critical points of disordered random-cluster models.

The paper by C. Douglas Howard and Charles M. Newman studies infinite geodesics in models of Euclidean first-passage percolation on R^d. It is shown that for any dimension d, almost surely, every semi infinite geodesic has an asymptotic direction and every direction has at least one geodesic staring from each Poisson particle.

Janko Gravener and David Griffeath describe in their paper a general theory of reverse shapes and apply it to first-passage percolation and related growth models. Yu Zhang considers the standard first-passage percolation model on the integer lattice Z^d. Let $c_{0,n}$ be the first-passage time from the origin to the boundary of the box $[-n, n]^d$, where the edges have passage times arising from an i.i.d. family of random variables with distribution F. It is shown that for $d = 2$, there are two curves F_a and G_b, with $F_a(0) = G_b(0) = p_c$ such that $\lim_{n\to\infty} Ec_{0,n}$ exists whenever $F(0) = p_c$ and $F \geq G_b$. It is also shown that the integrated super-Brownian excursion measure is the restriction of its closed support of a Hausdorff measure. Thomas M. Liggett in his paper obtains precise asymptotics on the critical value and on the extinction time as $n \to \infty$ for a branching random walk on the ball of radius n in a homogeneous tree.

The paper by Lawrence F. Gray obtains the continuous time analogue of Toom's theorem on the stability of discrete systems under one-sided random perturbations. Claudia Neuhausers paper sets up a model of an ecosystem based on interacting particle systems and, in that framework reviews some of the mathematical results related to the study of the importance of space on the outcome of competitive interaction in simple plant competition models. S. R. S. Varadhan considers the space time rescaling of a system consisting of a large number of particles interacting with each other and moving randomly in R^d or Z^d. A large deviation principle for the empirical process viewed as a random measure on the path space is obtained when there is a diffusive or parabolic relation between the space and time scales. The paper by Ana Meda and Peter Ney studies an irreducible Markov chain $X_1, X_2 \ldots$ taking values in a state space S. For $u : S^2 \to R^d$, let $U_n = \Sigma_{i=1}^{n} u(X_i, X_{i+1})$; then conditioned on $\{U_n \in nC\}$, for some open convex subset C of R^d, it is shown that under certain conditions on $\{X_n\}$, it converges to a Markov chain.

RAHUL ROY

Indian Statistical Institute,
7 SJS Sansanwal Marg,
New Delhi 110 016, India

Mutational and Morphological Analysis – Tools for Shape Evolution and Morphogenesis. Jean-Pierre Aubin. Birkhauser Verlag AG, P.O. Box 133, CH-4010, Basel, Switzerland. 1998. 472 pp. Price: SFr 148/DM 178.

Many problems in diverse areas such as biology, economics, engineering, numerical analysis, physics and population dynamics involve the analysis, evolution, optimization and control of shapes and images. Due to the familiarity that mathematicians have with the analysis of functions and mappings, one often associates a suitable function with a set (or a subset) and analyses the same. To apply classical differential calculus, we require, therefore, the sets to be smooth. However, shapes and images are basically sets, and, most often, are not smooth. Thus we are led to set-valued analysis and to the construction of a differential calculus of mappings on a metric space whose elements are sets. This is the aim of mutational analysis.

Various approaches to set-valued analysis have been tried in the past. Viability theory deals with evolution equations, the solutions $t \mapsto x(t)$ of which are viable in *tubes* $t \mapsto K(t)$, i.e. $K(t)$ is a subset and it is required that for all $t \geq 0$,

$$x(t) \in K(t).$$

These tubes are first assumed to be given and the characterization of the viability condition led to the introduction of a class of derivatives of set-valued mappings, called *graphical derivatives*. However, if the evolution of the set $K(t)$ is itself governed by a kind of differential equation (called a *morphological equation*), then graphical derivatives are no longer sufficient to define the velocities of the tube, needed to design the morphological equation governing the evolution of subsets.

Shape optimization is concerned with problems of structural mechanics and optimal control of distributed systems. Many such problems can be formulated as the minimization of functionals over a class of subsets under some geometrical constraints. To study such problems, the concept of *shape derivatives* was introduced. This contains the seeds of the notion of mutations of a map.

The itinerary suggested in the book under review to deal with set-valued analysis – as against classical analysis – is to first study the properties of *power spaces* (i.e. families of subsets) and of set-valued or set-defined maps. The investigation of the properties of power spaces began at the same time as set theory at the beginning of the twentieth century. Painlevé (1902) introduced the notion of upper and lower limits of families of sets (now known as Kuratowski limits) and Pompeiu (1907) introduced a metric (distance) on families of non-empty compact subsets of a metric space (now known as the Hausdorff metric). However, set-valued analysis was neglected for nearly half a century and functional analysis gained importance.

The aim of mutational analysis is to provide new tools for set-valued analysis which complement the existing ones from classical analysis. As already mentioned, it aims to construct a differential calculus for set-valued and/or set-defined maps between power spaces of an arbitrary metric space. Special emphasis is given to the power space of nonempty compact subsets of a metric space. At the heart of the matter lie the notions of a mutation (generalizing the notion of a derivative) and of a morphological equation (a kind of differential inclusion).

We define the derivative of a function $f : V \to W$, where V and W are normed vector spaces, at a point $x \in V$, as a continuous linear map $f'(x)$ from V to W such that for all $v \in V$,

$$\| f(x + hv) - f(x) - hf'(x)v \| = o(|h|),$$

i.e. the image under f of the translation $x + hv$ of x in the direction v and the translation of $f(x)$ in the direction $f'(x)v$ should be 'close' for small values of h. Thus, the key to this definition lies in the notion of the translation of a vector, which, in turn, depends on the linear, i.e. algebraic, structure of the vector space.

