Another statement constantly made is that war is *inevitable*. This is to mistake the fundamental causation of things. Guns do not attack the enemy automatically, their construction and operation depend obviously on the living mental agent behind them. Therefore it is with this mind that we have to deal. Even if this be granted, however, another frequent statement is that *human nature does not change*. The evil power of mass suggestion is only too evident. In this connection one might refer to another apparent assumption, that men of science are superior to war suggestion. I have not observed that men of science are much less quarrelsome than other people. Personally I have not found that the study of chemistry has in itself enabled me more easily to control my temper or to suffer fools gladly.

Another assumption would seem to be that peace and democracy were desirable things to which all had a right much as we have to air and sunshine. On the contrary they are the rewards of great and persistent effort. Peace is the reward of righteousness. Democracy is the reward of self-discipline.

Actually the root cause of War lies deeper than these material difficulties. I would say that there are three lusts which are present in the subconscious mind of man, which lie at the basis of war, the lust of *power*, the lust of *prestige* and the lust of *possession*.

Let us not, therefore, as men of science make use of any of these half truths in our campaign for the abolition of war. I say for the abolition of *War* because the abolition of any particular weapon is only a part of our task. While it is arguable that there is little to choose between death or torture by poison gas and being smashed by a shell, it must be recognized that throughout the history of humanity certain limits have always been instinctively laid down. This was so even in the old Mosaic law and is implicit in the common expression 'hitting below the belt'. Most people would sooner face a revolver than a bottle of vitriol.

Any joint effort, therefore, which can be made by the scientific workers of the world to limit the use of methods of warfare which involve death and torture to innocent and unprotected people is to be welcomed.

Nevertheless, the intelligence which is capable of investigating the furthest star or the smallest molecule, that has discovered means of communication which have annihilated space and time, can surely produce some result if it will honestly and humbly set itself to the study of these infernal forces which lie in the subconscious mind of man, and which need to be controlled by some higher energy if all the fair prospects of humanity are not to be obliterated in one hideous ruin. The famous psychiatrist Dr Jung says 'it is the psyche of man which makes wars... the most tremendous danger that man has to face is the power of his ideas. No cosmic power on earth ever destroyed ten million men in four years but man's psyche did it *and it can do it again*. I am afraid of one thing only, the thoughts of people, I have means of defence against things'.

The following resolutions, put forward from the chair, were passed by the meeting:

'This meeting while pledging its support to every united effort which can be made to abolish methods of warfare which are repugnant to the common instinct of humanity recognizes that the more important objective is the abolition of war itself.

To attain this end it would urge constant and strenuous activity on the part of thinkers and men of science.

In particular it records its opinion that more attention should be given by them to the study of the new economic conditions, which of necessity, accompany the advance of scientific research.

Of equal or greater importance is the study of means for controlling the evil effects of 'mass suggestion' by the more powerful agency of widely disseminated right ideas through the adoption of an international system of education'.

SCIENTIFIC CORRESPONDENCE

Quantum statistics of an ensemble of harmonic oscillators

It is widely believed that the concept of distinguishability of entities in an ensemble is a classical attribute[1]. The origin of the belief may be traced in Boltzmann's argument[2] that when two identical atoms interchange their places in phase space, a new dynamical state is created, because we can follow the movement of each atom continuously during the process of interchange and hence the two states can be distinguished. In quantum mechanics, continuous tracking is impossible and hence Boltzmann's counting process is wrong. This leads to the concept of indistinguishability of atoms. But atomic ensembles are not the only ones we encounter in statistical mechanics. An alternative example is the ensemble of non-interacting harmonic oscillators of identical frequency, as we find in the Hamiltonian version of electromagnetic radiation field and of the elastic waves in a solid in the harmonic approximation. In working out the statistical mechanics of these ensembles, we have a dilemma. Do we consider them as distinguishable or indistinguishable entities? The question is significant because in this case the entities are not real objects, they are mathematical constructs. Here, appeal to mechanics does not help. The only way to decide the question is to work out both the distinguishability and indistinguishability statistics and see which one is consistent with thermodynamics. In the atomic systems we know that distinguishability leads to Gibbs' paradox. The correct statistics is either Bose–Einstein (B–E) or Fermi–Dirac (F–D).

Here, we show that a reverse situation arises for an ensemble of harmonic oscillators. If we treat the oscillators as indistinguishable, we arrive at a paradoxical situation – internal energy is no longer an extensive quantity. The correct statistical mechanics is arrived at by considering the oscillators as distinguishable and this is the procedure followed in the usual Debye theory.

