

ling techniques, and for workers in the specific system. One would have liked to have seen more examples like the retinal model where the editor's desire to 'bridge the gap between structure and function at the network level' could be realized.

The book as a whole is at a fairly advanced level and will be most useful to workers already familiar with the basics of neuronal modelling. It provides a number of potentially useful techniques for building models of various scales, from dendrites to networks. One may dispute the claim that 'reduced models of single neurons as discussed in this book will in the next few years constitute neural network models of the future'. Nevertheless, many of the results and analytical methods described in the book will be valuable for developing more realistic and predictive models of neuronal function.

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**New Trends in Dynamic Games and Applications.** G. J. Olsder (ed.) Birkhauser, Boston, 1995.

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Game theory is the analysis of conflict. In any conflicting situation there are two or more individuals who take decisions. Different sets of decisions taken by these individuals lead to different results. Each decision maker, called a player, values the result differently, leading to a conflict among the decision makers. The mathematical analysis of the conflict begins with the pioneering work of von Neumann and Morgenstern (1944), and over the years game theory has become a major interdisciplinary subject dealing with economic, social and political conflicting situations, worst case design in various fields such as communication networks, and in modelling war games. It may be noted that the Nobel Prize in Economics in 1994 went to three game theorists: Nash, Harsanyi and Selten, to appreciate the increasing relevance of

game theory in economics. A significant feature of game theory is that it has led to new branches of mathematical sciences such as statistical decision theory, linear complementarity theory, etc. This field is undergoing tremendous advances in recent years, most notably in the refinements of the concepts of equilibria, dynamical aspects of approach to equilibria, aspects of 'common knowledge' and 'learning' in games, to mention a few.

Dynamic game is a part of game theory where the game evolves over time. Its root can be traced back to mixed strategies in matrix games which incorporate repeated play of the game with a prescribed relative frequency allotted to each pure action. In such a case, however, the game is played with the same payoff matrices, and therefore the game does not really evolve over the stages. In a typical discrete time dynamic game, the game is played over finite or infinite number of stages. Corresponding to each stage there is a payoff matrix for each player together with a 'motion' among this collection of matrices from stage to stage governed by the current matrix game and the actions chosen there. The evolution of the state of the game which describes the motion is usually modelled by difference equations and thus this class of dynamic games is referred to as difference games. Sometimes there are uncertainties which influence the outcome that cannot be completely predicted. These situations lead to the formulation of stochastic (dynamic) games. There are many situations where the game evolves in continuous time, e.g. in economic games or war games. In these situations the evolutions of the game is described by differential equations. These dynamic games are known as differential games. Dynamic games are all pervasive today since multiagent decision making with conflicting interest has become a part of our life in every way.

The book under review is based on selected papers presented at the Sixth International Symposium on Dynamic Games and Applications, held in St Jovite, Quebec, Canada, during 13-15 July 1995. The importance and timeliness of this symposium cannot be overemphasized. It has brought together several scientists spreading across the disciplines of mathematics, economics and engineering under one umbrella. The present book contains 24 papers, 16 in zero-sum games and 8 in nonzero-sum games. I will briefly describe the main themes of these papers.

*Zero-sum games.* In this category there are 5 papers on minimax control, 8 papers on pursuit evasion games, and 3 papers are devoted to solution methods.

Minimax controls play an important role in worst case design strategies. In the first paper, Bernhard offers a parallel between stochastic and minimax control of distributed nonlinear systems with partial observation by studying an interesting morphism between ordinary algebra and the so-called max-plus algebra. In the next paper, Friedman studies the  $H^\infty$  problem for nonlinear singularly perturbed systems and obtains certain solvability conditions in terms of invariant manifolds. Altman and Gaitsgory study a hybrid differential stochastic game. They establish a pair of stationary strategies which asymptotically determine a saddle point. Pan and Basar also study a hybrid stochastic system, but they work in the framework of  $H^\infty$  control. The last paper on minimax control is by Fristed, Lopic and Sudderth. They analyse a stochastic game analogous to the 'big match'.

Pursuit evasion problems analyse war games. The first paper in this theme is by Pesch *et al.* They use a novel neural network framework to analyse the well-known cornered rat game. Lipman and Shinar study a pursuit evasion game with a state constraint arising in anti-ballistic point defence as well as in ship defence scenarios against highly manoeuvrable attacking missiles. In the next paper, Lachner *et al.* study a similar problem. But they obtain their results by analysing the corresponding Isaacs equation with multi-point boundary values. Le Menec and Bernhard employ artificial intelligence techniques for the solution of aerial combats. Kumkov and Patsko study a pursuit game with incomplete information where the pursuer employs impulse control. Olsder and Pourtallier study a similar pursuit game with costly and asymmetric information. The next paper by Neveu *et al.* compares the results of an analytical treatment versus a realistic simulation of the same game treated in the previous paper. In the last paper on this topic, Chikrii and Prokopovich study a conflict interaction of  $n$  pursuers and  $m$  evaders in a multi-dimensional Euclidean space.

In the solution methods section, Bardi *et al.* present a novel approximation scheme for the Isaacs equation based on viscosity solutions. In the next paper, Tidball proposes a discretization scheme

## BOOK REVIEWS

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for a zero-sum differential game with stopping times. Chikrii and Rapport present a solution method based on resolving function technique.

*Nonzero-sum games.* In this category there are 5 papers dealing with theoretical aspects and 3 papers on applications.

In the first paper on theoretical aspects, Li and Gajic develop an algorithm for solving algebraic Riccati equations arising in a nonzero-sum differential game. Carlson and Haurie study a class of open-loop differential games on the infinite planning horizon with overtaking payoff criterion. In the next paper, Xu and Mizukami study the

Stackelberg strategies for discrete-time descriptor systems. Ehtamo and Ruusunen study bargaining games with the feasible set consisting of utility gains evaluated over multiple time periods. In the last paper on this topic, Petrosjan extends the notion of Shapley value for cooperative static games to dynamic games.

In the applications part, Jorgensen's paper discusses the results in dynamic game models with a view to assessing their potential use as support in management strategy decisions. Clemhout and Wan study endogenous growth as a dynamic game. In the last paper, Hines also applies the results

of dynamic games to problems of biology, more specifically to sexual reproduction.

It should be clear from the above description that the papers in the present volume are quite rich and diverse. It would serve as a very good source of material for researchers in the field.

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