Musical notes from a veena

The musical notes produced on stringed instruments like the veena, are based on the physical and mathematical formulations associated with the vibrations of stretched strings. Such instruments operate over a vibrational frequency range of perhaps 60 to 1000 Hz in the fundamental, spanning over about four octaves. An octave is a range over which the frequency doubles, for example, 60 to 120 Hz, 120 to 240 Hz and so on, and in a veena, twelve notes are placed within an octave. The frequency span of 60 to 1000 Hz is covered with four tensed wires spread along the length of the veena.

The layout of a veena and a schematic of each of its wires are as shown in Figure 1 *a* and *b*, respectively. The wires rest over a curved metallic surface called the bridge, located on a resonator on the right and over a cross bar, fret, on the left. There are additional twenty-four frets spread out over ¾ length of the wire and their mutual separations-geometrically decrease to the right. Pressing the wires against any of these frets and plucking them on the resonator side produces a distinct musical note. l_0 is the total length of the wire; l_1, l_2, etc. are lengths to the 1st, 2nd, etc. frets, respectively, as shown in Table 1 (row 1). Lengths up to l_{12} only are included here for clarity.

Any book on musical science[1] states that the total length of the wire, l_0, when properly tensed and plucked, will produce the fundamental musical note Sa; l_1 will produce Ri_1; l_2 will produce Ri_2 and so on, as shown in Table 1 (row 3), with l_{12} producing Ṡa, which incidentally happens to be same as the second harmonic associated with Sa of l_0. The notes between l_{12} and l_{23} are designated by Ṡa $\dot{\mathrm{R}}\mathrm{i}_1 \ldots \dot{\mathrm{N}}\mathrm{i}_3$, etc. Notes lower than Sa are designated by Ṣa, Ṛi, etc.

The values of the lengths of the wires l_o to l_{12} can be calculated[1] using a mathematical relation

$$l_{n+1} = (l_n/2^{1/N}) = \{l_0/2^{(n+1)/N}\}, \qquad (1)$$

where N is the number of subdivisions interposed between Sa and Ṡa, or the number of notes contained in an octave and n are corresponding integer numbers. Table 1 (row 2) lists these values for l_0 = 100 cm and $N = 12$.

The following 'approximate' relations between the various lengths are readily verifiable:

$$l_{12} = l(\dot{\mathrm{S}}\mathrm{a}) = 0.5\ l_0 = (1/2)\ l_0 \qquad (2)$$

$$l_6 = l(\mathrm{Ma}_2) = (l_0 l_{12})^{1/2} \qquad (3)$$

$$l_7 = l(\mathrm{Pa}) = 0.67\ l_0 = (2/3)\ l_0 \qquad (4)$$

$$l_5 = l(\mathrm{Ma}_1) = 0.75\ l_0 = (3/4)\ l_0 \qquad (5)$$

$$l_4 = l(\mathrm{Ga}_3) = 0.8\ l_0 = (4/5)\ l_0 \qquad (6)$$

$$l_2 = l(\mathrm{Ri}_2) = 0.89\ l_0 = (8/9)\ l_0 \qquad (7)$$

$$l_0 = l(\mathrm{Sa}) = l_0. \qquad (8)$$

A few more exercises with relations (1)–(8) using a calculator, will show some interesting results.

a) If any fret is taken as the reference instead of the first one as above and the calculations are pursued, the relations (1)–(8) are found to be similar everywhere, i.e. $l_{13} = l\ (\dot{\mathrm{R}}\mathrm{i}_1) = (1/2)\ l(\mathrm{Ri}_1) = ½\ l_1 = (l_0/2^{(13/12)})$ and so on.
b) For various values of N, the approximations involved in relations (1)–(8) will be found to be closest for $N = 12$ and 24. For N values greater than 26, the subdivisions get so close to each other, that overlapping approximations result in confusing equalities.
c) Only the values of $N = 12$, 24 and 22 (values over 26 are ignored) satisfy the relations corresponding to (1)–(8) above, jointly. (It seems, Bharatha was not satisfied with $N = 12$ and has proceeded further to arrive at $N = 22$.)
d) $N = 17$ gives numbers that satisfy the relations (2) and (4) satisfactorily. (It seems ancient Arabian music[1] was centred around $N = 17$.)