For N oscillators, each of frequency, ν, the Debye theory of specific heat of solids is based on the following partition function

$$Z = \left(\sum_{m=0}^{\infty} \exp(-mh\nu / kT) \right)^N = (1 - \exp(-h\nu / kT))^{-N}, \quad (1)$$

and internal energy, U, is given by

$$U = kT^2 \left(\frac{\partial \ln Z}{\partial T} \right) = \frac{Nh\nu}{\exp(h\nu / kT) - 1}. \quad (2)$$

Thus U is extensive, consistent with thermodynamics. Reif[1] mentioned that the above result corresponds to an ensemble of *indistinguishable particles*, but he did not mention that the same corresponds to an ensemble of *distinguishable oscillators*. This becomes clear, if we consider the case for $N = 2$ and m is confined to values 0 and 1. Then $Z = (1 + e^{-x})^2 = 1 + 2e^{-x} + e^{-2x}$, where $x = -h\nu/kT$. It is easy to verify, that this value is obtained only if we consider oscillator 1 in state 0, oscillator 2 in state 1 and oscillator 1 in state 1, oscillator 2 in state 0 as distinct dynamical states. In other words, we regard the oscillators as distinguishable. If we consider the oscillators as indistinguishable, the partition function would be $Z = 1 + e^{-x} + e^{-2x}$.

If we consider N oscillators, each of frequency ν and consider them indistinguishable, then the partition function can be calculated in the following way.

In B–E statistics, we consider N independent harmonic oscillators each of frequency ν. The total energy is specified by the quantum number

$$m = \sum_{i=1}^{N} m_i, \quad (3)$$

where m_i denotes the energy state of the ith oscillator. In the canonical ensemble formalism, the partition function Z is defined by

$$Z = 1 + \sum_{m=1}^{\infty} g_m(N) x^m, \quad (4)$$

where $x = \exp(-h\nu/kT)$ and $g_m(N)$ is the weight factor for the energy state m. According to B–E statistics, $g_m(N)$ is the number of possible partitions of m into N integers, without counting permutation. From the theory of partition of numbers we can write:

$$Z = (1 - x)^{-1}(1 - x^2)^{-1} (1 - x^3)^{-1} \ldots N\ factors. \quad (5)$$

(see Barnard and Child[3], first equation, in p. 504, where the quantity $P(m, p)$ is identical to $g_m(N)$ in this paper.) We can, therefore, write

$$\ln Z = -\sum_{n=1}^{N} \ln(1 - x)^n. \quad (6)$$

The internal energy is given by

$$U = kT^2 \left(\frac{\partial \ln Z}{\partial T} \right) = h\nu \sum_{n=1}^{N} \frac{n}{\exp(nh\nu / kT) - 1}. \quad (7)$$

The term within the summation sign is a decreasing function of n. Hence, if N is changed to $2N$, the sum will not be doubled and U is not proportional to N, i.e. U is not an extensive quantity.

In the F–D statistics, it is more difficult to compute the partition function. But we can calculate the ground state energy, which is

$$U = \sum_{m=0}^{N} mh\nu = \frac{h\nu}{2} N(N+1) = \frac{h\nu}{2} N^2, \quad (8)$$

for large N. The above equation shows that U is not an extensive quantity.

The correct expression for U is obtained if we use distinguishability statistics for which the partition function is

$$Z = \left(\sum_{m=0}^{\infty} x^m \right)^N, \quad (9)$$

from which we get,

$$U = \frac{Nh\nu}{\exp(h\nu / kT) - 1}. \quad (10)$$

The main conclusion emerging from this analysis is that the concept of distinguishability is not necessarily a classical concept. The essential element of a quantum statistics is to use quantum expression for energy levels. Whether the entities are to be regarded as distinguishable or indistinguishable depends on the nature of the entities. For atoms and electrons, one uses the concept of indistinguishability; for oscillators, the concept of distinguishability is used. There is, however, an alternative procedure for oscillators, developed by S. N. Bose[4]. Instead of oscillators one can consider the system as an ensemble of particles called photons or phonons which must be regarded as *indistinguishable*. As entities these particles are peculiar, each particle has only one energy state $h\nu$. Variation in total energy is possible only by varying the number of these particles, which, therefore, is not a conserved quantity. This ensemble of indistinguishable particles and its significance in connection with the discovery of B–E statistics has been discussed elsewhere[5].

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