# Mathematics and the physicist's conception of nature*

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*The many roles of mathematics in the continual evolution of the physicist's picture of nature are recalled. Examples and situations from classical and modern physics, mathematical formulation versus physical interpretation, and mathematics as the language of nature are considered.*

SOME years ago, at an earlier annual meeting of the Indian Academy of Sciences, I had presented an invited lecture on the theme 'The Mathematical Style of Modern Physics'[1]. Two main ideas were explored: the growth of the ideas of symmetry and invariance in physical science, and the ubiquitous use of unobservable quantities in an essential way in modern physical theories. In the present article, intended to be both retrospective and introspective, historical examples and episodes from the growth of physics will be recalled to highlight some facets of the way mathematics enters in the development of the physicist's picture of nature. In the process it will be seen that there are implications for the nature and origins of mathematics itself.

It seems evident that the origins of mathematics can be traced to human and social needs, commerce, land survey and so on. That it is useful in describing natural phenomena was also realized very early, for example by Pythagoras in connection with music. As is well known, the empirical Egyptian knowledge of geometry was raised by the Greeks to a very high position, a self-contained logical system which came to be regarded as a product of pure reason. One of the things they studied in great detail was the geometry of the conic sections. But they found the circle so perfect a figure that all heavenly bodies were declared to follow circular paths. One of the greatest events in the history of science was Kepler's discovery that the planetary orbits are ellipses and not circles, given the level of accuracy of observations at that time. However this was still a descriptive stage. The real transition from description to explanation, prediction and understanding – the start of modern science – came with Galileo and Newton, and with it also the appreciation of the crucial role of mathematics. Here is a famous passage from Galileo's 'Il Saggiatore' in 1623 about the 'book of nature'[2]:

'Philosophy is written in this very great book which always lies open before our eyes ... but one cannot understand it unless one first learns to understand the language and recognize the characters in which it is written. It is written in mathematical language ...; without these means it is humanly impossible to understand a word of it ...'.

Already at this stage one is tempted to say – to the extent that the laws of nature are part of objective reality, mathematical structure is also an objective component of nature, of reality. That there exist laws at all, that nature is lawful, is of course a deep mystery. The full meaning of this statement – its depth – will sink into you only if you are precocious, or nearing retirement!

For a long time after Galileo and Newton, for almost two centuries, mathematics and physics progressed hand in hand. They reinforced one another, and often the same individuals contributed to both. There are many familiar examples of this: the development of calculus to describe motion; the growth of partial differential equations to handle continuum mechanics and later electromagnetism; the birth of Fourier series in solving problems of heat conduction; and so on. But then the paths began to sometimes diverge a little – and independent developments in mathematics were found to have profound uses in physics somewhat later. Here are just a few outstanding examples. The theory of groups – one of the most beautiful developments in mathematics in the 19th century – has seen its deepest applications in the physics of the 20th century, particularly in the framework of quantum theory. Another major advance within 19th century mathematics was the discovery that Euclidean geometry was just one of many geometries, not an inevitable product of pure reason, that non-Euclidean geometries could be consistently set up. This culminated in the development of Riemannian geometry around 1850. While Riemann himself had remarkable ideas in physics, it was in the 1910s that Albert Einstein made splendid use of Riemannian geometry in formulating his general theory of relativity. In the case of quantum mechanics, the theory of infinite dimensional vector spaces or Hilbert space was developed inde-

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pendently within mathematics, and then put to use by physicists, though in this case the gap in time was quite short. One must also say that the uses of mathematics in relativity and in quantum mechanics have greatly stimulated further advances in mathematics itself; this is so even to this day. All this being so, yet one has to acknowledge that mathematics is more self-contained, depending on inner coherence and consistency, and in that sense does not depend on physics; while physics, for which mathematical tools are essential, has ultimately to deal with experiment and nature. Here is a recent statement of C. N. Yang recognizing the differences in the wellsprings of creativity in the two domains[3]:

'It would be wrong, however, to think that the disciplines of mathematics and physics overlap very much; they do not. And they have their separate aims and tastes. They have distinctly different value judgements, and they have different traditions. At the fundamental conceptual level they amazingly share some concepts, but even there, the life force of each discipline runs along its own veins.'

Against this general background, I would now like to recall a few instances from the development of physics, which highlight important facets of the relationship to mathematics. However it is important to understand and continually remind oneself that the physicist's picture of nature is always evolving and never final; as experimental techniques improve, new phenomena become accessible and they affect, modify and become part of the overall picture. So the necessary mathematical language also constantly evolves.

