

Transition zone in constant pressure converging flows

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The effect of lateral streamline convergence alone on the transitional flow characteristics has been studied towards modelling the transition zone. Apart from a thicker boundary layer, the transitional flow characteristics are found to be similar to those in two-dimensional constant pressure flows. The transition zone has been modeled using the linear-combination model for two-dimensional flows.

Introduction

It is now a well-accepted fact that the problem of laminar-turbulent transition in viscous flows is one of the most challenging ones in fluid dynamics today. Consequently, transition zone modelling remains a critical phase in the prediction of viscous flow past bodies¹. For example, the transition region in a gas turbine blade can be as much as 80% of the blade surface² and so requires careful design consideration, as the peak heat transfer rate occurs within this region. The most disquieting aspect about transition is that there is no unique route to transition. Also, parameters like the pressure gradient, streamline convergence/divergence, streamline curvature (longitudinal/lateral), all acting simultaneously or individually, influence the transition process, thus making our understanding of the process more difficult.

Although there is no unique route to transition, the spot-wise transition theory of Emmons³ provides a reasonable explanation for a large class of boundary layer transitional flows. According to this theory, flow breakdown occurs via the appearance of turbulent spots, which then propagate downstream, making the flow intermittent in nature (with the laminar intervals slowly disappearing) and the flow finally attains a fully turbulent state, further downstream. Narasimha⁴ in proposing the hypothesis of *concentrated breakdown* suggested that the flow breakdown occurs at a fixed stream-wise location (x_t), called the 'onset of transition'. Thus a finite transition zone length exists between the onset of transition and the fully turbulent state. Though the overall process of transition may be gradual, the onset of transition may be relatively sudden⁵. A measure of

the extent of turbulent region in transitional flow is the 'fraction of time for which the flow remains turbulent' at any streamwise station. This is given by the intermittency factor (γ). Various γ -distributions have been proposed in the past for two-dimensional (2-D) flows^{4,6,7}. Similarly, a variety of transition zone models are now available⁸.

Most of the earlier works on the transition zone were undertaken with 2-D mean flows⁸. But three-dimensional flows are more important as they are frequently encountered. However, these flows are complex owing to the presence of many parameters like the pressure gradient, streamline curvature (lateral and longitudinal), streamline convergence/divergence, etc. For a better understanding and modelling of the transition zone, we need to have enough knowledge on the effect of each of these parameters individually on the transitional flow characteristics. Among the various flow parameters affecting the transitional flow characteristics, the effect of pressure gradient (favourable and adverse) has already been studied^{6,9-13}. Kohama¹⁴ has investigated the effect of curvature on a concave wall. Streamline convergence/divergence causes an additional strain rate and is known to produce a large effect on turbulent boundary layer growth, and is likely to affect transition characteristics as well. Studies carried out on internal flows in turbo-machines show¹⁵ that the start and length of transition are affected by the presence of lateral convergence and divergence to a far greater degree than by pressure gradient alone. Recently, Saddoughi and Joubert¹⁶ and Panchapakesan *et al.*¹⁷ have investigated laterally strained turbulent boundary layers. While a significant reduction of the skin friction and a thicker boundary layer, compared to 2-D flows, have been observed by Panchapakesan *et al.*¹⁷ for converging flows, Saddoughi and Joubert¹⁶ find a thinner boundary layer and increased skin friction in a laterally diverging flow. Also, a considerable reduction in the Reynolds stresses in the inner region and an increase in the turbulent kinetic energy in the outer part of the flow have been reported¹⁷, along with the result that the mean velocity profile is similar to that in a 2-D adverse pressure gradient flow. Furthermore, in terms of a newly-defined convergence/divergence parameter¹⁷, a diverging flow attains equilibrium while this is not so for converging flows. One may therefore expect some effect of streamline convergence on transitional flows also. In this pa-

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One of the earliest and best-known transition zone models has emerged from the transition studies initiated by Prof. S. Dhawan at IISc. This paper, which has reference to this models, is dedicated to Prof. Dhawan on his 80th birthday.

