Statistical analysis of ensembles of strong motion records

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Spatial variations of strong motion are generally studied in terms of attenuation laws for peak ground accelerations with reference to the epicentre, located from long distance records. The question arises, how a set of strong motion records obtained from arbitrarily located stations are inter-related among themselves and with the epicentre. To answer this question, a new approach is proposed in this paper by determining a force centre statistically from the set of recorded accelerograms.

NEAR-source earthquake records provide valuable information for hazard estimation and for aseismic design of structures. In the past such data have been used to propose stationary and nonstationary stochastic models[1,2] for ground acceleration during earthquakes. However the analyses have been largely based on a few isolated samples drawn from separate sources on different dates.

In recent years many strong motion (SM) arrays have been operating in seismic zones. Several samples, from the epicentral region, originating from the same event are available. This provides an opportunity to understand spatial variability of strong motion going beyond attenuation of peak ground acceleration. It may be noted here that for finding spatial correlation of ground motion at very short distances, strong motion instruments will have to be placed very closely which may not always be possible. In the present paper, ensemble of strong motion data available from the Himalayan region for five different events are statistically analysed to arrive at some new results.

One purpose of strong motion instrumentation is to find out the forces acting on engineering structures during earthquakes. This has been the primary reason for investigating the relation between peak ground acceleration (PGA) and structural damage. Thus there has been interest among engineers to study the relationship between isoseismals (MMI) and ground acceleration. Importance of such studies for hazard estimation and zonation is well known. It is usually believed that the PGA is maximum at the epicentre and so is the structural damage. Further, the orientation of the isoseismals is thought to indicate the directions of the fault and its rupture.

When a set of records from the same shock are available, it would be interesting to ask whether it is possible to substantiate or verify such intuitive concepts. To this end, a new concept, namely, of a force centre is proposed here. First it is observed that whereas the SM data are point measurements, the ground vibration is spatially varying. Thus the station measurements have to be attributed to an area surrounding the specific location. This problem is similar to the one faced in meteorology, where point rainfall values have to be attributed to an area through area weights. One approach of weight assignment is through the Theissen polygon method[3]. As an example the station area weights (a_i) and the areal extent represented by the SM stations of a particular array are shown in Figure 1. When an event is recorded by the array, the epicentre may or may not lie within the representative area. However each station is associated with an area weighted acceleration time history. An engineering interpretation of this quantity would be that it is proportional to the force experienced by a rigid body in that area.

Alternatively if as an approximation the intervening medium is taken as a rigid body with a plane surface, the area weighted horizontal accelerations represent time dependent co-planar forces. It is clear that following this analogy it is possible to arrive at an area weighted resultant force in two orthogonal directions for a given set of data. But a more interesting problem would be to find the point of application of this resultant force. Since the forces are time-dependent and random, the location of the force centre will also randomly change with time. However, this will be an important strong motion parameter inasmuch as it is derived from the ensemble of all the records.

Let the coordinates of the stations ($i = 1, 2, \ldots N$) be represented as (x_i, y_i) with reference to any arbitrary origin. The area weighted forces at these points $[u_i(t), v_i(t)]$ are co-planar and acting in two orthogonal directions. It is convenient to take the coordinate axes in the E–W and N–S directions. The resultant forces are easily obtained as

$$F_x(t) = \sum_{i=1}^{N} u_i(t) \quad \text{and} \quad F_y(t) = \sum_{i=1}^{N} v_i(t) \,. \tag{1}$$

