Labyrinth of Thought: A History of Set Theory and its Role in Modern Mathematics. Jose Ferreiros. Birkhauser Verlag AG, P.O. Box 133, CH-4010, Basel, Switzerland. 1999. 464 pp. Price: SFr 168/DM 198.

Mathematics, more than any other discipline, is transmitted ahistorically, both in pedagogy and research. Most often, the historical and philosophical ideas that inhabit the sphere of mathematical activity are erased and the subject is presented as if history and philosophy are but footnotes to mathematical ideas. While it is obvious that history and philosophy of mathematics cannot be the primary concerns of a mathematician, it is also important to find a space for incorporating these disciplines into a more critical understanding of mathematics and the practice of mathematicians. By analysing the historical growth of the subject of set theory, the book under review once again points to the importance of inculcating historical and philosophical reflections into the practice of mathematics.

Ferreiros' historical study of set theory illustrates the difficulty in formulating fundamental concepts like sets, manifold, classes, mapping, magnitude, infinity, continuous and discrete in a rigorous and consistent manner. The author discusses three stages of this development: the emergence of the idea of sets; creation of abstract set theory; and the search for an axiomatic basis for it. Through this approach, we see how set theory has contributed to the development of many disciplines like analysis, function theory and topology, thus exhibiting the fertile cross-pollination of ideas from different fields in the search for fundamental structures of mathematics. Particularly interesting examples are the genesis of point-sets and topological considerations in integration theory as a consequence of problems in rigorously defining Fourier Series, and the attempt by Cantor to get a grasp on the notion of infinity through ideas of cardinality and ordering. Set theory has also seminally contributed to the development of modern logic, a subject that was a 'part' of mathematical concerns in the early stages of the formation of set theory. For a mathematician, such an analysis cannot but enlarge the understanding of the foundations of these fundamental concepts of mathematics.

One important strand that is present through the length of the book, as is perhaps to be expected in a historical treatment, is the role of institutions in the development of modern mathematics. Mathematics does not develop as a coherent discipline through the efforts of individual genius alone. It needs the creation and sustenance of powerful institutions that nurture these ideas and propagate them. In the earlier parts of the book, Ferreiros discusses the various schools that were developed, such as the Gottingen Group and the Berlin School. It is pertinent to note that the reform of German universities was a response to those in France, 'particularly the creation of the Parisian *Ecole Polytechnique* in 1794' (p. 5). The creation of Berlin University in 1810 was a landmark in the growth of mathematics in Germany. From about the mid-19th century onwards, the presence of prominent mathematicians like Dirichlet, Riemann and Dedekind at Gottingen made it one of the most important centres of mathematics. All these people had a strong predilection for rigorous conceptualization of the foundation of mathematics and thus were very influential in the growth of 'abstract' mathematics. The Berlin School around the same period (1855–1870) also had its share of powerful mathematicians, notably Kummer, Weierstrass and Kronecker; Cantor obtained his doctorate under Kummer in 1867.

These institutions had a profound influence on the growth of mathematics during this period, although the approach to mathematics by both these groups was quite different. The Berlin group was much more suspicious of the turn towards abstraction, as in the works of Riemann and Dedekind. Cantor's difficulties in getting some of his later works published are also traced to the philosophical difference between these groups of mathematicians. The history of set theory once again illustrates what many of us well know: institutionalization (in the form of universities, research groups, journals, etc.) is the primary catalyst for the development and diffusion of new ideas and disciplines, but it is also prone to developing ideologies that then prohibit a free development of ideas.

I would also like to illuminate here the close relationship between philosophy and mathematics in these formative years of modern mathematics. Not only was this relationship encouraged in the universities but it was also an important motivating factor that shaped the thoughts of mathematicians like Riemann, Cantor and others. The history of set theory shows the influence of Kant's influential writings on the philosophy of mathematics throughout its development (pp. 13–18). Riemann, in setting out the notion of an n-dimensional manifold, considered the problem to be fundamentally philosophical in nature (p. 41). Cantor's influential book *Foundations of a General Theory of Manifolds*, was a combination of 'mathematical, foundational and philosophical considerations' (p. 259) and its subtitle was 'a mathematico-philosophical attempt to contribute to the theory of infinity'. Further, in order to argue for the validity of transfinite numbers, he drew upon the philosophies of Locke, Descartes, Spinoza and Leibniz (p. 260). Most, if not all, of the pioneering mathematicians, including those of the later generation like Hilbert and Weyl, had a deep and continued interest in the philosophical foundations of mathematics. I believe that it is possible to stake a strong claim that the philosophical motivation for rigorously articulating the foundations of mathematics played a defining role in the growth of set theory.

While it would be presumptuous to claim that this trend is fast vanishing among mathematicians, I would still venture to suggest that teaching and research in India today exhibit a most unwelcome distinction between mathematics (and in general, science) and philosophy (as well as history). Can an institutional attempt to open up common avenues between these disciplines prove beneficial to the creation of mathematicians of the stature of Riemann, Cantor and others? This is a question that mathematicians (and scientists), especially in India, may need to consider. I believe that Ferreiros' analysis strongly suggests that such an attempt is well worth the effort. Here, it is pertinent to remember that history is not just about the past, but it is also the condition for creating a future.

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