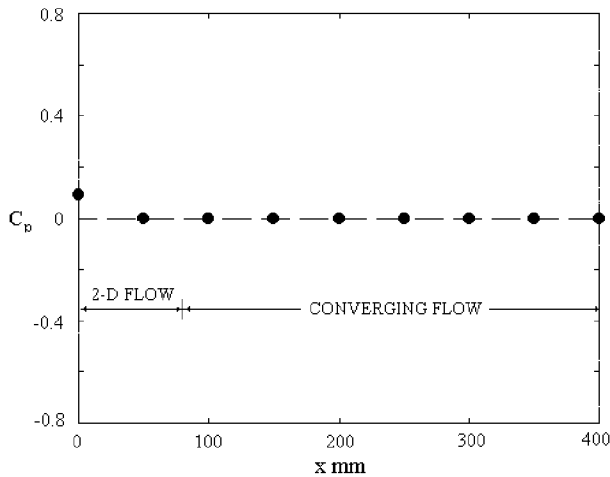


Figure 2. Stream-wise variation of C_p .

Results and discussion

Both the laminar and transitional boundary layer flows have been investigated. In addition, the propagation characteristics of an artificially-generated turbulent spot are examined. All the measurements reported here are carried out corresponding to a free-stream velocity of $U = 10.3$ m/s.

Laminar flow

The laminar boundary layer measurements have been made both in the 2-D region ($0 \leq x \leq 80$ mm) and in the converging section at various stream-wise locations along the line of symmetry. The normalized velocity profiles (u_L/U) are compared with the Blasius profile for 2-D constant pressure flow²² in Figure 3 (for the sake of clarity, the velocity profiles are shown for five x -locations); here (and hereafter) the subscript L denotes the laminar condition. It can be seen that the measured velocity profiles in the converging section are close to the Blasius profile. The investigation by Ramesh¹⁸ also shows that the laminar velocity profiles in a constant pressure diverging flow is the Blasius one but with a thinner boundary layer. The stream-wise variation of the momentum thickness (θ_L) and the shape parameter (H_L) are shown in Figure 4. It can be seen that in the 2-D region ($0 \leq x \leq 80$ mm), the measured θ_L agrees well with the Blasius values (Figure 4). In the converging flow region ($80 \leq x \leq 400$ mm), θ_L values are higher than the corresponding Blasius values (Figure 4), thus indicating a thickening effect on the boundary layer. This thickening effect can be attributed to the streamline convergence. The shape factor has a constant value of $H_L \approx 2.6$ (Figure 4) corresponding to the Blasius value. This has to be so since the velocity profiles are of similar shape at all these locations²³.

Apart from the thickening effect, the observed similarity with the Blasius flow is expected due to the similarity between a converging/diverging boundary layer flow and a 2-D flow shown by Ramesh *et al.*²⁴. Following Kehl²⁵, a diverging/converging flow can be approximated by a source/sink flow, as shown in Figure 5. Let u , v , and w be the velocity components in the x , y and z directions, respectively. The span-wise velocity (w) is taken as

$$w = uz/(a + x).$$

For the coordinate system shown in Figure 5, $w = 0$ along $z = 0$ (i.e. on a streamline), but

$$\left(\frac{\partial w}{\partial z}\right)_{z=0} \neq 0.$$

This non-zero component causes the extra lateral strain rate. The governing equations under the boundary layer approximation are²²,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{u}{a+x} = 0, \tag{2}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2},$$

$$-\frac{1}{\rho} \frac{\partial p}{\partial x} = U \frac{dU}{dx}. \tag{3}$$

The boundary conditions are $u = v = 0$ at $y = 0$, $u \rightarrow U$ as $y \rightarrow \infty$.

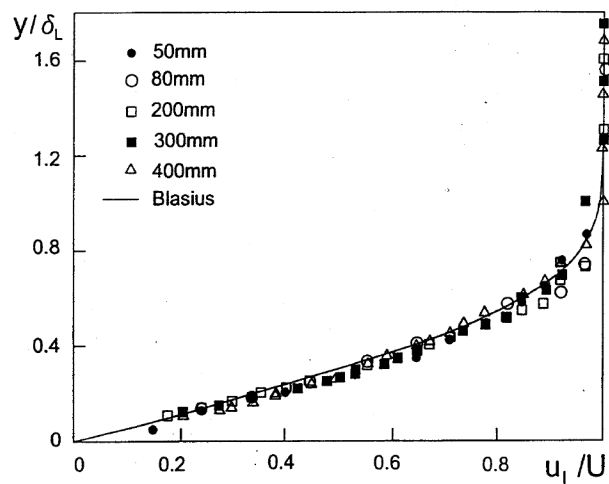


Figure 3. Comparison of the measured laminar velocity profiles with Blasius profile.

