Photonic gap materials

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The concept of a photonic material is explained in terms of Bragg diffraction. Analogies are also made with electrons in semiconductors. Possible routes to manufacture of these structures are explored as some naturally occurring micro-periodic structures. The article concludes by giving some background to computational modelling of these materials.

The concept of a photonic crystal

As the world demands more computers and communication we turn increasingly to optical devices whose bandwidth and speed of execution offer great potential. However materials science has lagged behind in this rush into the optical domain. Optical properties of materials are not always well-matched to the functions we seek. This is in contrast to the vast range of electronic properties available to us. In the electronic domain we can almost make materials to order. The main cause of this richness in electronic properties is the interaction of electrons with the periodic structure of the materials. It is this interaction that decides whether a material will be a metal, a semiconductor, or an insulator, and can be further exploited to fine-tune the detailed electronic properties. Change the structure change the properties.

It was this concept that led Yablonovitch[1] to propose that we try the same trick with light. Interaction of electromagnetic waves with periodic structures goes back to Bragg and his observation that planes of atoms can act like perfect mirrors to X-rays when the Bragg condition is met (Figure 1):

$$\lambda = 2d\sin(\theta \pm \delta).$$

In fact the perfect mirror behaviour holds over a range of angles, $\pm\delta$, determined by how strongly the atoms scatter the X-rays. Any given arrangement of atoms will contain many possible Bragg planes reflecting X-rays from different directions. As a crystal is turned in a beam of X-rays it will briefly 'glint' as each Bragg condition is met. Figure 2 illustrates the point.

Thus we know that a crystal rejects X-rays for certain limited angles of incidence. The concept of Bragg reflection applies equally well to visible radiation except that we cannot rely on atoms to do the work for us. The material must have some periodic structure on the scale of the wavelength of light – a fraction of a micron. In fact, nature has made this arrangement for us in the case of gemstone opal. Some years ago, using an electron microscope it was shown that the brilliant colours observed in precious opal were due to mesoscale structure within the mineral. The structure had eluded previous workers because the scale was too fine to be seen with optical instruments, and too coarse to be observed by X-rays. However, it is now possible to make beautiful electron micrographs that reveal the face-centred cubic close-packed arrangement of the microscopic silica spheres that constitute opal. Figure 3 shows such a micrograph.

Nearly all of biology is 'engineered' on the micron scale and with the help of DNA very complex structures are manufactured. Not surprisingly the optical properties are frequently exploited, nowhere with more spectacular effect than in the butterfly. Many species show iridescent green or blue patches and these owe their colouring to diffraction from periodic materials in the scales of the wing. Figure 4 shows a photograph of a Peacock butterfly where the 'eyes' exhibit the characteristic blue metallic sheen of a photonic material. Figure 5 shows a somewhat enlarged section of an almost entirely blue butterfly, the Adonis Blue, in which the tiny submillimetric scales are visible, and in Figure 6 we see an electron micrograph of a scale, the entity responsible for the colour. Diffraction from a three-dimensional array of holes in the scale gives rise to its characteristic colour. Man has not yet completely mastered the art of manufacturing three-dimensional optical structures which nature produces with such ease.

So, just like electrons, photons can be pushed around by the structure of a material. Yablonovitch wanted to find out if just as a semiconductor has a forbidden band of energies within which no electron could enter the crystal, is it possible to make a periodic dielectric such that in a forbidden range of frequencies no photon could enter the crystal. What we need to do is find a periodic material in which the scattering of light is so strong that the rejection windows of the different Bragg planes overlap (Figure 7). When this happens photons will be rejected whatever direction they attempt to penetrate the material. In fact, such a material could truly be called a 'photonic insulator'.

Realizing the concept

Although the *concept* is a simple one, *realizing* a material structured on a sub-micron scale in three-dimensions

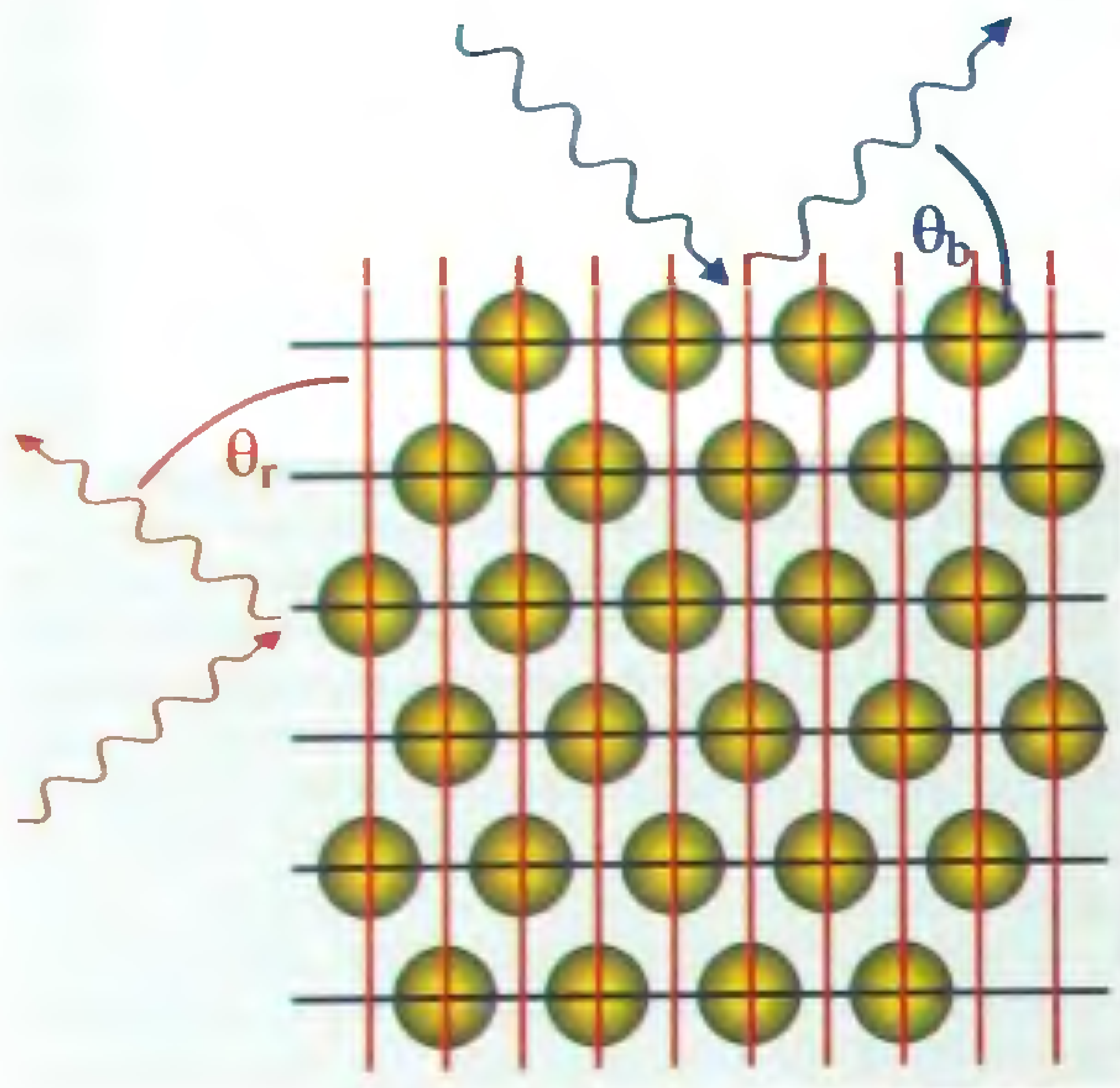

Figure 1. A regular array of atoms diffracts X-rays when the Bragg condition is met. For incident X-rays of a given wavelength different planes reflect at different Bragg angles.

