

Professor Srinivasa R. S. Varadhan

K. B. Athreya

Professor Srinivasa R. S. Varadhan of the Courant Institute of Mathematical Sciences, New York University is an outstanding mathematician who specializes in probability theory and stochastic processes. He was elected member of the National Academy of Sciences, USA in 1995 and a Fellow of the Royal Society, UK in 1998. He is the recipient of numerous other awards and honours some of which are listed later. We give below a brief biographical sketch of Varadhan followed by accounts of his contribution to two of the many areas that have interested him.

Biographical sketch

Srinivasa Varadhan was born in Madras (Chennai) on 29 July 1941. His father was a government school teacher in the Madras Presidency and was transferred periodically. He became a headmaster when Varadhan was still young. Varadhan says that his teachers treated him well but he could also not do any mischief without his father finding out right away. In his final year at school his mathematics teacher noticed Varadhan's abilities and introduced him to the pleasures and fun of solving mathematical problems and puzzles. He moved back to Madras for his Intermediate at the Madras Christian College (MCC). He found the transition difficult. Up to this time his medium of instruction was not English. At MCC it was not only English but the accent of some of the teachers who were from UK, that was difficult for Varadhan. Over and above this, these teachers insisted on students thinking for themselves. Despite these difficulties Varadhan quickly made the adjustments and did very well. Now it was time to enter the university. Varadhan heard about statistics for the first time in his life. Due to the policies of reverse discrimination it was important for him to do well academically and get a degree quickly. This meant trying for the BA (Honours) that would save a year and yield an equivalent MA degree. The statistics honours was offered only at the Presidency College and had only fourteen seats. His father managed to get Varadhan a seat in the Presidency College and he now had a choice of subjects: physics or statistics. He opted for the latter. What was surely a big loss to physics became a big gain for mathematics and probability theory in particular. Varadhan sailed through this with great ease spending a good deal of

his time at the beautiful Marina Beach across from the Presidency College in Madras and at the movies and indulging in a lot of general discussions with his friends. He got his BA (Honours) in June 1959 having established a new record for the marks scored in all his subjects. At this point there was some pressure from his family to sit for the competitive civil service examinations such as the IAS. Varadhan was keen on research and his family relented. Varadhan then went to Indian Statistical Institute (ISI) in Calcutta that had become a world famous institution for the theory and practice of statistical methods under the leadership of P. C. Mahalanobis. Varadhan says that at the ISI he was given a desk and was expected to produce a thesis in three to four years. There was no formal course work for research scholars. There were a couple of seminars, one by V. S. Varadarajan (now at UCLA) on point set topology and one by R. R. Bahadur (University of Chicago, passed away in 1997) on analysis and measure theory. Soon he and other research scholars organized their own seminars and found it a wonderful learning experience and started to do 'research'.

The atmosphere at the ISI Calcutta in the late fifties and early sixties was very stimulating. Mahalanobis was very good at spotting and nurturing talent and he had managed to assemble at the ISI a brilliant set of statisticians and mathematicians that included C. R. Rao, R. R. Bahadur, D. Basu, S. K. Mitra, G. Kallianpur, J. Roy, V. S. Varadarajan, R. Ranga Rao, K. R. Parthasarathy and J. Sethuraman. There was also a stream of visitors from abroad during the pleasant winter months. The statistician and geneticist, R. A. Fisher, came regularly. The mathematician, Norbert Wiener, spent four months. The great biologist and

general scientist J. B. S. Haldane had just moved to the ISI from England. In 1961 the Russian mathematician, A. N. Kolmogorov, spent two months at the ISI. By that time Varadhan had learned a fair amount of probability theory, functional and harmonic analysis that helped him write a nice Ph D dissertation on infinitely divisible probability distributions on Hilbert spaces. The real line case had been treated extensively earlier by Y. A. Khinchine and P. Levy. A probability distribution P is *infinitely divisible* if for each integer n there exist independent and identically distributed random variables (iidrv) $X_{n1}, X_{n2}, \dots, X_{nm}$, such that their sum $\sum_1^n X_{nj}$ has distribution P . Examples include the Gaussian, Cauchy, and Poisson but not uniform $[0, 1]$ or binomial. C. R. Rao who was the head of the Research and Training School of the ISI requested Kolmogorov to evaluate Varadhan's thesis and he sent his approval from Moscow after a few months.

Varadhan was now looking for post-doctoral opportunities. Varadarajan who had just returned from the Courant Institute of Mathematical Sciences at the New York University wrote a letter of recommendation but nothing happened. Then in April 1963, C. R. Rao wrote to Cornell about Varadhan and promptly an offer of a tenure track position came. Varadhan accepted it but the very next day an offer came from Courant, where he had wanted to go in the first place. He sent a letter to Cornell requesting to be released from his acceptance and went to Courant in September 1963. Cornell's loss was Courant's gain.