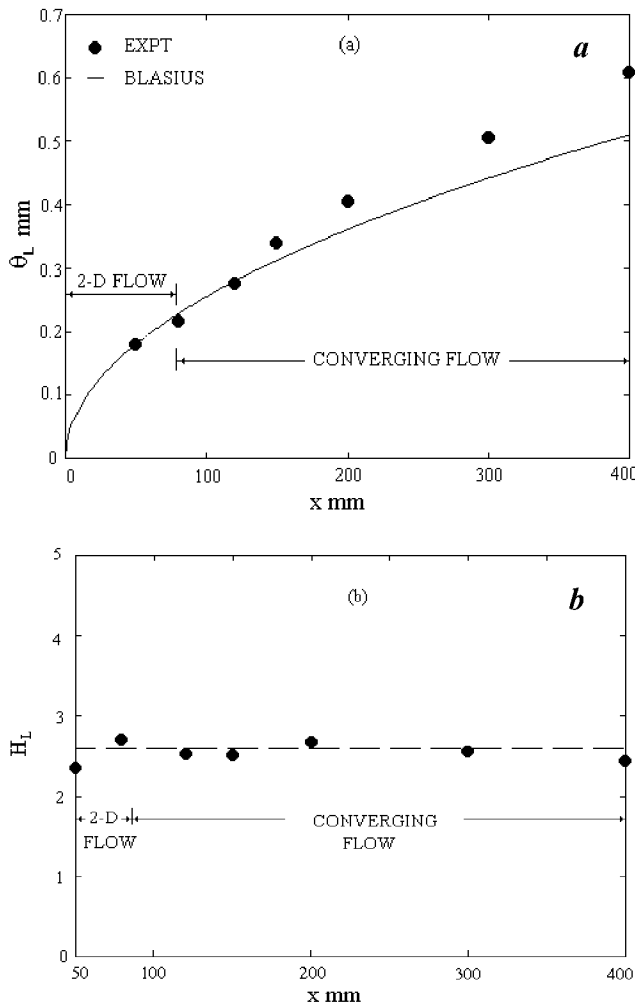


Figure 4. Stream-wise variation of the laminar momentum thickness (a) and laminar shape factor (b).

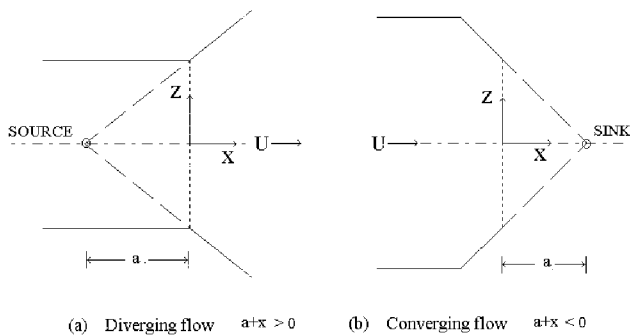


Figure 5. Source/sink approximation of diverging/converging flow.

Ramesh *et al.*²⁴ have shown that the Mangler type transformation,

$$X = \frac{(a+x)^3}{3a^2}, \quad Y = \left(\frac{a+x}{a}\right)y \quad (4)$$

$$u \rightarrow U_1, \quad V_1 \rightarrow \frac{a}{a+x} \left[v + \frac{yu}{a+x} \right], \quad U \rightarrow U_{1\infty},$$

reduces eqs (2) and (3) to 2-D flows; although Ramesh *et al.*²⁴ have extended the Mangler transformation to turbulent flows, only the laminar flow is considered here for the sake of simplicity. A straight-forward reduction of (2) and (3) to the Falkner–Skan equations is also possible by means of the following transformations²⁶. The velocity scaling,

$$u \rightarrow U_2 \left(\frac{a}{a+x} \right), \quad v \rightarrow V_2 \left(\frac{a}{a+x} \right),$$

$$u_\infty \rightarrow U_{2\infty} \left(\frac{a}{a+x} \right) \quad (5)$$

reduces (2) and (3) to

$$\frac{\partial U_2}{\partial x} + \frac{\partial V_2}{\partial y} = 0, \quad (6)$$