Perhaps the single most important point to be made is that in physics the interpretation of a mathematical formulation and mathematical equations is an essential component of complete understanding, as important as the mathematical component. And there have been numerous instances, some of which I will recount presently, where the mathematical part has been discovered first, new equations have been found, and this has then been followed by a prolonged struggle to see exactly what it all means! This situation has been well described by Bertrand Russell:

'It may be said generally that, in the mathematical treatment of nature, we can be far more certain that our formulae are approximately correct than we can be as to the correctness of this or that interpretation of them.'

It is like catching a tiger by its tail, or better still, riding a tiger! You cannot let go or get off, and yet you do not know where you are being taken!

When Newton proposed his Law of Universal Gravitation in the *Principia* in 1687, it was a law of action at a distance – any two masses in the universe, howsoever far apart, would instantaneously exert inverse square law forces upon each other. The prevailing idea at that time, especially on the Continent, was that natural influences could only be by material contact, not across intervening empty space. Yet, taken together with his laws of motion, his law of gravitation worked beautifully, and explained all of Kepler's Laws and much more. But was nature really so? Here is Newton himself writing to Richard Bentley on this question in 1692–93, a few years after the *Principia*[4]:

'That one body may act upon another at a distance through a vacuum, without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity, that I believe no man, who has in philosophical matters a competent faculty of thinking, can ever fall into.'

Twenty years later, in 1713 in the second edition of the *Principia*, he said[4]:

'I have not been able to discover the cause of those properties of gravity from phenomena, and I frame no hypotheses; ... And to us it is enough that gravity does really exist, and act according to the laws which we have explained, and abundantly serves to account for all the motions of the celestial bodies, and of our sea.'

One can see that while Newton trusted his mathematical law, he was not willing to physically interpret it literally as an action at a distance! But over the next century or so, the successes of Newtonian dynamics were so great that people got used to this idea and it was extended also to electricity and magnetism. In the case of gravity, the return to the action by contact point of view took a long time. First the field concept had to be developed by Faraday and Maxwell for electromagnetism, and then after special relativity, Einstein went on to the general theory which is the classical field theory of gravitation.

The next example has to do with Maxwell's equations for electromagnetic fields, formulated around 1865. When he wrote to Faraday about this, the latter is supposed to have replied that while he could not grasp all the mathematics, he was glad that his physical ideas had survived the process of being expressed in equations! But this 'mathematization' by Maxwell was essential – he found an inconsistency in the equations, discovered the displacement current, then went on to predict electromagnetic waves. All this is a triumphant story. Remember also that both special relativity and quantum theory were first 'seen' through the 'window' of electromagnetism, and later understood to have much wider applicability. But as for their physical interpretation – for a long time Maxwell was ambivalent and tried to invent mechanical 'gears and wheels' models for electric and magnetic fields; and the ether concept was also adopted because of the prevailing attitude that all understanding in physics had to be on mechanistic lines. It took a long time for this point of view to be overcome, for the ether to be given up and for electric and magnetic fields to be regarded as primary constituents of nature, not made up of anything else. The most eloquent expression of this is from a 1907 review of special relativity by Einstein[5]:

'... electromagnetic forces appear here not as states of some substance, but rather as independently existing things that are similar to ponderable matter and share with it the feature of inertia.'

This story does not end here! By 1905 – thanks to Planck's work and his own light quantum idea – Einstein was convinced that the classical Maxwell theory had to be changed, and he foresaw and said in effect: 'I have shown that the Maxwell field on its own "must be quantized"'. And indeed that is what happened when in 1927, Paul Dirac showed how to apply the principles of the then new quantum mechanics to the Maxwell field. Here we have electric and magnetic fields which are not numbers any more but operators, new life breathed into old symbols. The equations were the same as before, but the interpretation had changed completely – from a mechanistic interpretation to primitive classical fields to quantum field operators – in the period from 1865 to 1927. One is reminded here of Hertz's statement about Maxwell's equations:

'One cannot escape the feeling that these mathematical formulae have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than was originally put into them.'

My next example concerns the Lorentz transformation equations of special relativity, which as you know are closely connected to Maxwell's equations. As early as 1887 Waldemar Voigt, and soon after Lorentz, showed that in going to a moving reference frame it was *mathematically convenient* to introduce a new time variable $t'$ distinct from the *physical time* $t$ (ref. 6). Then in 1904 Lorentz himself, and later Poincaré, found the complete set of transformation equations for space and time such that the Maxwell equations maintained their forms. But they left it at that mathematical level. And it was the genius of Einstein that found the correct physical interpretation, the new kinematics of space and time, the relativity of simultaneity and of length measurements, all of which were in fact contained in Lorentz's equations! Only then was their significance for all of physics, going beyond electromagnetism, clearly recognized and stated[7]:

'The principle of relativity is a principle that narrows the possibilities; it is not a model, just as the second law of thermodynamics is not a model.'