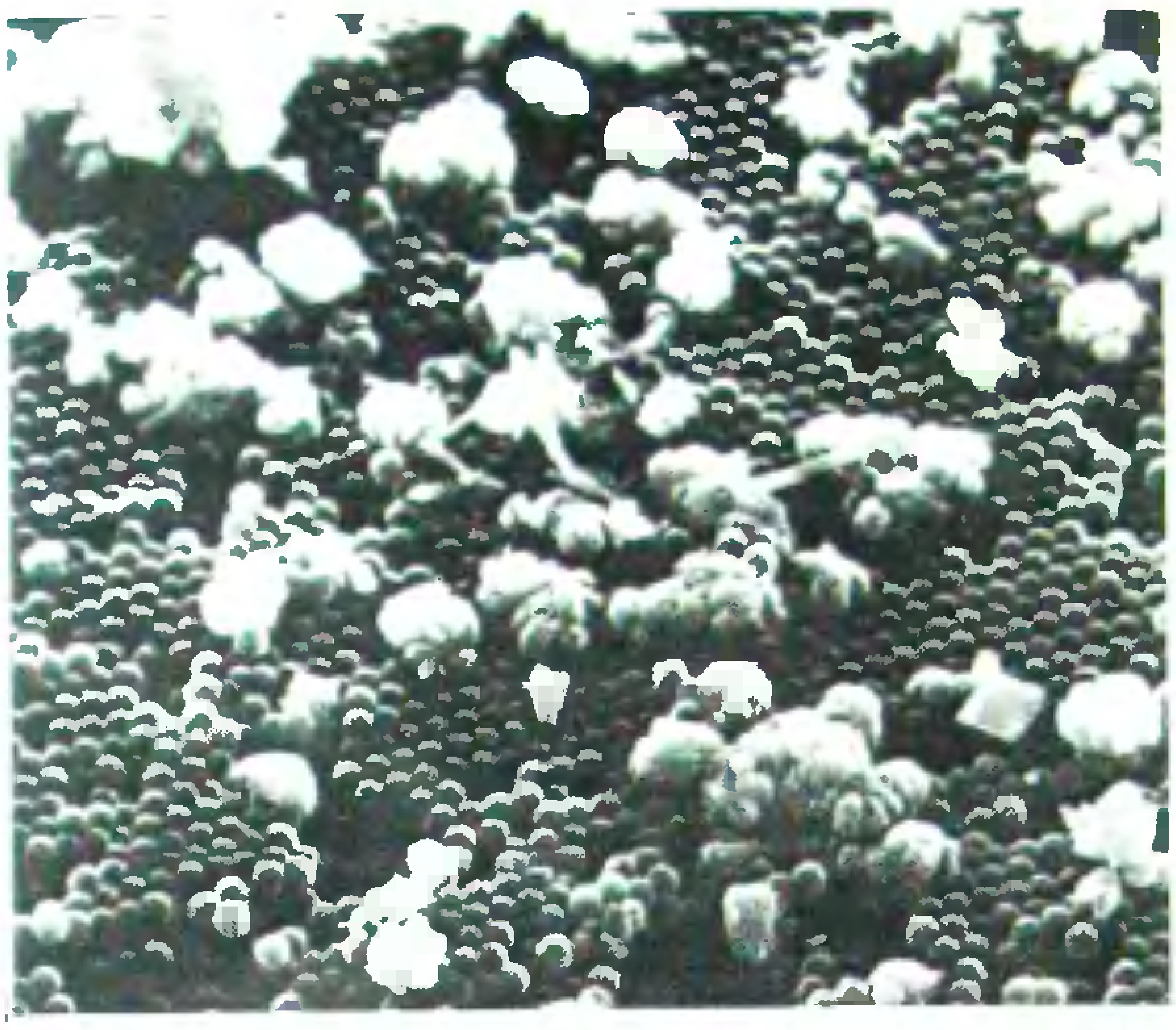

Figure 3. An electron micrograph of precious opal comprising sub-micron-sized silica spheres arranged in a face-centred cubic close-packed structure. The individual spheres are clearly visible under a layer of surface clutter.

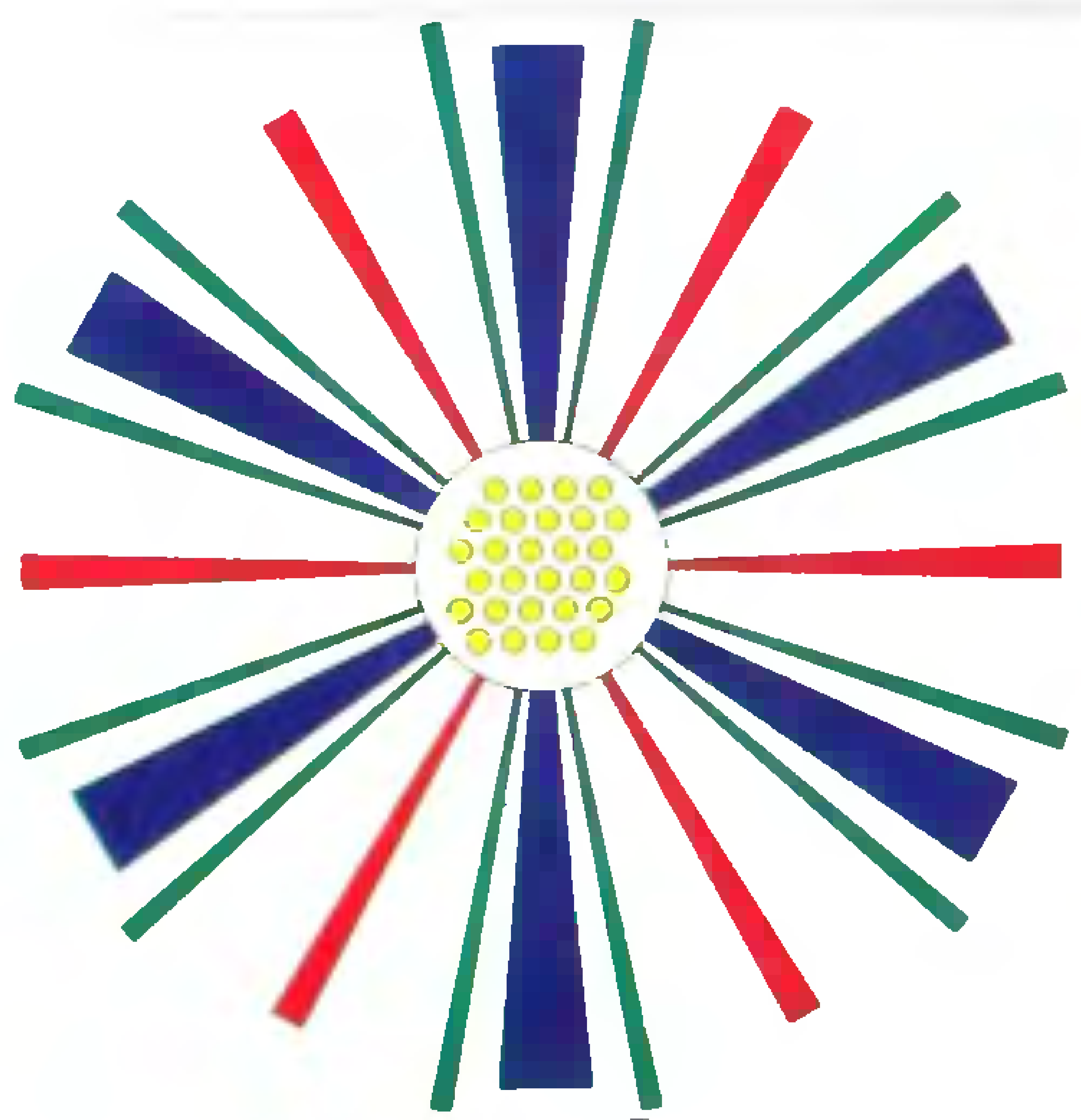

Figure 2. Bragg scattering for X-rays. Each set of planes acts like a mirror in a *small* range of angles which scarcely overlap.

Figure 4. Many butterflies owe their colour to diffraction by three-dimensional micro-structures within the wings. This is particularly true of blues or greens which exhibit a metallic sheen, as in the 'eyes' of this Peacock (*Inachis Io*).

is a highly non-trivial exercise. In opals, nature does the work for us by close packing spheres. Unfortunately this material, although beautiful, falls short of perfection – it is not a photonic insulator. The dielectric contrast between silica and air is too small, and the structure consisting of close-packed spheres is not sufficiently favourable. However Yablonovitch has shown[2,3] that there is a structure manufactured by drilling holes that does behave as a photonic insulator. This structure has been christened 'Yablonovite' in honour of its inventor, and its construction is given in Figure 8.

If the refractive index of the host material is of the order of 3.6 or greater, this material is a photonic insulator. This was demonstrated in an elegant manner by observing that all structured materials obey a very general scaling law – if a given structure is a photonic insulator for incident light of wavelength λ, then another

Figure 5. The wing of a butterfly consists of a series of scales each a fraction of a millimetre long. In this photograph of an Adonis Blue (*Lysandra bellargus*) the small scales are visible as striations of the blue section of the wing. The black spots are caused by the occasional missing scale.