$$U_2 \frac{\partial U_2}{\partial x} - \frac{U_2^2}{(a+x)} + V_2 \frac{\partial U_2}{\partial y} = U_{2\infty} \frac{dU_{2\infty}}{dx} - \frac{U_{2\infty}^2}{(a+x)} + v \left(\frac{a+x}{a} \right) \frac{\partial^2 U_2}{\partial y^2}, \quad (7)$$

respectively. Using the similarity variables²⁶,

$$\eta = y \left[\frac{U_{2\infty}(1+m)}{v(a+x)} \right]^{\frac{1}{2}} \left(\frac{a}{a+x} \right)^{\frac{1}{2}},$$

$$\psi = \left[\frac{U_{2\infty}v(a+x)}{(1+m)} \right]^{\frac{1}{2}} \left(\frac{a+x}{a} \right)^{\frac{1}{2}} F(\eta), \quad (8)$$

(6) and (7) reduce to the Falkner–Skan similarity equation²⁶,

$$\left(\frac{m-1}{m+1} \right) (F'^2 - 1) - \frac{1}{2} FF' = F''', \quad (9)$$

under the condition $U_{2\infty} = K(a+x)^m$, where K is a constant; the stream function ψ is such that $U_2 = \partial\psi/\partial y$ and $V_2 = -\partial\psi/\partial x$. The choice $m = 1$ gives the Blasius solution. The above transformation shows that the velocity scaling (5) is essential for reduction to 2-D flows. Therefore, the observed similarity with the Blasius flow is not surprising.

Transitional flow

Transition is initiated with the help of a grid with a 1 cm × 1 cm mesh (of 3 mm circular rods). The grid is so fixed that the entire transition zone is within the measurement region.

The intermittency distribution in the transition zone is measured by the method employed by Ramesh *et al.*²⁷. In this method, the signal is double differentiated and then squared to sensitize it. A probability distribution plot of the sensitized signal is then obtained, which in turn provides a threshold to distinguish between the turbulent and non-turbulent parts of the signal. The sensitized signal is averaged over a time scale of 5 sample points (0.0025 s) to ensure better congruity of the intermittency function with the sensitized signal. This scale is of the order of the Kolmogorov length scale²⁸. The intermittency distribution along the longitudinal line of symmetry has been measured at various stream-wise locations ($x = 150, 200, 250, 300, 350, 400, 450, 500, 550, 600$ mm) with the hot-wire probe kept at a height of 0.5 mm with respect to the flat plate. The measured γ -distribution in the transition zone is compared with the universal 2-D constant pressure law of Narasimha⁴,

$$\gamma = 1 - e^{-0.4\zeta^2}, \quad \zeta = (x - x_t)/\lambda, \quad (10)$$

in Figure 6. Here x_t is the transition onset location, and $\lambda = ([x]_{\gamma=0.75} - [x]_{\gamma=0.25})$ is a measure of the transition zone length. It can be seen that the γ -distribution follows the 2-D universal law. This is indeed a very interesting result, as the 2-D γ -distribution can be used in a flow with streamline convergence. The γ -distribution in a diverging flow is also found to follow the same 2-D law, as reported by Ramesh *et al.*²⁷.

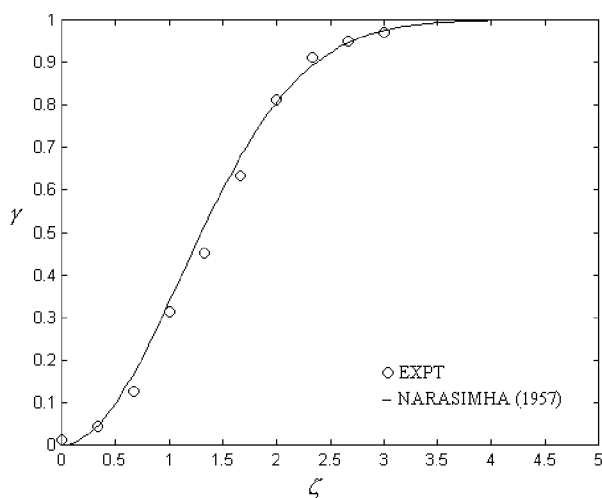


Figure 6. Comparison of the measured intermittency distribution with 2-D distribution.