The fourth episode in my list is a stunning one – it is the case of the Dirac equation which was evidently discovered by him in 1928 while staring at a fireplace in St. John's College in Cambridge and trying to combine quantum mechanics and special relativity[8]! Earlier Pauli had shown how to incorporate spin in nonrelativistic quantum mechanics; Dirac's approach however was radically different and led to one of the most beautiful equations in physics. As Michael Atiyah said recently[9]:

'The differential operator introduced by Dirac in his study of the quantum theory of the electron has turned out to be of fundamental importance both for physics and for mathematics.'

This equation introduced spinors into physics in a basic way, and from it came tumbling out as consequences the spin and anomalous magnetic moment of the electron and the fine structure of hydrogen. But it also had negative energy solutions which seemed unphysical and made the interpretation very difficult. For a while Dirac suggested that somehow these solutions could be reinterpreted as protons, partly he says as a lack of courage and reluctance to increase the number of elementary particles from three to four. Soon, following arguments advanced by Oppenheimer and by Weyl, it became clear that this interpretation was untenable, and then in 1931 Dirac introduced the concept of positrons and the general idea of antimatter. Soon after, positrons were experimentally found, and this became another success of the Dirac equation. Today we know that really the only consistent interpretation of this equation is in terms of a quantum field.

Time now for an interlude. All these examples show how difficult and nontrivial the problem of physical interpretation of mathematical structure can be, how far from being self-evident or straightforward. Within mathematics the most important things are structures, operations, relationships and consistency – the symbols one operates with need not have any meanings at all. (It is like saying in biology that the key things are structure and function.) The advantage of course is that the same mathematical tools can be used in widely different contexts – look at the use of Fourier's theory in wave propagation, communication, signal processing and even the quantum mechanics of position and momentum. But the situation in physics is much more demanding – in each given context we *must* invest symbols with *some* meanings, even if provisional, and check against nature. Physics needs mathematical consistency *and* experimental verification, and is incomplete without rules of interpretation, even if this is an ongoing task. In all the examples I have given, the mathematical structures come through unscathed or intact, while the physical meanings pass through 'phase transitions' in time. This means then that at each stage our understanding was partial and evolving. But now one must realize also that on a longer time scale the equations of physics themselves are transitory, as each has a domain of validity and has then to be superceded by something more comprehensive. At the risk of possibly upsetting mathematicians, let me quote Dirac on this problem of interpretation[10]:

'The situation of a formalism becoming established before one is clear about its interpretation should not be considered as surprising, but as a natural consequence of the drastic alterations which the development of physics has required in some of the basic physical concepts. This makes it an easier matter to discover the mathematical formalism needed for a fundamental physical theory than its interpretation, ... the number of fundamental ideas in pure mathematics being not very great, while with the interpretation most unexpected things may turn up.'

Now to the most profound example of the problem of interpretation, namely quantum mechanics. As is well known, the basic mathematical structures and equations were worked out in a brief period during 1925–27. In contrast to classical mechanics, here physical variables of a system are represented by noncommuting operators, while the states are represented by complex wave functions $\psi$ obeying the Schrödinger equation. This is the replacement for the old Newtonian equations of motion. Out of all this come quantized energy levels, scattering cross-sections and a host of other properties which are all experimentally checked in physics and chemistry. In a practical sense quantum mechanics is enormously successful; we can say most parts of it are relatively well interpreted and linked to experiment. But as to quantum mechanics as a whole, the wave function itself and its objective status, there have been diverging views and even now after more than seventy years the debates continue. At first Schrödinger wanted to interpret his wave function $\psi$ as a physically existing objectively real field like the electromagnetic field; in this way he thought one could have a pure continuum theory with no point particles at all. But pretty soon, after intense discussions with Bohr and Heisenberg in 1927, he had to give this up; and as Heisenberg says[11]:

'After this time Schrödinger at least understood that it was more difficult with the interpretation of quantum theory than he had thought.'