What is the significance of these approximate relations? We refer to a textbook on physics[2], with a chapter on vibrations of stretched strings. If a stretched string is plucked, it produces a sound wave in the air around with a fundamental wavelength λ_1, that is proportional to the length of the string, l. It also produces a second harmonic sound with $\lambda_2 = \lambda_1/2$ that is proportional to 1/2 l as well as 3rd, 4th and 5th, etc. harmonics, with wave lengths proportional to the respective fractional lengths of the string. Consequently, it is obvious that plucking the veena wire for Sa also produces a Ṡa as second harmonic, a Ṗa as third harmonic, (for $l(\dot{\mathrm{P}}\mathrm{a}) = l_{19} = (1/2)\ l\ (\mathrm{Pa}) = (1/2)\ (2/3)\ l\ (\mathrm{Sa}) = (1/3)\ l$) and S̈a, $\ddot{\mathrm{G}}\mathrm{a}_3$, etc. as fourth, fifth, etc. harmonics. Similarly, plucking the wire for Ri_1, produces the harmonics, $\dot{\mathrm{R}}\mathrm{i}_1$ as the second, $\dot{\mathrm{D}}\mathrm{ha}_1$ as the third, and so on.

For each fundamental note produced on the instrument within the musical scale, numerous higher harmonics, which are produced simultaneously,

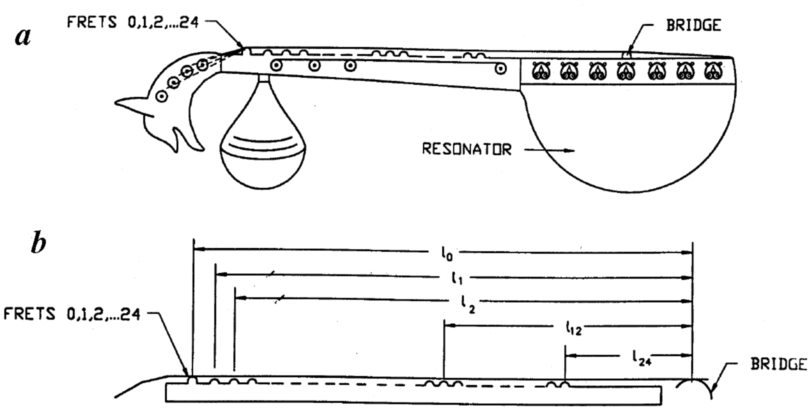

Figure 1. ***a***, Typical layout of a veena; ***b***, schematic of a veena wire.

Table 1. Relative wire lengths and associated standard designation of the notes

	l_0 cm	l_1 cm	l_2 cm	l_3 cm	l_4 cm	l_5 cm	l_6 cm	l_7 cm	l_8 cm	l_9 cm	l_{10} cm	l_{11} cm	l_{12} cm
l_n cm	100	94.40	89.09	84.09	79.37	74.92	70.71	66.74	63.00	59.46	56.12	52.97	50
D	Sa	Ri_1	Ri_2	Ri_3	Ga_3	Ma_1	Ma_2	Pa	Dha_1	Dha_2	Dha_3	Ni_3	Sa
F			8/9	5/6	4/5	¾		2/3					1/2
			0.9	0.83	0.8	0.75	$(l_0\ l_{12})/2$	0.67					0.5

l_n, Normalized wire lengths calculated for l_0 = 100 cm using the equation $l_{n+1} = l_n/2^{1/N}$: N = 12, n = integers 0 to 12; *D*, Standard designations of the notes; *F*, Lengths l_1 to l_{12} as fractions of the total length l_0. These fractions approximate close to integer ratios. Consequently, specific higher harmonics of the concerned notes get inter-related; e.g., 8th harmonic of Ri_2 is identical to 9th harmonic of Sa and so on.