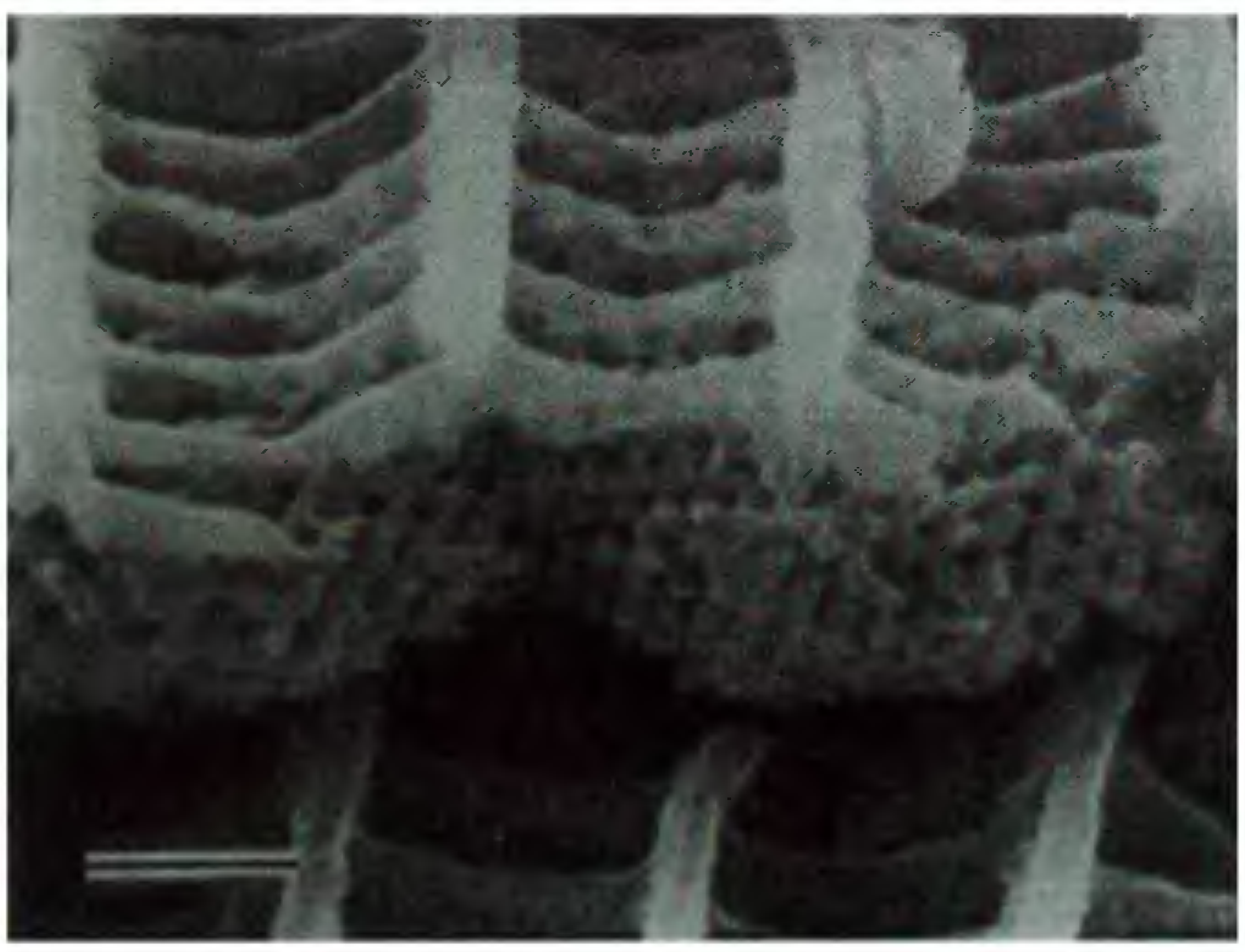

Figure 6. Electron micrograph of a broken scale taken from the butterfly *Mitoura grynea* revealing a periodic array of holes responsible for the colour. The bar in the bottom left is one micron long. (Taken from ref. 10).

Figure 7. Bragg scattering for a photonic material. Each set of planes acts like a mirror in a *large* range of angles which overlap completely. Under these circumstances no radiation can penetrate into the material which is a *photonic insulator*. For a limit range of frequencies, the band gap, radiation is rejected whatever the incident angle.

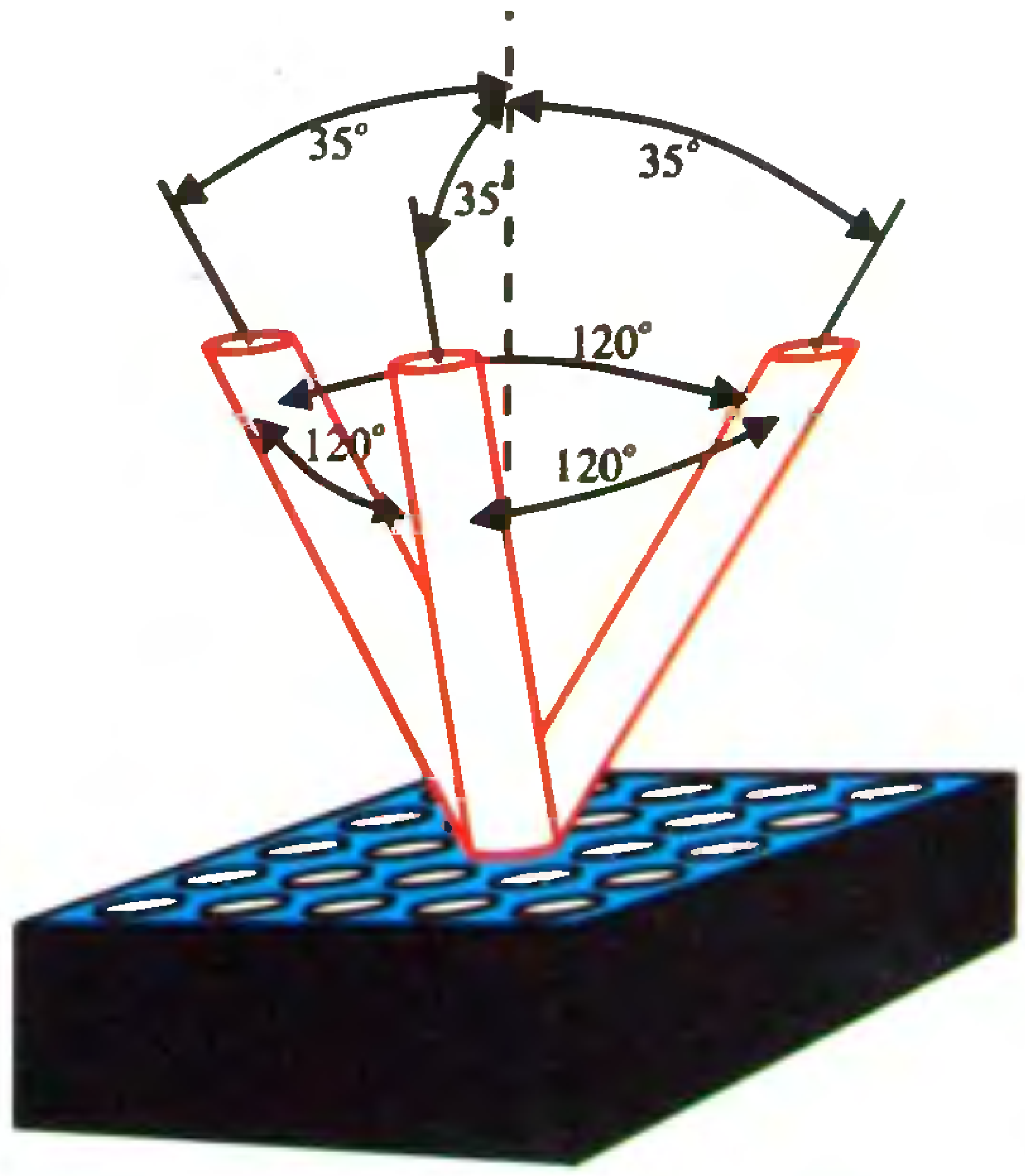

Figure 8. 'Yablonovite': a slab of material is covered by a mask consisting of a triangular array of holes. Each hole is drilled through three times, at an angle 35.26° away from normal, and spread out 120° on the azimuth. The resulting criss-cross of holes below the surface of the slab produces a fully three-dimensionally periodic *fcc* structure.

material structured on double the length scale of the first, will also be a photonic insulator, but at incident wavelength 2λ. This is true provided that the dielectric function does not depend on frequency. Therefore, it should be possible to make a proof of principle experiment on a much larger structure, active in the microwave range, and much more easily constructed, than to invoke the scaling theorem to prove that if only a small structure could be manufactured, it would function as an insulator in the visible.

