India and China, plan to remain in the United States. Almost 30% of papers published in core journals of the world involve international collaboration. Industrial firms are increasingly using global research partnerships as a means of strengthening core competencies. Cross-national patenting and foreign direct investment in R&D are also on the rise.

Mathematics teaching at school level continues to be a matter for concern. Students often need remedial math and science preparation when they enter college. Industry invests two out of every three of the nation's R&D dollars and performs three-fourths of the nation's R&D effort. As the growth in federal support of academic R&D is slowing, new funding mechanisms are gaining prominence and cooperative