The point of application of the resultant is given by

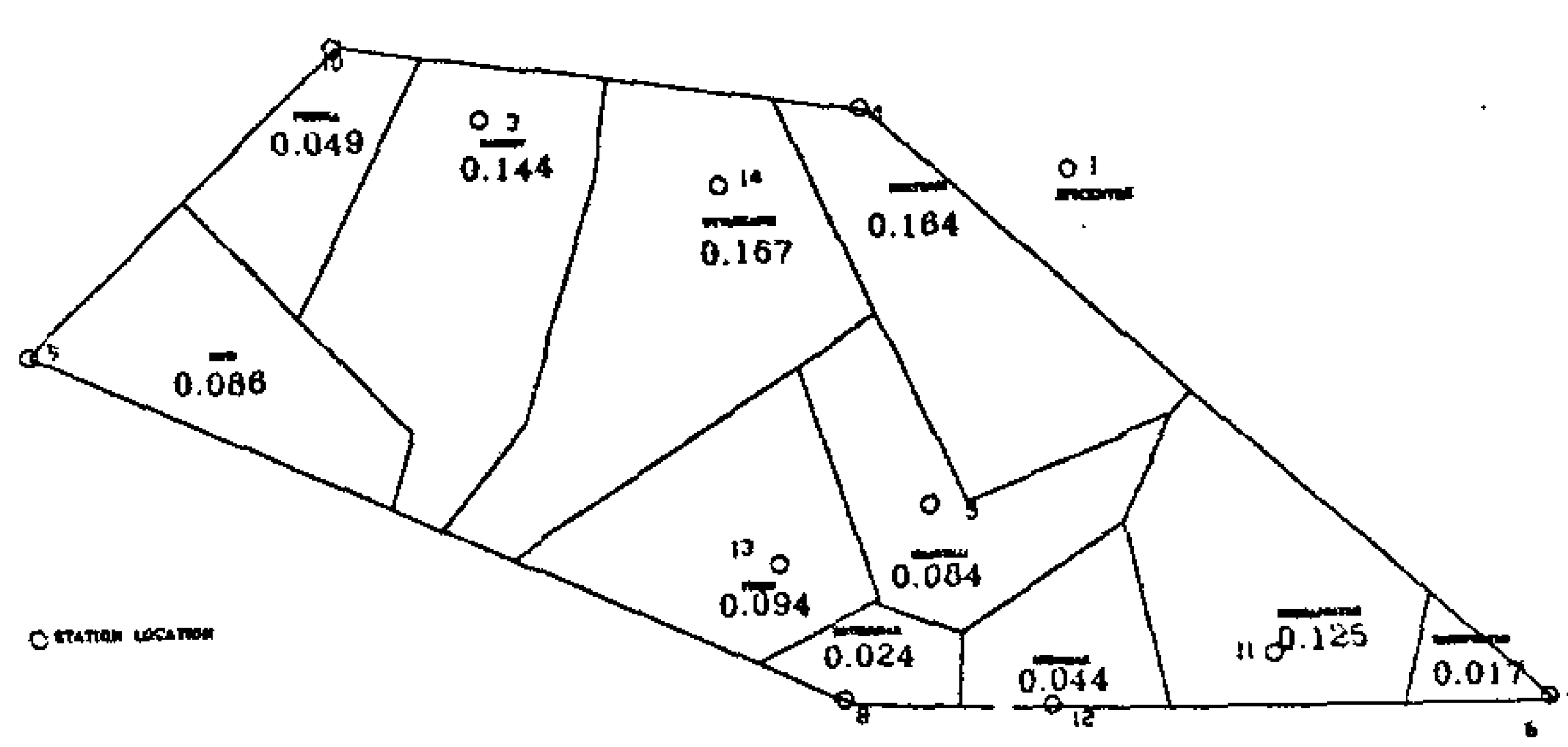

Figure 1. Area weight of stations (Uttarkashi event).