Figure 9 shows Yablonovitch's measurements for a sample of Yablonovite made with unit cell dimensions of a few millimetres and the band gap is clearly visible.

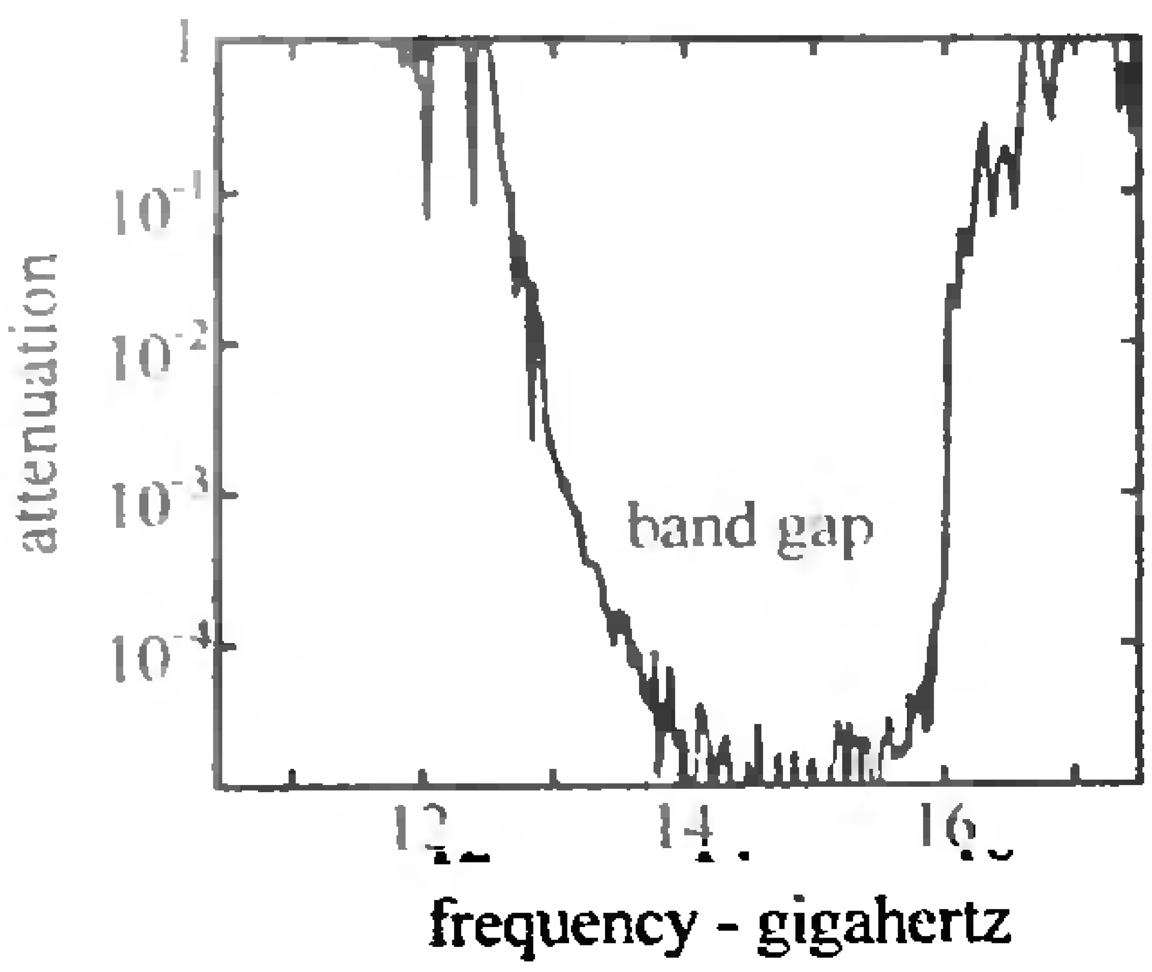

Figure 9. The transmission coefficient to microwaves of a sample of Yablonovite that shows a large band gap at frequencies around 14 GHz.

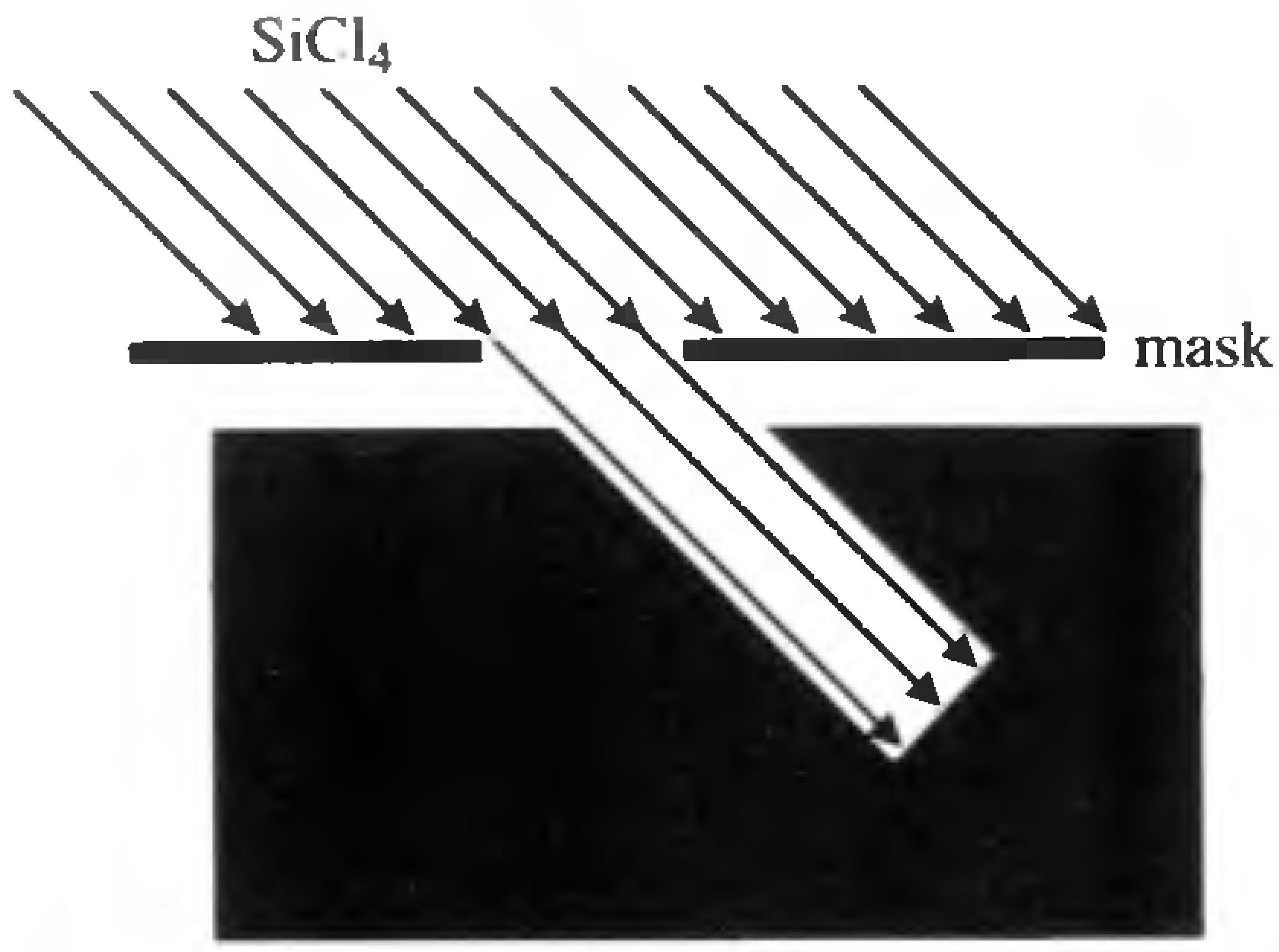

Figure 10. Hole drilling by reactive ion etching. Reactive ions, $SiCl_4$ in this case, erode a surface at the point where they strike. A mask selectively protects the surface from unwanted erosion. Holes as small as 100 nm in diameter can be drilled.