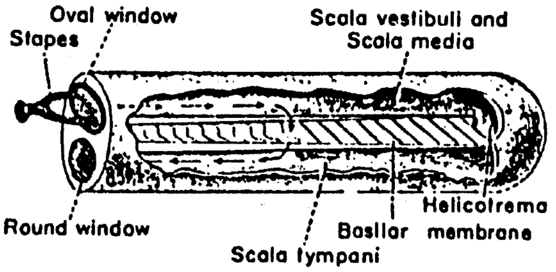

Figure 2. Schematic of the unwound cochlea showing over 20,000 basilar reeds (ref. 3).

also happen to be notes contained within the scale. In other words, the higher harmonics associated with any of the fundamental notes do not fall out of the musical scale for the values of N considered here. For all values of N other than 12, 24 and 22, the relations (1)–(7) are not satisfied and thereby the higher harmonics of any of the notes do not constitute satisfactory members of the scale.

Thus when one produces a fundamental tune of say, Sa Ri_2 Ga_3 Pa Dha_2 $\dot{S}a$, a second harmonic tune in the next octave $\dot{S}a, \dot{R}i_2, \dot{G}a_3, \dot{P}a, D\dot{h}a_2, \ddot{S}a$, a third harmonic tune of $\dot{P}a$ $D\dot{h}a_2$ $\dot{N}i_3$ $\ddot{R}i_2$ $\ddot{G}a_3$ Pa, a fourth, fifth and sixth, etc., harmonic tunes starting with $\ddot{S}a, \ddot{G}a_3, \ddot{P}a$ and so on are also simultaneously produced. These tunes are said to be in concordance (samvada) with the fundamental tune. With the Sa to $\dot{S}a$ separations and the octaves divided into twelve parts as described above, even if the fundamental tune spreads over two or more octaves, the concordant tunes follow it faithfully. In the case of a musical scale with the octaves divided into 22 parts (i.e. $N = 22$), it will be found that such concordant tunes up to the order of twelfth harmonic accompany each fundamental tune, reasonably well. That Bharatha[1] could arrive at this number, ($N = 22$) among all other numbers using perhaps, only his trained ears is particularly commendable.

Besides, in an extended musical tune spanning over more than an octave, if a mistake is committed, say, in the example quoted above, a single note $\dot{R}i_2$ is replaced by $\dot{R}i_1$, even an untrained ear can recognize the resulting discordance in the tune. This suggests that the higher harmonic tunes accompanying the fundamentals are also recognized by our ears, although we may be aware of having heard only the strongest, fundamental tune.

What is meant by the statement that the musical tune along with its harmonics is 'heard'? Some details on the internal construction of our ear are relevant here (see ref. 3).

The human ear is an inverted, biological harmonica. From the eardrum, a three-bone lever mechanism leads the incident sound to the base of a compact, fluid-filled, spirally-wound cochlea tube. If spread out, this cochlea would open into a 35-mm long, folded tube as shown in Figure 2. Along a muscle fibre at the joint of the two folds are located numerous soft bone reeds, thick, short rods near the base, thinning and elongating towards the other end. Long-wavelength, low-frequency sound received from the eardrum, penetrates deep into the cochlea and causes resonating vibrations of the thin reeds[3]. Small-wavelength messages are attenuated closer to the base of the cochlea and are picked up by the reeds here. Numerous hair cells and vibration-to-electrochemical activity transductors located at the roots of each of the reeds, in association with fine nerve fibres convey the message picked up to the brain.

Thus we notice that the design of the musical instrument veena and the human ear form a one-to-one compatible pair and thereby the ear can perhaps 'hear' and follow a veena recital with least 'cross-talk' and rejection and hence with least 'efforts'.

In view of these considerations, some interesting questions might however be raised. It is unlikely that other means of production of musical notes, say vocal or by wind instruments, follow the same mechanism of sound production and hence produce the same pattern of the notes as those obtained from a veena. It might be interesting to analyse the basis of commonalities musicians adopt, while they tune themselves or their instruments to accompany a veena recital.

1. Bhat, K. K., *Bharathiya Sangeetha Shastra*, published by Murthy, D. V. K., Mysore, 1971.
2. Feynman, R. P., Leighton, R. B. and Sands, M., *The Feynman Lectures on Physics*, Addison Wesley, NY, 1963, pp. 49–50.
3. Guyton, A. C., *Text Book of Medical Physiology*, Saunders & Co., Tokyo, 1981, VI edn, p. 760, figs 61.4, 61.5.

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