Spherically symmetric empty space and its dual in general relativity

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In the spirit of the Newtonian theory, we characterize spherically symmetric empty space in general relativity (GR) in terms of energy density measured by a static observer and convergence density experienced by null and time-like congruences. It turns out that the space surrounding a static particle is entirely specified by the vanishing of energy and null convergence density. *The electrograv-dual[1] to this condition would be* the vanishing of time-like and null convergence density which gives the dual-vacuum solution representing a Schwarzschild black hole with global monopole charge[2] or with a cloud of string dust[3]. Here the duality[1] is defined by interchange of active and passive electric parts of the Riemann curvature, which amounts to interchange of the Ricci and Einstein tensors. This effective characterization of stationary vacuum works for the Schwarzschild and NUT solutions. The most remarkable feature of the effective characterization of empty space is that it leads to new dual spaces and the method can also be applied to lower and higher dimensions.

THE Newtonian gravitational field equation is given by $\nabla^2\phi = 4\pi G\rho$ and empty space is characterized by $\rho = 0$. It is well-known that the measure of energy is an ambiguous issue in GR primarily because of the inherent difficulty of non-localizability of gravitational field energy. However there is no difficulty in defining various kinds of energy density, signifying different aspects. The analogue of the Newtonian matter density is the energy density measured by a static observer and defined by $\rho = T_{ab}u^a u^b$, $u^a u_a = 1$, where $u_a = ((g)^{1/2}, 0, 0, 0)$ and T_{ab} is the matter–stress tensor of non-gravitational matter field. Then there is the convergence density experienced by time-like and null particle congruences in the Raychaudhuri equation[4]. They are defined as the time-like convergence density, $\rho_t = (T_{ab} - \frac{1}{2}Tg_{ab})u^a u^b$ and the null convergence density $\rho_n = T_{ab}v^a v^b$, $v^a v_a = 0$, $v_a = (1, (-g_{11}/g_{00})^{1/2}, 0, 0)$. The energy density ρ refers to all kinds of energy other than the gravitational field energy, while the time-like and null convergence densities act as active gravitational charge densities. For a perfect fluid they are given by $\rho_t = \frac{1}{2}(\rho + 3p)$ and $\rho_n = \rho + p$. It is important to recognize that these three densities represent different aspects of energy distribution and its gravitational linkage. They would thus in general not be equal. Obviously all the three can never be equal unless space is flat. However $\rho = \rho_t$ implies vanishing of scalar curvature (radiation), $\rho = \rho_n$ indicates vanishing of pressure (dust) and $\rho_t = \rho_n$ gives $\rho = p$ (stiff fluid). It may be noted that the weak field and slow motion limit of the Einstein non-empty space equation is $\nabla^2\phi = 4\pi G\rho$, while its limit in weak field and relativistic motion is $\nabla^2\phi = 8\pi G\rho_t$.

In the following, we shall always refer ρ and ρ_t relative to a static observer, and ρ_n to radial null geodesic. This does, however, bring in a particular choice for the time-like and null vectors but the choice is well motivated by the physics of the situation. The radial direction is picked up by the 4-acceleration of the time-like particle, identifying the direction of gravitational force, and so is the static observer for measure of energy and time-like convergence densities.

The main question we wish to address in this note is, can we characterize empty space solely in terms of these densities?

The answer is yes for the space surrounding a static particle. This may in general be true for an isolated particle with some additional conditions which would specify the additional physical character of the problem. It is clear that any specification of empty space must involve density relative to both time-like and null particles. That means ρ_n must vanish in any case and in addition one or both of ρ and ρ_t must vanish. Of course there should be no energy flux, $P^c = h^{ac}\ T_{ab}u^b = 0$, $h^{ac} = g^{ac} - u^a u^c$. It turns out that for spherical symmetry the effective equation for vacuum is $\rho = \rho_n = P_c = 0$, the solution of which would imply $\rho_t = 0$ and vanishing of the all Ricci components. Thus vanishing of energy and its flux, and null convergence density is sufficient to characterize empty space for spherical symmetry as these conditions completely determine the unique Schwarzschild solution. The effective vacuum equation is less restrictive than the vanishing of the entire Ricci tensor.

What actually happens is, for the spherically symmetric metric in the curvature coordinates, $P_c = 0$ and $\rho_n = 0$ lead to $R_{01} = 0$ and $R^0_0 = R^1_1$ which imply $g_{00} = f(r) = -g^{11}$, and then $\rho = 0$ means $R^2_2 = 0$ which integrates to give the Schwarzschild solution completely with $g_{00} = 1 - 2GM/r$ (we have set $c = 1$). Thus instead of $R_{ab} = 0$, the less restrictive effective equation $\rho = \rho_n = P_c = 0$ also equivalently characterizes empty space for a static particle. It is a covariant statement relative to a static observer and in the curvature coordinates it takes the form $R^0_0 = R^1_1$, $R^2_2 = 0 = R^0_1$.