Thereafter Schrödinger remained mainly a critic of the orthodox Copenhagen interpretation, without proposing any serious alternative. The Copenhagen view, basically fashioned by Bohr and Heisenberg with their Complementarity and Uncertainty Principles, rests ultimately on Max Born's interpretation of $\psi$ as a probability amplitude – its absolute square $|\psi|^2$ gives physically measurable probabilities. This is the most widely accepted view, so in quantum mechanics one has quantitative probabilistic descriptions of ensembles alone, not of individual physical systems. Sometimes people draw a distinction between this ensemble interpretation, attributed to Einstein, and the view – attributed to Bohr – that the wave function actually describes the state of an individual system, but the lines are very faint. As early as 1927 de Broglie attempted to interpret $\psi$ as a 'guiding wave' or 'pilot wave' influencing the paths of individual particles; but Pauli criticized him so ferociously that de Broglie withdrew. Many years later such ideas were revived by David Bohm and John Bell, under the general notion of 'hidden variable interpretations'. Late in his life Pauli expressed his own views; he was one of the keenest minds ever and was bold enough to carry Bohr's point of view to its logical conclusion. He felt that ultimately at the level of individual quantum systems and events there is an 'element of irrationality' in nature, and we have to be satisfied with statistical casuality[12]. Add to all this Dirac's early warning that quantum mechanics deals with a level or substratum of nature, of which we cannot and should not try to form any mental pictures but just go by the mathematical equations – and you see how complex the whole situation is! In recent times people have suggested *ad hoc* modifications in the Schrödinger equation, like a bit of stochasticity here or a bit of nonlinearity there. However these attempts seem rather *ad hoc*, and remind one of the phrase 'throwing the baby out with the bath water ...'.

At this stage it should be pointed out that the struggle for physical interpretation is partly tied up with psychological factors and human fears. Let me introduce my examples with this remark of Dirac[13]:

'The research worker is only human and, if he has great hopes, he also has great fears.'

In the case of Lorentz, he had done all the difficult mathematical analysis and found the correct transformation equations for space and time coordinates, but he could not take the next decisive step to special relativity. Dirac conjectures[13]:

'I think he must have been held back by fears, some kind of inhibition. He was really afraid to venture into entirely new ground, to question ideas which had been accepted from time immemorial. He preferred to stay on the solid ground of his mathematics.'

Well, at that time Lorentz was fifty two and Einstein was half that age, and where one failed, the other succeeded.

Next we come to Heisenberg's discovery of matrix mechanics in June–July 1925. His basic idea was to give up picturing electronic orbits in the atom, since they were not observable. So he said we must not treat position and momentum in the old numerical way, but should replace them by abstract arrays. Later they were recognized to be matrices. But then he found that these arrays for which he invented a law or process of multiplication would in general not commute, and he became afraid. Again to Dirac[13]:

'Now when Heisenberg noticed that he was really scared .... He was afraid this was a fundamental blemish in his theory and that probably the whole beautiful idea would have to be given up.'

However when Dirac saw Heisenberg's paper and the noncommutativity, he had no fears at all – he realized this was the key new idea and took off from there and built up the whole structure of quantum mechanics!. Three years later, though, in 1928, he fell victim to the same syndrome. After he had found his beautiful relativistic wave equation for the electron which I mentioned earlier, he calculated the hydrogen spectrum but only to the first order of relativistic corrections and not exactly. Many years later he explained why he stopped there[13]:

'... simply because I was scared .... Perhaps the whole basis of the idea would have to be abandoned if it should turn out that it was not right to the higher orders and I just could not face that prospect.'

Soon after, C. G. Darwin (the grandson of the Darwin of evolutionary theory) came along and solved Dirac's equation exactly and it agreed perfectly (at that time!) with experiment.

So we see that in these situations two kinds of difficulties can appear – the inhibiting effect of prevailing ways of thinking and prejudices; and fear that a beautiful, new idea may fail upon detailed examination. In both Newton's and Maxwell's cases described earlier, it was the former difficulty. As Heisenberg once said[14]:

'I think the greatest effort in the developments of theoretical physics is always necessary at those points where one has to abandon old concepts.'

Anyway these situations are few and far between, but they have over-riding importance in building up our picture of nature. It would be instructive to find parallels to all this in chemical and life sciences as well.