$$\bar{x} = \frac{M_x}{F_x} = \frac{\Sigma x_i u_i}{\Sigma u_i}, \quad (2)$$

$$\bar{y} = \frac{M_y}{F_y} = \frac{\Sigma y_i v_i}{\Sigma v_i}. \quad (3)$$

The above equations, as they are, are ill conditioned, since the records may not have the same starting time and also due to phase differences the resultants may be vanishingly small at some instants. Thus, it is essential to recognize the random nature of u_i and v_i. It is well known that ground accelerations can be taken to be Gaussian processes. Thus to find the probability distribution of $(\bar{x}, \bar{y})$ at any instant, we need to know the four-dimensional Gaussian probability distribution $p(F_x, F_y, M_x, M_y)$. The construction of this 4-D probability density function is straightforward in terms of the mean values and the covariance matrix of the four random variables. The point probability density function of $\bar{x}, \bar{y}$ at any instant is given by

$$p(\bar{x}, \bar{y}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(F_x, F_y, \bar{x}F_x, \bar{y}F_y) \mid F_x F_y \mid \mathrm{d}F_x \, \mathrm{d}F_y. \quad (4)$$

One integration in this expression can be carried out in closed form. It may be noted here that since $(\bar{x}, \bar{y})$ are ratios of Gaussian random variables their moments are not defined. However the probability density function can be obtained numerically.

Five sets of strong motion data from the Himalayan region are analysed for illustrating the computation of the locus of the force centre. The details of the data considered[4] are shown in Table 1. Uniformly digitized data are available at 50 samples per sec. In the computations, four mean values and ten covariance values are needed. Since the processes are nonstationary, these values would vary with time. In practice these can only be statistically estimated from the data by short-time averaging. The values of the moments will have to be taken as constant over the interval of averaging. For the present data the interval of averaging has been kept as one second, which amounts to fifty samples of the data.

$p(\bar{x}, \bar{y})$ and its mode have been computed at every one-second interval to arrive at the most probable location $(\bar{x}, \bar{y})$ of the force centre. In Table 2, a typical set of results are shown. The duration of the data available was 35 sec and thus 35 time-dependent locations for the force centre have been computed. For all the events considered here the epicentral location is available as reported by various seismological observatories. The location of $\bar{x}, \bar{y}$ in Table 2 has been reported with reference to the epicentre as the origin. For all the five sets of data the locus of the force centre as the strong motion progressed is shown in Figure 2 *a–e*.

The concept of the epicentre for an earthquake is well known. In a vague sense it is the point on the surface of the earth from which the motion originates. It is also established that strong motion is invariably associated with fault rupture, the length of which is well related to the magnitude of the shock. As the fault

Table 1. Strong motion data

Earthquake	Magnitude	No. of recording stations	Epicentre latitude and longitude
Dharmashala (26 April 1986)	5.7	9	32.193°N 76.290°E
N–E India (18 May 1987)	5.7	14	25.498°N 93.450°E
N–E India (6 February 1988)	5.8	18	25.504°N 91.342°E
Shillong (10 September 1986)	5.5	12	25.562°N 92.187°E
Uttarkashi (20 October 1991)	6.6	13	30.750°N 78.860°E

Table 2. Temporal location of force centre with respect to epicentre–Uttarkashi event

Time (sec)	x (km)	y (km)
1	0.8	1.2
2	0.8	0
3	-20.0	0.8
4	-7.6	0
5	-0.8	0
6	-3.6	0
7	-2.4	0
8	-4.0	0.8
9	-2.8	0.4
10	-4.4	1.6
11	-0.8	0.8
12	-0.4	0.8
13	0	1.2
14	-2.8	0.4
15	-7.6	0.8
16	-2.0	1.2
17	-2.0	2.4
18	1.6	0.8
19	1.6	2.0
20	-4.0	1.6
21	1.2	1.6
22	3.2	1.2
23	-1.6	0.8
24	-6.8	1.2
25	-6.8	0.8
26	-2.4	0
27	-9.2	0.8
28	-16.4	1.2
29	-5.2	4.0
30	-10.8	0.4
31	-2.0	0.4
32	0.8	1.2
33	-6.0	0.4
34	1.2	1.2
35	-13.6	1.6