At Courant, Varadhan collaborated with Monroe Donsker who had earlier proved a very general functional version of the central limit theorem that goes as follows. If X_1, X_2, X_3, \dots are iidrv with

mean 0 and variance 1 then the distribution of the random continuous functions $W_n(t)$ or $[0, 1]$ obtained by interpolating

$$\frac{1}{\sqrt{n}} \sum_{i=1}^j X_i \text{ at } \frac{j}{n}$$

converges to the standard Brownian motion on $[0, 1]$. This collaboration lasted for 25 years until 1991 when Donsker passed away. Together they produced a series of landmark papers on the subject of large deviations in Markov processes. An example of large deviations is the following. Let X_1, X_2, X_3, \dots be iidrv with mean μ . Then the law of large numbers says that for each $\epsilon > 0$ probability $p_n(\epsilon)$ that $|\bar{X}_n - \mu| > \epsilon$ goes to zero as $n \rightarrow \infty$ where \bar{X}_n is the sample mean

$$\frac{1}{n} \sum_{j=1}^n X_j.$$

The central limit theorem says that $(\bar{X}_n - \mu)$ is roughly of the order magnitude $1/\sqrt{n}$ so that the deviation $|\bar{X}_n - \mu| > \epsilon$ for fixed $\epsilon > 0$ is a very large deviation. The subject of large deviations deals with the decay of rates of $p_n(\epsilon)$ with n . It has been shown under some regularity conditions that $(1/n) \ln p_n(\epsilon)$ can be bounded above and below away from ∞ and 0. Varadhan is considered as the Grand Master of the subject of large deviations. He was one of the earliest to see the connections between large deviations, diffusions and elliptic partial differential equations. Varadhan says that some of his work as large deviations started with seeking a probabilistic explanation of the variational formula for the ground state energy of the Schrödinger operator. Varadhan's lecture notes on the subject is a beautiful and illuminating account of this work.

The Rockefeller University is located not far away from the Courant Institute in New York City. The mathematician, Mark Kac (physicists may know of him from the Feynman-Kac formula) was a professor there and he had a student, Daniel Stroock. Varadhan and Stroock started to work together on the subject of diffusions which are Markov processes with continuous sample paths. These are locally approximately Gaussian. The Varadhan-Stroock collaboration led to a revolution in the way diffusions were treated. They

introduced the notion of martingale formulation that has now become the standard tool. Their book¹ has become the standard reference on the subject. A nice account by S. Ramasubramanian of Varadhan's work on diffusions is given in an accompanying article.

During the mid-seventies Wall Street discovered that probability could be useful. The Black and Scholes Model became popular. An economist Harry Markowitz and Varadhan were consulted by an investment company in New York that managed pension funds. Markowitz later won the Nobel prize in economics. Varadhan says that the consulting was fun although the company went under not because of poor performance but because the partners had a fallout and the lawyers ate up everything.

In the eighties a fellow mathematician at Courant, George Papanicolau was interested in establishing what he called bulk diffusion under rescaling for interacting diffusions also referred to as a problem of hydrodynamic scaling. Varadhan, Papanicolau and his student, Guo, worked on this and were able to solve it completely. Varadhan went on to develop this area and it is now an active area of research. A former student of Varadhan, Sunder Sethuraman, has written a nice account of this that also appears in an accompanying article.

Varadhan who joined the Courant Institute in 1963 as a post-doctoral fellow became an assistant professor there in 1964, associate professor in 1966 and a full professor in 1972 at a very young age of thirty-one. He served as the Director of Courant Institute from 1980 to 84 and again from 1992 to 4. He has had over 25 doctoral students.

The list of honours received by Varadhan is a long one. Here are some of them.

1. Alfred P. Sloan Fellowship in 1969-70.
2. Invited speaker at the International Congress of Mathematicians in Helsinki in 1978.
3. Guggenheim Fellowship in 1984-85.
4. Fellow of American Academy of Arts and Sciences, 1988.
5. Fellow of the Institute of Mathematical Sciences, USA, 1991.
6. Elected to the National Academy of Sciences, USA, 1995.

7. Elected Fellow of the Royal Society of UK, 1998.
8. Awarded the Leroy P. Steele Prize of the American Mathematical Society (joint with D. Stroock), 1996.
9. Received the Margaret and Herman Sokol Award from NYU.
10. Plenary Speaker at the International Congress of Mathematicians in Zurich, 1994.
11. Editor, *Annals of Probability* and a number of prestigious journals.

On the personal side Varadhan got married in 1964 to Vasandhara. She got a Ph D from New York University and is currently an administrator. They have two sons, Gopal and Ashok, both of whom now work for the Wall Street.

Varadhan has kept up his close ties with India. Last year he and his wife were in Tambaram for the dedication of a wing of a hospital to which they had made a large donation. Varadhan has given courses of lectures at various Indian Universities and Institutes.

Varadhan besides being an outstanding mathematician is also an extraordinarily fine human being. He is very helpful to students and scholars alike and is always willing to explain his great work. He is an inspiring lecturer. His writings are models of clarity. When he lectures on a topic everything seems so natural and easy. Varadarajan once said of Varadhan that he was the most naturally gifted probabilist. Today he is ranked as one of top two or three probabilists in the world. Recently he was nominated for the Presidency of the American Mathematical Society. Varadhan has been constantly working for nearly forty years and he continues to be very active. It is safe to say that much new and profound work will come from him over the next several decades. We wish him well.

1. Stroock, D. W. and Varadhan, S. R. S., *Multidimensional Diffusion Processes*, Springer Verlag, New York, 1979.

K. B. Athreya is in the Department of Mathematics, Iowa State University, Ames, Iowa 50011, USA. (e-mail: kba@iastate.edu)