At last I come to the deepest and most difficult aspect of my topic. This has to do with epistemology or the theory of knowledge. Probably all working physicists (and, for that matter, other scientists too) believe in an externally existing objective world which has properties we can investigate, and which obeys laws we keep unravelling. Our understanding and pictures and theories are *about* something out there independent of us. Though science and understanding are human creations and possessions, the whole universe cannot be a play just for our benefit – as Feynman said in *The Pleasure of Finding Things Out*, it seems so out of proportion to think so. Now where does our knowledge about the external world come from? Surely our senses play a key role; and all the sophisticated instruments invented and used based on earlier understanding of science are ultimately refinements and extensions of our senses. But is that the sole source of knowledge? The extreme empiricist view of David Hume in the middle of the 18th century assumed so, and ended up in an impasse. Certain key notions basic to scientific understanding of phenomena – the notions of time, space, continuity of existence, cause and effect, ... – do not come through direct sensory experience. Faced with this problem, the philosopher Immanuel Kant proposed that there are two components to knowledge, what we may call internal and external; and only when they come together and act in synchrony do we achieve understanding and make progress. The internal part is the a priori component of knowledge, what we as individuals are endowed with in advance of our experience of the external world. This includes the notions of time, space and its geometry, ideas of causality and determinism, and so on. The other external part is what reaches our minds via the senses when we encounter the world. When that happens, we filter or process all incoming information with the help of the a priori categories of thought which are already within us, and which in a sense are ready and waiting for sensory experience to arrive. Such a beautiful picture – we can only understand the world in terms of the a priori notions built into us, there is no other way. But this leads to the question – how does the a priori component of knowledge get built up, where does it come from, and how is it that it is able to handle sensory experience so well?

A convincing answer to this question has been offered only in this century. It is due to Konrad Lorenz, has been beautifully elaborated by Max Delbruck, and is based on the theory of evolution[15]. In brief the idea is this: in the course of evolution governed by natural selection, living organisms retain and develop those capacities that respond best to and help cope with the most important features of the external world. This is the environment in which they have to survive. This is a very slow 'learning process' by the species, not by the individual. On the other hand, each individual member of the species is born, so to say, with these capabilities 'ready made'. And the actual acquisition of knowledge is completed, to a great degree, in early infancy by using these inborn capacities. What seems a priori, given in advance of experience, to the individual member of the species within one life time is actually a posteriori, the result of experience, from the point of view of the species as a whole! In this way our intuitive concepts of space, geometry, time, causality, the capacity for making mental pictures of natural phenomena – they are all related to features of that part of the world directly relevant for biological survival, namely the world of phenomena roughly at our own scales of length, time, mass and motion. But when we move away from this range of phenomena – the world of middle dimensions – and explore the microscopic scale at one end or the macroscopic scale at the other – no wonder at all that all so often our intuitions fail us, and we have to rely almost entirely on mathematics for our understanding. Away from the biologically familiar, mathematics really becomes our sixth sense and guide.

I have hinted earlier that the very fact that phenomena obey definite laws indicates that there is 'a mathematical quality in nature', which however we cannot reach through the ordinary senses. Within mathematics itself there are those who are realists, and those who believe mathematics is entirely a human creation[16]. The realists deeply believe that there is an objective external 'continent of mathematics' independent of us, which we keep exploring as time goes on. Is our capacity for mathematical thinking linked to this mathematical quality inherent in nature? Has it been acquired by us as an a priori category of thought because then we can recognize this aspect of nature and so at least in a rudimentary sense it is useful for biological survival? Even before Lorenz and Delbruck, the great mathematician David Hilbert did express such ideas relating mathematics to the a priori. In a famous 1930 lecture he says[17]:

'I even believe that mathematical knowledge depends ultimately on some kind of such intuitive insights.... Thus the most general basic thought of Kant's theory of knowledge retains its importance.... But the line between that which we possess a priori and that for which experience is necessary must be drawn differently by us than by Kant.... Kant's a priori theory contains anthropomorphic dross from which it must be freed. After we remove that, only that a priori will remain which also is the foundation of pure mathematical knowledge.'

So where does all this leave us? Hilbert's ideas were expressed well before the ideas of Lorenz and Delbruck explaining the origins of the a priori. But if we believe both, and put them together, we seem to reach some definite points of view. Mathematical structure is surely an essential component of nature, though it lies beyond space, time and the reach of the senses. Echoing Heisenberg[18]:

'If nature leads us to mathematical forms of great simplicity and beauty that no one has previously encountered, we cannot help thinking that they are 'true', that they reveal a genuine feature of nature.'

And we have acquired over evolutionary time an a priori capacity and sensitivity to it, a sixth sense, though the subsequent enlargement of mathematical ideas makes it a world of its own. A very great deal of the physicist's picture of nature can only be expressed in mathematical language. Mathematics and physics keep coming close to each other and to touch one another, then draw apart, only to meet each other again.


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