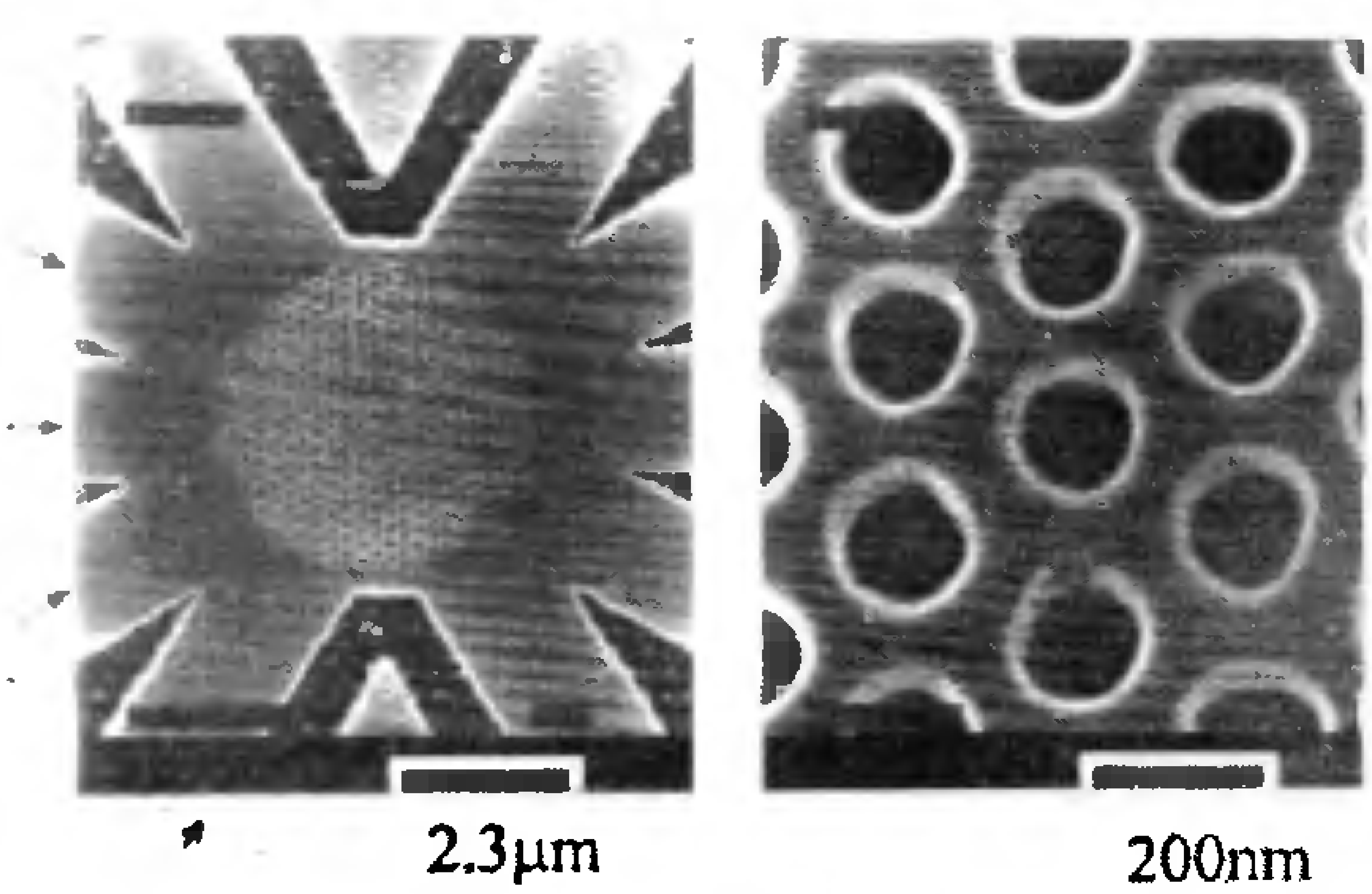

Figure 11. A photonic structure after Krauss *et al.*[14], drilled into a 0.4 µm thick sample of AlGaAs. On the left an overview is given with the photonic structure in the centre surrounded by connecting waveguides. On the right we see a 10× enlargement of the central photonic region.

There still remains the challenge of sub-micron engineering which is currently being addressed in laboratories around the world. One tool that has shown great promise is reactive ion etching. Ions incident on a sample will react with the first surface atom they meet and carry it away into the vacuum. Thus, shaping the beam by passing it through a shadow mask, and choosing its orientation, we can drill holes of a chosen cross-section in any direction we choose. Figure 10 illustrates the principle of this technique. Krauss *et al.*[4] have employed this technique to make the structures shown in Figure 11.

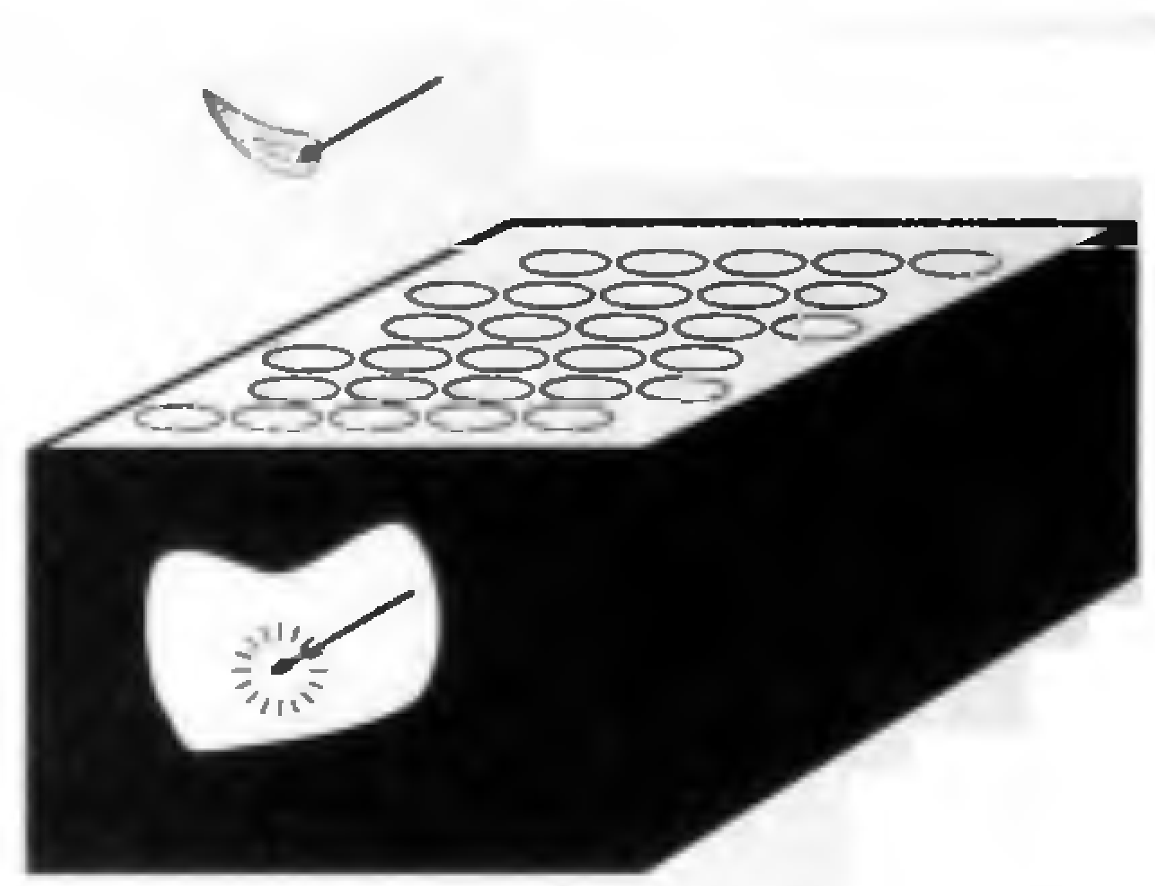

Figure 12. Trying to make light inside a photonic insulator by striking a match. The match will not light because photons are rejected by the photonic insulator. It is so dark that not even the quantum zero point fluctuations are present. We can exploit this effect to preserve atoms in their excited state to make lasers more efficient.