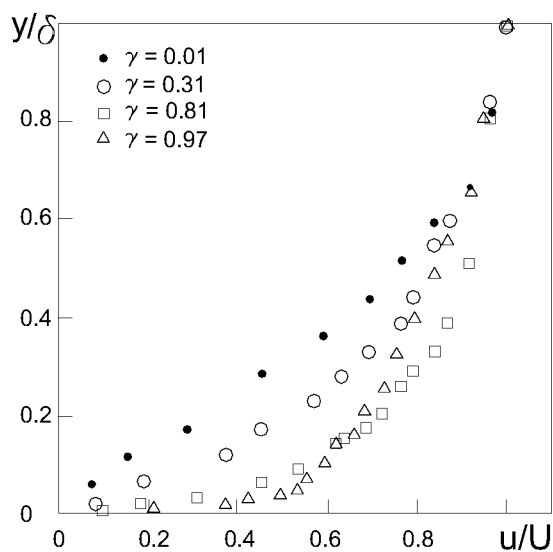


Figure 7. Transitional velocity profiles.

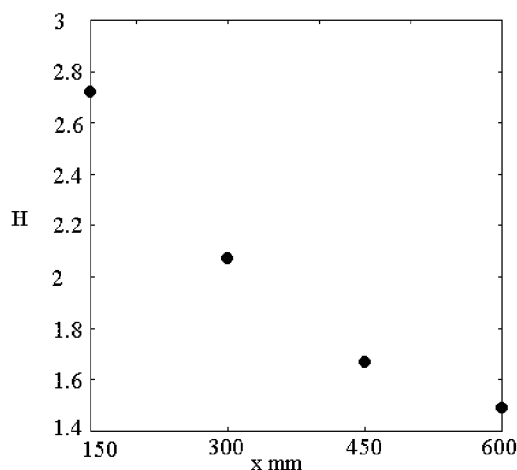


Figure 8. Variation of the shape-factor in the transition zone.

The measured transitional velocity profiles at various stream-wise locations ($x = 150, 300, 450, 600$ mm) along the longitudinal line of symmetry are shown in Figure 7. At the x -location where the intermittency is only 1%, the velocity profile is very similar to that for a laminar flow. As one goes downstream, γ increases and the velocity profiles become fuller, thus indicating a turbulent flow development. This is expected since, at the beginning of the transition zone, the flow is nearly laminar and is fully turbulent at the end of transition zone. It can be seen in Figure 8 that the shape factor changes from the near-laminar value (of $H \approx 2.7$, at $\gamma = 0.01$) to the nearly fully turbulent value (of $H \approx 1.5$, at $\gamma = 0.97$) within the transition zone. Interestingly, these values are similar to those for the 2-D transitional flow over a flat plate^{22,29}.

As mentioned earlier, many transition zone models are now available⁸. We have used the Linear-Combination Model (LCM) of Dhawan and Nara-

simha³⁰, as it is simple and still very effective in predicting various transitional boundary layer parameters. This model takes the mean velocity profile in the transition zone as a linear combination of the laminar and the turbulent profiles, weighted by the flow intermittency,

$$\frac{u}{U} = \frac{u_L}{U}(1-\gamma) + \frac{u_T}{U}\gamma. \quad (11)$$

Here (and hereafter) the subscript T denotes the turbulent value. This model assumes⁸ that the laminar boundary layer originates at the leading edge of the flat plate, and the turbulent boundary layer originates at the transition onset location x_t . Apart from the simplicity associated with the LCM of Dhawan and Narasimha³⁰, the momentum imbalance of this model is small³¹. Interestingly, among the other models in this class, the momentum balance aspect is known only for this model. Briefly, for the velocity distribution (11), the expression for the momentum thickness used by Dhawan and Narasimha³⁰ is,

$$\theta = \gamma(1-\gamma) \int_0^\delta [u_L - (1-u_T) + u_T - (1-u_L)] dy + (1-\gamma)^2\theta_L + \gamma^2\theta_T. \quad (12)$$