The notion of linearity is not indispensible for defining a differential calculus, although it greatly simplifies the definition of the maps and their study. It suffices to replace the notion of a 'translation' by that of a *transition* – if E is a metric space, we consider a space of transitions, i.e. mappings $(h, x) \in \mathbb{R} \times E \mapsto \theta\ (h, x) \in E$, which satisfy a certain number of axioms. These play the role of $x + hv$. For instance, in a vector space, transitions could be defined using the solution trajectories, starting at x, of nonlinear ordinary differential equations. The value of the solution at time h would be $\theta(h, x)$.

A metric space with a space of transitions is called a *mutational space*. Given two mutational spaces E and F, and a single valued map $f : E \to F$, we say that a transition (on F) $\tau \in \overset{\circ}{f}(x)(\theta)$ is a *mutation* of f at x in the direction of the transition θ if the transition $\tau\,(h, f(x))$ of $f(x)$ and the image $f(\theta(h, x))$ of the transition of x are 'close', i.e.

$$\lim_{h \to 0_+} \frac{d(f(\theta\,(h, x)), \tau\,(h, f(x)))}{h} = 0.$$

This simple structure allows one to adapt, in the framework of mutational spaces, a large number of important results of differential calculus and analogues of differential equations called *mutational equations*. This idea can easily be extended to cover set-defined and/or set-valued maps between power spaces. In particular, one considers various mutational structures on the space $\mathcal{K}(X)$ of non-empty com pact subsets of a finite-dimensional vector space X called a *morphological space*.

In the recent decades, various parallel approaches (mathematical morphology, shape optimization, graphical derivatives and mutations) have been developed. There is a deep unity of basic mathematical concepts and tools buried in these competing, yet complementary concepts. Mutational and morphological analysis offers a structure that embraces and integrates the underlying framework of these approaches and reveals that their apparent differences stem from the differences in the sources of motivation.

The monograph under review provides a fairly self-contained mathematical treatment of this subject. It is divided into four parts.

The first part is devoted to mutational analysis providing the abstract tools for studying set evolution. Most of the standard results on differential equations are adapted to the case of mutational equations.

The second part deals with morphological and set-valued analysis. The third part presents geometrical morphology and algebraic morphology. The latter connects algebraic techniques characterizing mathematical morphology with general morphological concepts arising in set evolution.

The last part is an appendix that provides a summary of the statements of basic theorems on differential inclusions used in the book.

Though the motivations come from diverse sources of real life problems, the book is written in a very rigorous mathematical style and will thus be accessible to those with a taste for, and training in, abstract and formal mathematical exposition. To ease the way, each chapter is provided with an outline that serves to orient the reader by providing a summary of the principal concepts introduced and the principal results proved in it. The book contains a fairly exhaustive bibliography.

S. KESAVAN

The Institute of Mathematical Sciences,
CIT Campus, Taramani,
Chennai 600 113, India
e-mail: kesh@imsc.ernet.in

Flora of The District Garhwal North West Himalaya (with Ethnobotanical Notes). R. D. Gaur. TransMedia, Bhandari Bag, Srinagar (Garhwal) 246 174, India. 1999. 811 pp. Price: Rs 1600/US$ 100.

The mighty Himalayan range has lured pilgrims, practitioners of Ayurveda, geographers, mountaineers and botanists from all parts of India and the world. The Indian Himalayan Region (IHR) covers an area of 5,91,000 km^2 and extends over 2800 km in length and 220 to 330 km in width, with an altitudinal range of 300 to 8000 m asl. It is represented by 21 forest types and provides an enormous diversity of habitats, enabling the occurrence of a wide diversity of microbes, plants and animals. The mountain dwellers have depended on the resource of the region for subsistence for millennia. The increasing exploitation of Himalaya for industry, defence, agriculture, construction of roads and dams, over-collection of economic plants, grazing and development of townships has created severe environmental problems. The foremost among these has been the loss of biodiversity and valuable top soil caused by deforestation, mining, landslides and tourism. The acute shortage of water, feed for livestock, firewood for cooking and unavailability of minor timber for implements have caused intense hardships for the Himalayan people.

What has been already recorded through the preparation of Himalayan floras and faunas is impressive. The understanding of the strategies for survival of organisms under inclement climatic conditions has added a great deal to our biological knowledge. What is still not known is enormous, as the Himalayan region is vast and formidable to explore. It is in this context that the book by R. D. Gaur comes as an excellent contribution to our knowledge of the floristic wealth of north-west Himalaya.

District Garhwal has a wide range of habitats from Teraibhabar tracts at the foot of the Siwaliks to Dudhatoli (3114 m asl) in the north-eastern parts and includes flortistic elements of the Himalayan, Indo-Malyasian and Indo-Japanese elements. 57.89% of the district is covered by forests. Rajaji National Park and Corbett National Park are located in the study area. There have been earlier botanical explorations in the north-west Himalaya. What makes Gaur's flora unique? Most floras are written by taxonomists for other taxonomists (whose numbers are regrettably dwindling as their work is highly undervalued), drawing heavily on herbarium collections. While retaining the rigour expected of a professional taxonomist, Gaur's flora meets the needs of environmental managers, conservationists, wildlife biologists, agricultural scientists, foresters, anthropologists, planners, sociologists and above all common people.

The information provided in the flora has come from painstaking work done over the last 24 years by the author and his students, involving extensive field