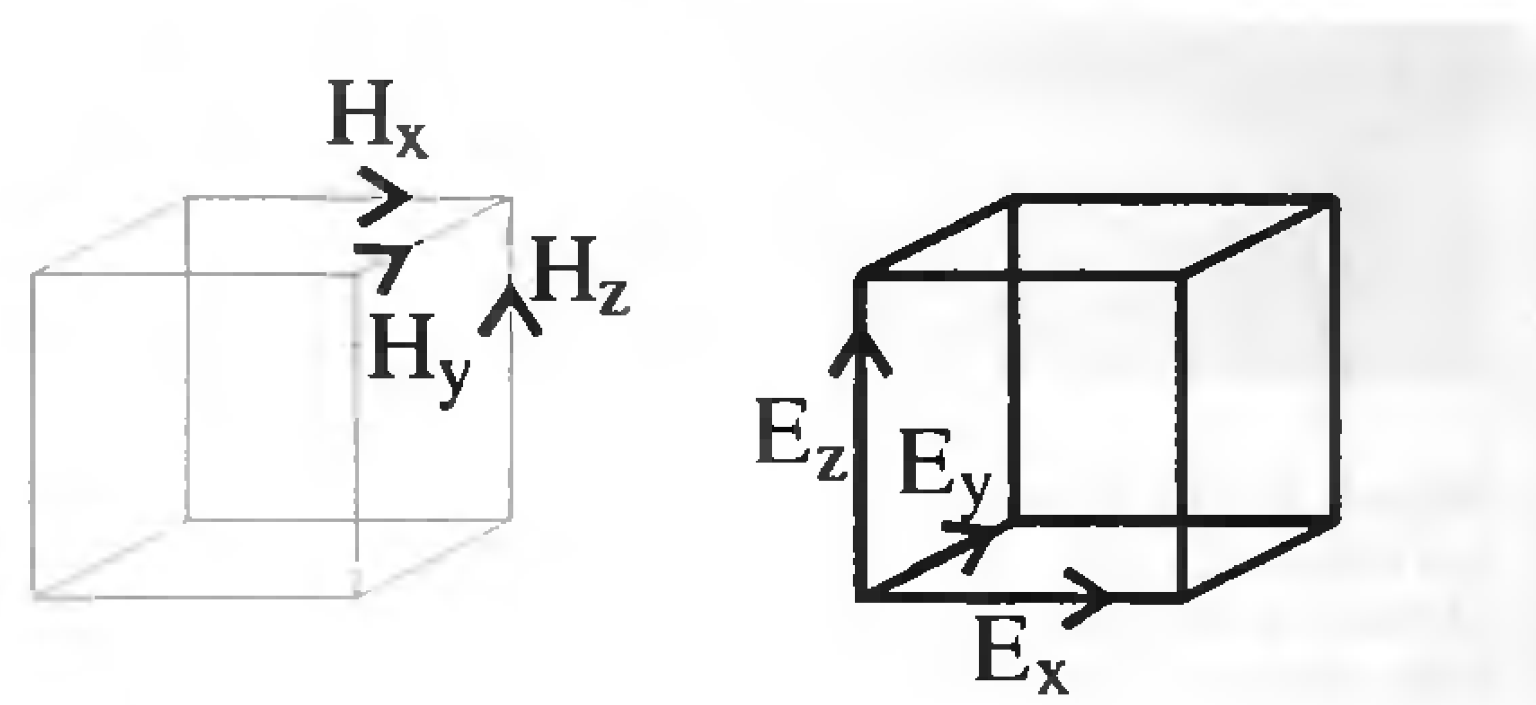

Figure 13. The unit cells of the H and E lattices. Note that the origin is different in each case: at the top right of the cell of the H-field, and at the bottom left for the E-field.

In fact the Glasgow structure, though a technical *tour de force*, is still only a two-dimensional material, and therefore is not a true photonic insulator. Nevertheless, technical progress is sufficiently encouraging that we can expect these structures to be part of a materials scientist's resources in future.

Why photonic materials are special

Why should a material that excludes all light in a given frequency range be special? After all there are other ways of excluding light – a metal box is quite dark inside! I want to emphasize how very special a photonic material is – it rejects light not because light cannot penetrate through the surface, but because there is just no place for the light to go once it is inside. There are no electromagnetic states available inside the material. Take a practical illustration of this fact. The interior of a sealed metal box is dark, but can be illuminated by, say, striking a match inside. However, if we introduced a suitably miniature match down one of the excavated cylinders in Yablonovite and attempted to strike it, we could not do so. The structure would return the photons to the reacting atoms refusing to accept the energy (Figure 12).

Although no one is interested in striking matches inside these structures, they are interested in manufacturing lasers from them where similar principles may help the efficiency of the laser. In most lasers there is a power threshold for lasing action, because there are other ways of emitting light than into the desired lasing mode. The demands of these spurious modes must be met before the critical amplitude can be established in the lasing mode. Encasing the laser in a suitably structured photonic material can forbid the power sapping modes and increase the laser efficiency moving us further towards thresholdless lasers.

The contribution of the theorist

At this challenging time when the photonic concept is established, but practical realization remains so difficult, it is extremely valuable to have computer programs that can predict the properties of these materials, avoiding costly experimentation with difficult new technology. Perhaps the most successful approach to computation has been the discrete sampling of the electric and magnetic fields on a fine lattice of points. Maxwell's equations present particular difficulties in that they are concerned with vector fields and geometry is an ever present subtlety. However one successful approach[5,6] is to define two simple cubic lattices – unit cells shown in Figure 13. Along the edges of one are arranged the **E** fields, and along the edges of the other, the **H** fields. Next arrange the two lattices so that they interpenetrate – one lattice centres the other as we see in Figure 14. Note that each *face* of the **H** lattice is pierced by a *line of force* of the **E** lattice.

Applying Stokes theorem around the edges of the face and using Maxwell's equation,

$$\frac{\partial}{\partial t}\varepsilon_0\varepsilon\cdot\mathbf{E} = +\nabla\times\mathbf{H}$$

gives the discrete form of the first Maxwell equation. Similarly, the second equation can be obtained by considering faces of the **E** lattice and applying,

$$\frac{\partial}{\partial t}\mu_0\mu\cdot\mathbf{H} = -\nabla\times\mathbf{E}.$$

The resulting difference equations have been much exploited to calculate photonic properties and to publish

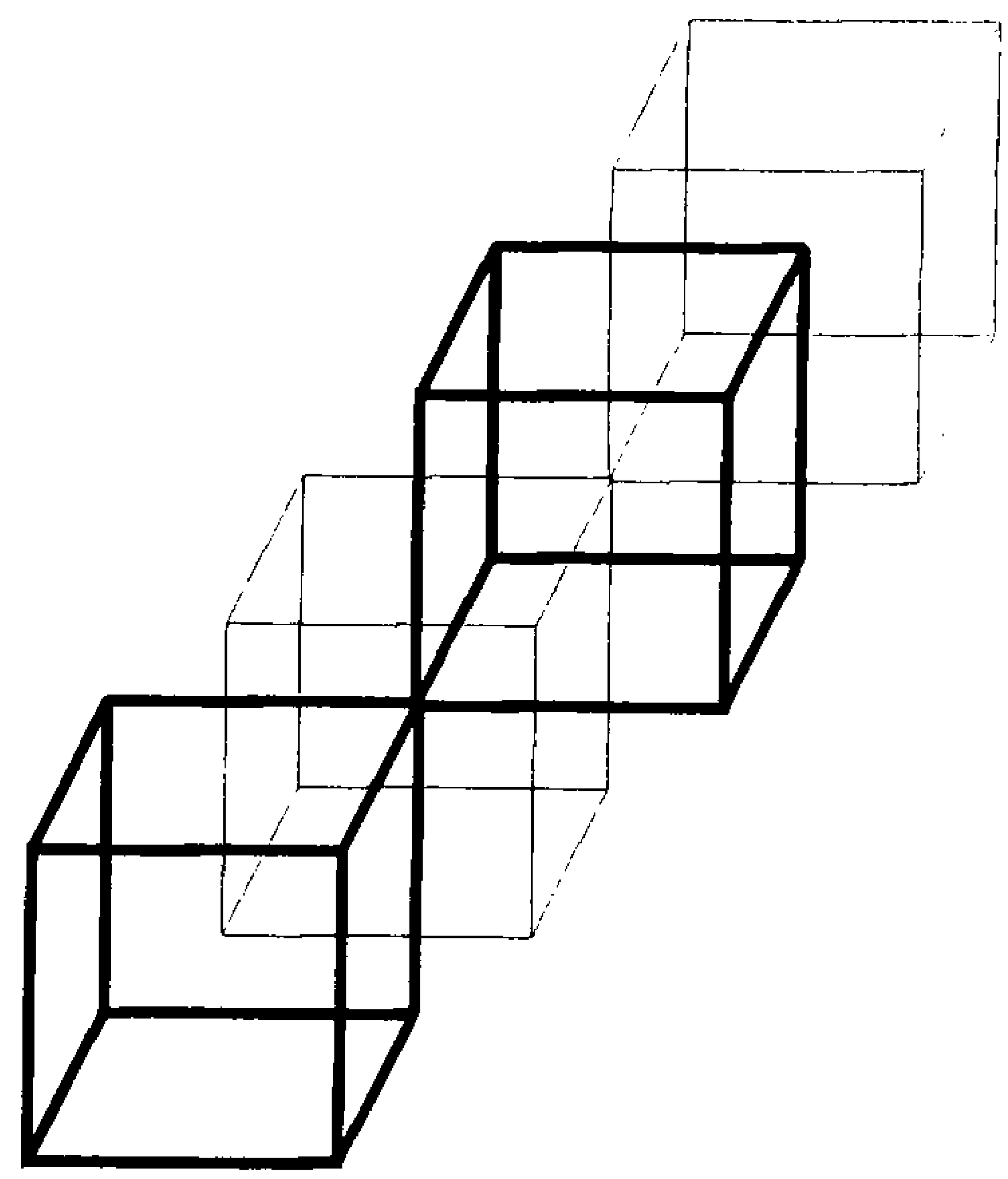

Figure 14. The **H** and **E** fields comprise two interpenetrating simple cubic lattices: the origin of the **H**-lattice is displaced by $(\mathbf{a} + \mathbf{b} + \mathbf{c})/2$ relative to the origin of the **E**-lattice. Note that each bond carrying a component of the **E**-field is encircled by four bonds defining the side of one of the **H**-field cells, and correspondingly for each bond of the **H**-field.