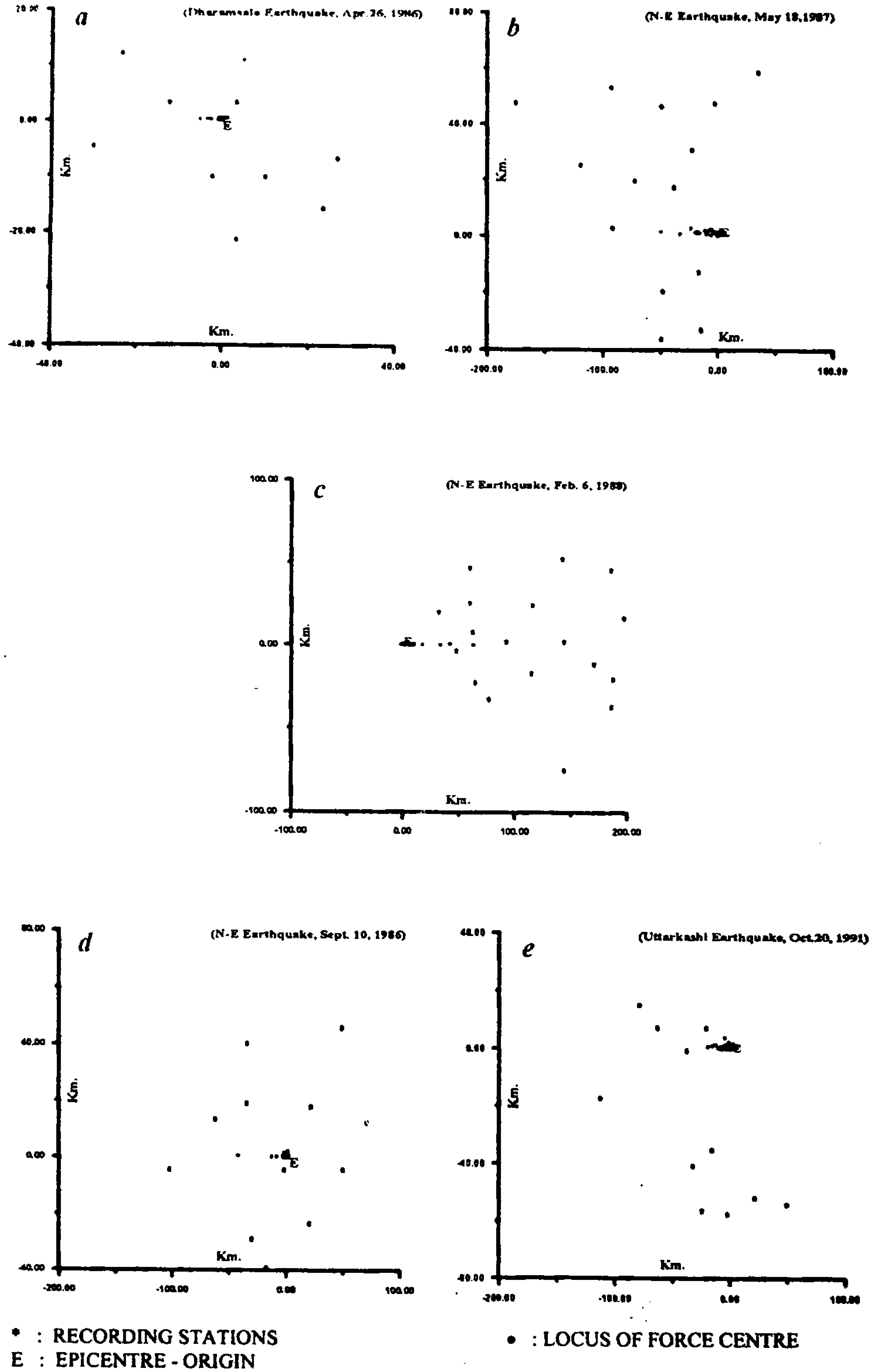

Figure 2 *a–e*. Force centre estimation.