Here $\overline{u_L}$ and $\overline{u_T}$ are non-dimensional. With this expression for θ , it is slightly difficult to estimate the momentum imbalance although not impossible. However, a simpler expression³¹ for θ that also follows from the velocity distribution (11),

$$\theta = (1-\gamma)\theta_L + \gamma\theta_T + \gamma(1-\gamma) \int_0^\delta (\overline{u_L} - \overline{u_T})^2 dy, \quad (13)$$

makes it easier to estimate the momentum imbalance from the momentum integral equation for constant pressure flows,

$$2 \frac{d\theta}{dx} - C_f = 2 \left(I_1 + \frac{dI_2}{dx} \right), \quad (14)$$

$$I_1 \equiv \frac{d\gamma}{dx} (\theta_T - \theta_L), \quad I_2 \equiv \gamma(1-\gamma) \int_0^\delta (\overline{u_L} - \overline{u_T})^2 dy.$$

Here, following the velocity distribution (11), the skin friction coefficient (C_f) also is a linear combination type, i.e. $C_f = (1-\gamma)C_{fL} + \gamma C_{fT}$. As shown in Figure 9, the momentum residual ($I_1 + dI_2/dx$) is small compared to $d\theta/dx$; I_1 and dI_2/dx are of opposite signs and tend to cancel each other³¹; the data of Schubauer and Klebanoff²⁹ are used to estimate these quantities. This fact and the associated simplicity of the model render it an attractive one.

For the present transition zone model, the laminar profile is the Blasius profile, as mentioned earlier. Although a log plus wake profile is desirable for turbulent profiles, the wake parameter is not fully defined for low Reynolds number flows³². As such, we have used a simple power law velocity profile,

$$\frac{u_T}{U} = \left(\frac{y}{\delta_T} \right)^{\frac{1}{n}}, \quad (15)$$

where δ_T is the turbulent boundary layer thickness. The value of the exponent n varies between 5 and 8, depending on the flow Reynolds number. In the present analysis, $n = 7$ is taken, as it is found that the measured velocity profile is in good agreement with the profiles predicted by LCM. δ_T is obtained from the momentum integral equation for converging/diverging flows. Under the source/sink approximation of Kehl²⁵, the momentum integral equation is

$$\frac{d\theta_T}{dx} + \frac{\theta_T}{a+x} = \frac{C_{fT}}{2}. \quad (16)$$

In order to solve (16), a correlation between C_{fT} and the Reynolds number ($Re = Ux/\nu$) is built using the data of Panchapakesan *et al.*¹⁷ for turbulent flow in a converging channel, as the range of the convergence parameter ($D = 1/(a+x)$) in their experiment and in the present case is nearly same²¹. The correlation used is

$$C_f = 0.0084(Re)^{-0.0578}; \quad (17)$$

the number of data points is limited (only four). As shown in Figure 10, the measured transitional velocity

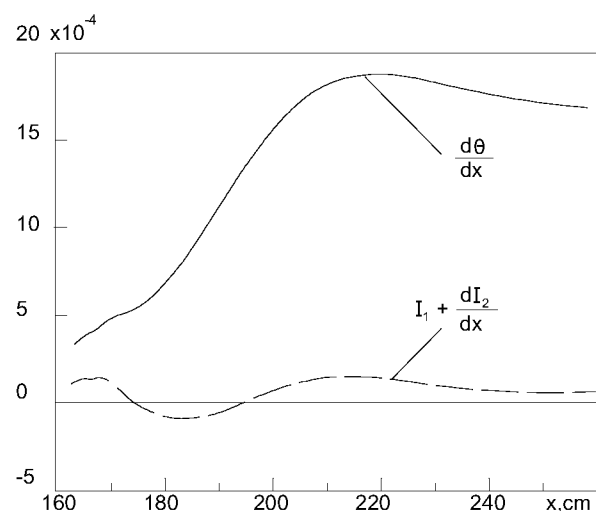


Figure 9. Momentum residual in comparison to the gradient of the momentum thickness in the transition zone.