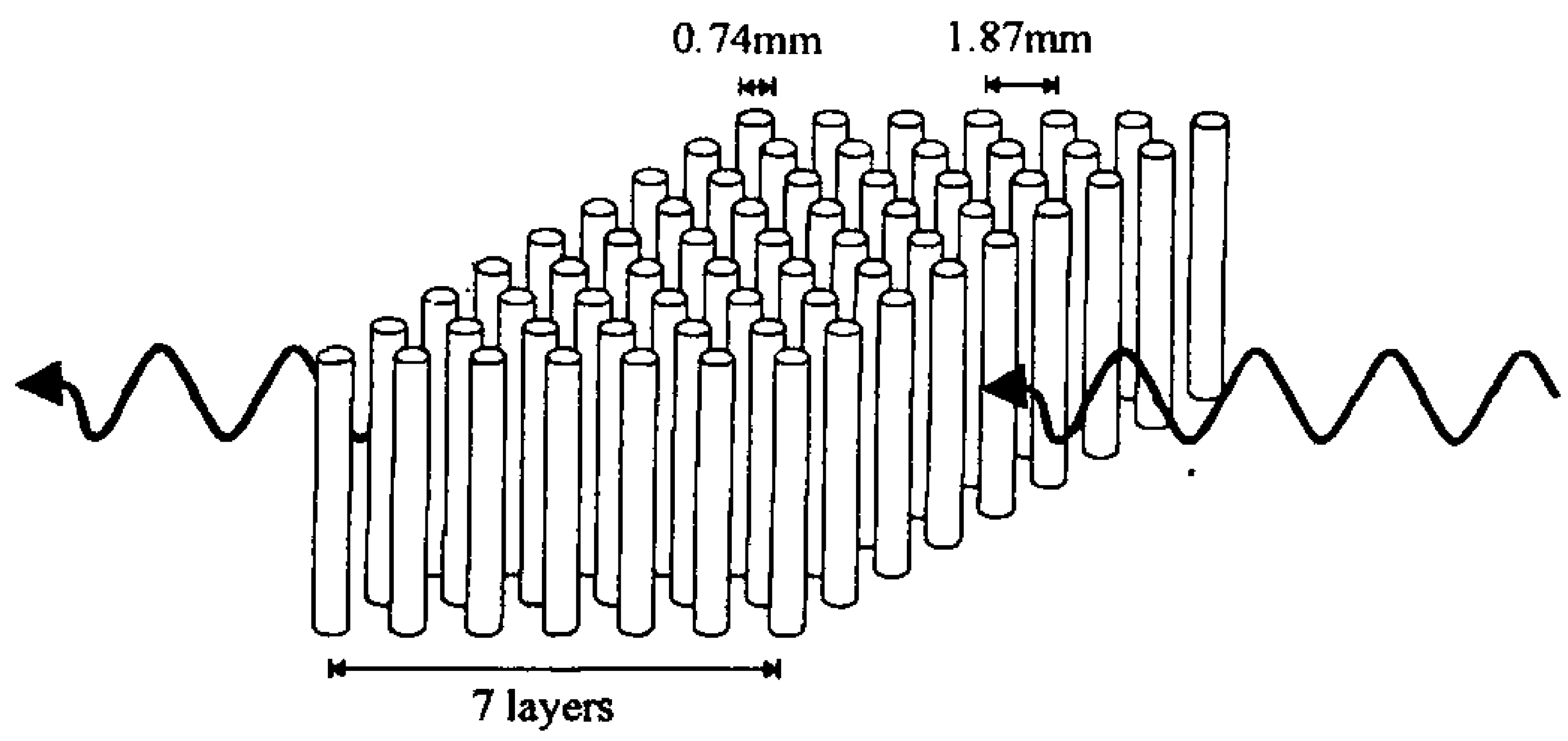

Figure 15. Schematic version of the microwave experiment by Robertson *et al.*[8] Microwaves are incident on a simple cubic array of cylinders, $\varepsilon = 8.9$, 7 layers deep but effectively infinite in the directions perpendicular to the beam. The amplitude of the transmitted beam is measured.

computer codes that are freely available[7]. One of the first such calculations was a 'proof of principle' for an array of dielectric cylinders shown in Figure 15. The calculations[6] shown in Figure 16 are in good agreement with experiment[8].

Even if the lattice is distorted, we can still apply Stokes theorem and get some equations. These can be transformed back to the original Maxwell's equations on the *undistored* lattice provided that we modify ε and μ and the fields[9],

$$\frac{\partial}{\partial t}\varepsilon_0\hat{\varepsilon}\cdot\hat{\mathbf{E}} = +\nabla\times\hat{\mathbf{H}},$$

$$\frac{\partial}{\partial t}\mu_0\hat{\mu}\cdot\hat{\mathbf{H}} = -\nabla\times\hat{\mathbf{E}}.$$

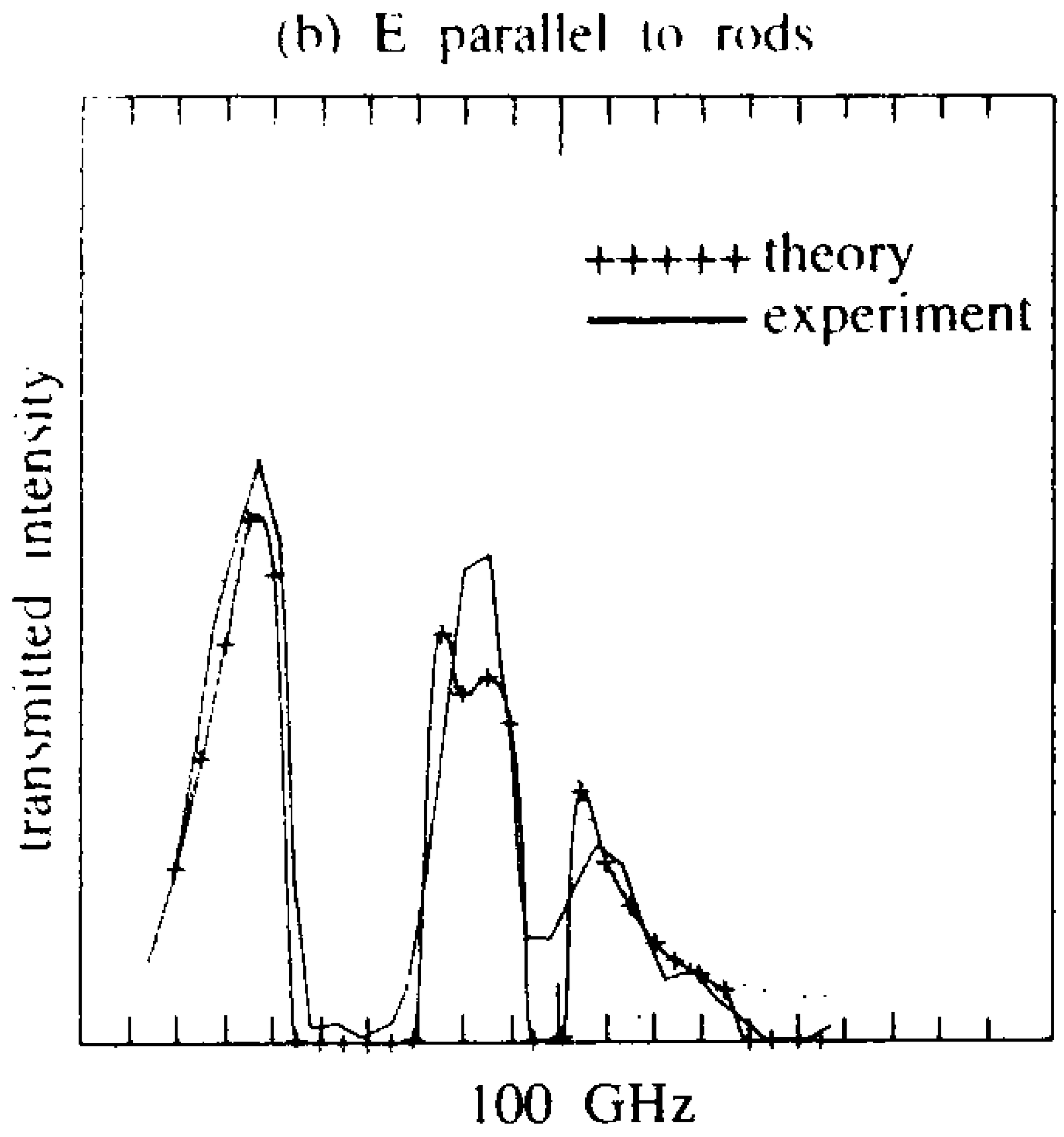

Figure 16. Transmitted power for an array of seven rows of dielectric cylinders. The dotted curve shows the instrument response in the absence of the cylinders. **E** is parallel to the cylinders.