Since there are three kinds of density, which could vanish with two at a time in three different ways, it is then natural to ask what would the other two cases give rise to?

The first thing that comes to mind is to replace ρ by ρ_t in the effective equation to write $\rho_t = 0 = \rho_n = P_c$, which would imply $G^0_0 = G^1_1$, $G^2_2 = 0 = G^0_1$. That is replacing Ricci by Einstein, which represents a duality relation between the two. Remarkably this duality transformation is implied

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at a more fundamental level by interchange of the active and passive electric parts of the Riemann curvature[1]. (Active and passive electric parts of the Riemann curvature are defined by the double (one for each 2-form) projection of the Riemann tensor and its double (both left and right) dual on a time-like unit vector, and dual is the usual Hodge dual, $*R_{abcd} = 1/2\varepsilon_{abmn}R^{mn}_{cd}$.) That is, interchange of active ($E_{ab} = R_{acbd}u^c u^d$) and passive ($\tilde{E}_{ab} = *R*acbdu^c u^d$) electric parts implies interchange of the Ricci and Einstein tensors because contraction of Riemann gives Ricci while that of its double dual gives Einstein tensor. We have defined the electrogravity duality transformation[1] by interchange of the active and passive electric parts, $E_{ab} \leftrightarrow \tilde{E}_{ab}$, $H_{ab} \rightarrow H_{ab}$. Under this duality transformation it is clear that $\rho \leftrightarrow \rho_t$, $\rho_n \rightarrow \rho_n$, $P_c \rightarrow P_c$.

Then the condition $\rho_t = \rho_n = P_c = 0$ is electrograv-dual to the effective empty space equation given above, and its solution would give rise to the space dual to empty space. It can be easily verified that it integrates out to give the general solution given by $g_{00} = -g^{11} = 1 - 8\pi G\eta^2 - 2GM/r$, where η is a constant. This is an asymptotically non-flat non-empty space which reduces to the Schwarzschild empty space for $\eta = 0$. At large r, the stresses it produces accord precisely to that of a global monopole of core mass M and η indicating the scale of symmetry breaking[2]. Alternatively it can exactly for all r represent a Schwarzschild black hole sitting in a cloud of string dust[3]. It is remarkable that here it arises as dual to empty space, i.e. dual to the Schwarzschild black hole[1]. A global monopole is supposed to be produced when global symmetry $O(3)$ is spontaneously broken into $U(1)$ in phase transition in the early Universe. The physical properties of this space have been investigated[5] and it turns out that the basic character of the field remains almost the same except for scaling of the Schwarzschild's values for the black hole temperature, the light bending and the perihelion advance[6]. The difference between the Schwarzschild solution and its dual can be demonstrated as follows. Both the solutions have $g_{00} = -g^{11} = 1 + 2\phi$ with $\nabla^2\phi = 0$, which would have the general solution $\phi = k - M/r$. The Schwarzschild solution has $k = 0$, while the dual does not. This is the only essential difference between the two. It is this constant, which is physically trivial in the Newtonian theory, that brings in the global monopole charge, a topological defect.

Let us also consider the remaining possibility, $\rho = \rho_t = P_c = 0$ which would in terms of the Ricci components imply $R = 0$, $R^0_0 = 0$. This integrates out to give the general solution, $g_{00} = (k + \sqrt{1 - 2GM/r})^2$, $g_{11} = -(1 - 2GM/r)^{-1}$, where k is a constant. It is an asymptotically flat non-empty space with the stresses given by

$$T^1_1 = \frac{2kGM/r^3}{k + \sqrt{1 - 2GM/r}} = -2T^2_2.$$

Obviously, these stresses cannot correspond to any physically acceptable matter field because $\rho = 0$. On the other hand, the space-time unlike the dual solution remains asymptotically flat. It will admit a static surface only if $k < 0$ at $r_s = 2GM/(1 - k^2)$. However $r \geq 2GM$ always for g_{00} to be real. The region lying between r_s and $2GM$ would define an ergosphere where negative energy orbits can, as for the Kerr black hole, occur. The Penrose process[7] can be set up to extract out the contribution of k only if it is negative. However we do not know the physical source for k.

On the other hand, when $k > 0$, there occurs no horizon and it can represent a wormhole[8] of throat radius $r = 2GM$. It is remarkable that it has the basic character of a wormhole which needs to be further investigated. Pursuing on this track, we are presently working out a viable wormhole model[9].

This space is certainly empty relative to time-like particles as both ρ and ρ_t vanish but not so for photons as $\rho_n \neq 0$. At the least, it can be viewed as an asymptotic flatness preserving perturbation to the Schwarzschild field.