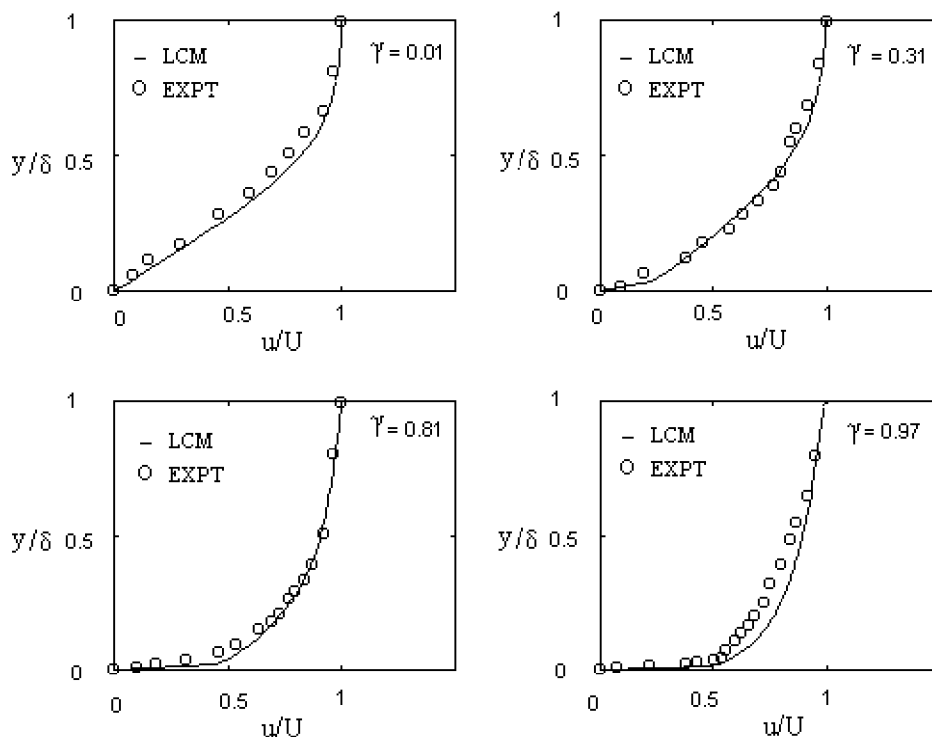


Figure 10. Comparison of the measured velocity profiles with the LCM prediction.

profiles can be predicted using the LCM. For diverging streamlines, Ramesh¹⁸ reports that the same LCM adequately represents the transition zone. Thus a 2-D model can be used even in flows with converging/diverging streamlines.

The spot propagation characteristics of an artificial spot are also found to be similar to those in 2-D constant pressure flows³³ like, for example, the conical similarity of Cantwell *et al.*³⁴ shown in Figure 11; here x_c and z_c are the similarity co-ordinates.

Conclusions

The effect of lateral streamline convergence alone on the transitional flow characteristics has been studied as part of the ongoing project at the Dept. of Aerospace Engineering, IISc, on transition zone modelling. A constant pressure lateral streamline convergence is achieved by converging the side-walls and appropriately diverging the roof; the half angle of convergence is chosen as 10° , which is approximately the same as the half of the turbulent spot envelope in constant pressure two-dimensional flows. The important findings of the present work are the following.

Except for a thicker boundary layer, the laminar boundary layer is of the Blasius type for 2-D constant pressure flows. The intermittency distribution in the transition zone follows the 2-D universal constant pressure law. The 2-D linear combination model for the

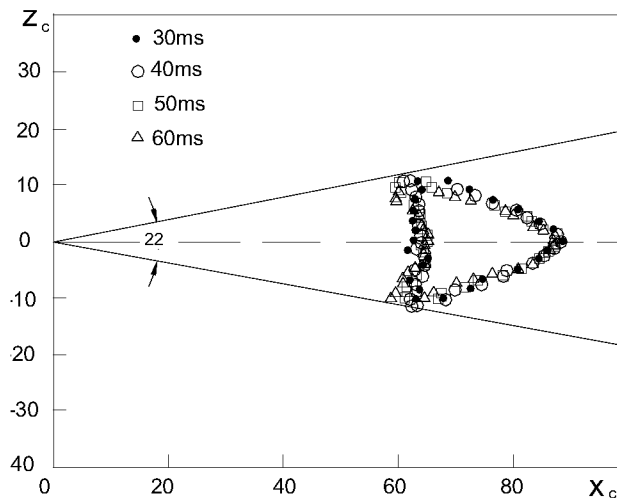


Figure 11. Conical similarity of turbulent spots.

transition zone has been found to be satisfactory in predicting transitional velocity profiles, even in converging streamline flows. The spot propagation characteristics are also similar to those in 2-D constant pressure flows.

To conclude, the present study shows that apart from causing a thicker boundary layer, the streamline convergence has a negligible effect on the transitional flow characteristics.

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