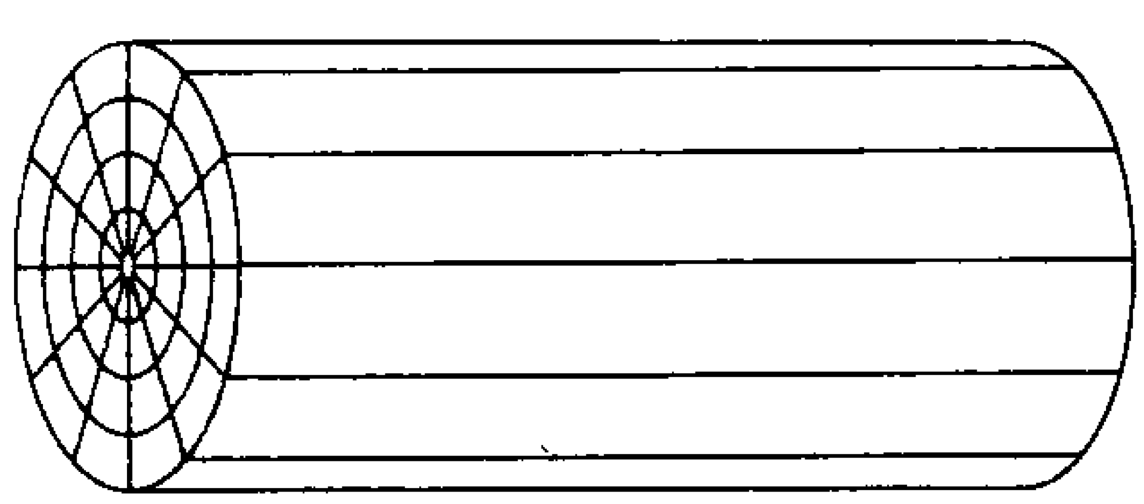

Figure 17. An adaptation of the simple cubic mesh to the geometry of a fibre. The z-component of the mesh can be taken to be uniform.

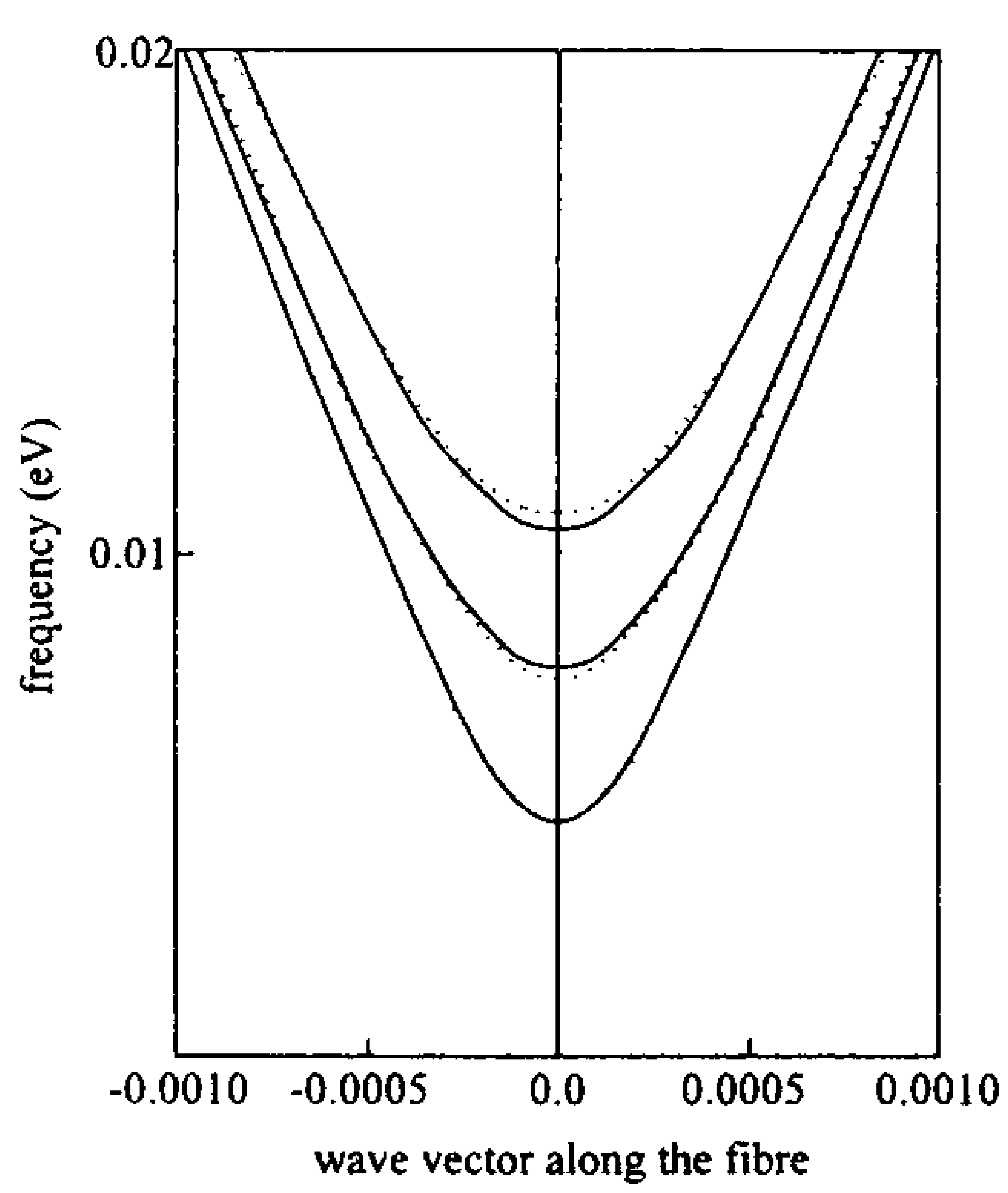

Figure 18. The lowest three modes of a 10^{-4} m radius cylindrical metal waveguide for angular momentum $m = 0$, plotted against wave vector. The full lines are the results of a numerical calculation using a cylindrical mesh; the dotted lines are the exact results of analytic theory.

Remarkably, we find that the *form* of Maxwell's equations is invariant to coordinate transformations. We can choose any coordinate system, adapted to the particular geometry in which we are working, and the only effect on the equations we solve is to modify ε and μ. This is a remarkable and valuable simplification. It means that once we have written our computer codes for one coordinate system, all we have to do to work with another system is to plug in different values of ε and μ. For example, suppose that we are examining structured optical fibres, we can choose a cylindrical coordinate system compatible with the cylindrical symmetry of the fibres, rather than try to fit a square peg in a round hole which is what the original cubic lattice would require us to do. Figure 17 shows the new coordinate system.

Finally we show this scheme in action for a cylindrical wave guide with perfect metal boundaries. One advantage of the cylindrical mesh is that angular momentum is now a good quantum number in the computation, and in addition cylindrical symmetry can be exploited to speed up the calculation. The computations taken from Ward and Pendry[9] and presented in Figure 18 are compared to the analytic result demonstrating that the distorted mesh shows the same excellent convergence as the original uniform version.

Conclusions

The idea of photonic materials is at an exciting stage – the concept is well established and at the same time powerful numerical techniques enable us to foresee many new applications. However we have still to meet the challenge of manufacturing these structures. Existing sub-micron technology is well-developed for two-dimensional objects, but photonic materials demand a fully three-dimensional facility. When it arrives we can expect the rapid incorporation of this new technology into optical devices. In the meantime, we can draw inspiration from the manner in which nature structures her works on exactly the scale we seek.

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