Further it is also possible to characterize the Reissner–Nordström solution of a charged black hole by $\rho = \rho_t$, $\rho_n = P_c = 0$, and the de Sitter (Λ-vacuum) space by $\rho + \rho_t = 0$, $\rho_n = P_c = 0$. In the Ricci components, the former would translate into $R = 0$, $R^0_0 = R^1_1$. This is clearly invariant under the duality transformation. It is a non-empty space with trace-free stress tensor. The de Sitter space is given by $\rho + \rho_t = P_c = 0$, which implies $R_{ab} = \Lambda g_{ab}$. Of course under the duality transformation the sign of Λ would change indicating that the de Sitter and anti de Sitter are dual of each other.

The next question is, could other empty space solutions representing isolated sources be characterized similarly?

It turns out that it is possible to characterize the NUT solution and its dual[10,11] in a similar manner. However an additional condition would come from the gravomagnetic monopole[12] character of the field. The most difficult and challenging problem would be to bring the Kerr solution in line. That is an open question and would engage us for some time in the future. The crux of the matter is to identify the additional condition corresponding to gravomagnetic character of the field and solving the resulting equations. Once that is achieved, our new characterization of vacuum would cover all the interesting cases.

In conclusion, we would like to say that it is always illuminating and insightful to understand the relativistic situations in terms of the familiar Newtonian concepts and constructs. Relating empty space to the absence of energy and convergence density is undoubtedly physically very appealing and intuitively soothing. The most remarkable aspect of this way of looking at empty space is that it gives rise, in a natural manner, to the new spaces dual to the corresponding empty spaces. The dual spaces only differ from the original vacuum spaces by inclusion of a topological defect, a global monopole charge.

Note that the characterization of empty space and its dual is by the covariant equations. Earlier the dual space-times[1,13] were obtained by modifying the vacuum equation, so as to break the invariance relative to the electrogravity duality transformation, in a rather ad-hoc manner. Now the effective vacuum equation has the direct physical meaning in terms of the energy and convergence density. This characterization could as well be applied in lower and higher dimensions to find new dual spaces. For example, in three-dimensional gravity the dual space represents a new class of black hole spaces[14] with a string dust matter field. For higher dimensions, the method would simply go through without any change for n-dimensional spherically symmetric space and the dual space would represent a corresponding Schwarzschild black hole with a global monopole charge. It can be further shown that a global monopole field in the Kaluza–Klein space can be constructed similarly[15] as dual to the vacuum solution[16]. It is thus an interesting characterization of empty space which leads to new spaces dual to corresponding empty spaces.

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Comparative efficacy of Ayush-64 vs chloroquine in vivax malaria

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A phase II prospective comparative randomized clinical trial was conducted in patients of vivax malaria to compare the efficacy of Ayush-64 vs chloroquine. Ayush-64, a herbal formulation patented by Council of Ayurveda and Siddha was compared with chloroquine. Patients received an oral dose of either 1 g Ayush-64, three times a day for 5–7 days or a total dose of 1500 mg chloroquine over 3 days. Peripheral smears were examined everyday for 3 days or till they were negative and then weekly up to 28 days.

The results of the study showed that at day 28, only 23 of 47 (48.9%) patients in the Ayush group and all the 41 in the chloroquine group were cured ($p < 0.05$). Even in these 23 patients in the Ayush group parasite clearance time was longer than chloroquine (3.16 vs 1.5 days). Both regimens were generally well tolerated. In conclusion, Ayush-64 in a dose of 1 g three times a day for 5–7 days is not as effective for treatment of vivax malaria, as standard chloroquine therapy.

MALARIA is a major health problem in India. The annual incidence was between 2 and 3 million during the last decade[1]. The problem of drug resistance and treatment failures in *P. falciparum*[2] and recently in *P. vivax*[3,4] malaria using chloroquine has focused interest on new drugs/drug combinations/indigenous drugs or remedies.

Ayush-64 is a combination of four Ayurvedic drugs namely *Alstonia scholaris* R. Br. (aqueous extract of the bark – 1 part), *Picrorhiza kurroa* Royle (aqueous extract of the rhizome – 1 part), *Swertia chirata* Buch-Ham (aqueous extract of the whole plant – 1 part) and *Caesalpinia crista* Linn (fine powder of seed pulp – 3 parts)[5]. The ingredients after mixing are formulated into tablets of 500 mg each. The drug/formulation is patented and registered by Central Council for Research in Ayurveda and Siddha (CCRAS) and was reported to cure malaria[5,6]. Since the studies were conducted more than 15 years ago and susceptibility of parasites to drugs keeps changing, it was considered essential to evaluate the efficacy of the drug before introduction in the national programme. Moreover, in these studies, patients were included on the basis of clinical diagnosis of malaria and assessment criteria were not uniform[5], therefore the present comparative

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