Holography, black holes and string theory

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Recently, a remarkable idea called holography has gained prominence in string theory. It says that a system containing gravity can be described by degrees of freedom living on its boundary rather than in the bulk. Recent progress has led to concrete realizations of this idea in string theory revealing deep and unexpected connections between gravity and gauge theories. It seems clear now that holography will profoundly change our view of space–time. It could also have important applications in other branches of physics, like the study of the strong interactions in some limits.

RECENTLY, a remarkable idea called holography has gained prominence in string theory. The idea has led to a lot of excitement and resulted in some rapid progress. It is now clear that holography will have long-lasting impact on string theory, profoundly changing our view of space–time. It is also likely to lead to interesting connections between string theory and other branches of physics, like the study of the strong interactions, where the strongly-coupled behaviour of gauge theories is of interest.

Holography has its origins in the study of black holes in the 1970s, especially the work of Beckenstein and Hawking. Its significance for the study of quantum gravity was emphasized by 't Hooft and for string theory by Susskind. More recently, concrete examples implementing this idea have been uncovered in string theory, in particular due to a remarkable conjecture by Maldacena. This has brought holography into sharp focus.

We start this article, by introducing the idea of holography in general terms, in the next section. To gain a better appreciation for it, we discuss next some of the historical motivations provided by the study of black holes. Recent developments which have incorporated holography into string theory are discussed next. In particular, we discuss one specific example of holography in string theory, the evidence in support of it and some of the consequences. We end with some general comments on possible future directions.

This article is meant as a general introduction for the non-specialist. No prior knowledge of string theory or the General Theory of Relativity is assumed.

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What is holography

Let us begin by noting two features about the world around us:

- We live in 4 space–time dimensions, 3 dimensions of space and 1 dimension of time.
- There is gravity in the world around us.

Roughly speaking, holography says that there is an equivalent description of nature:

- In 3 space–time dimensions
- Without gravity.

Equivalent means that the two descriptions agree on the outcome of any experiment.

More precisely, holography says that all physical phenomenon inside a region of space–time can be equally well described by a theory, without gravity, living on the boundary of the region. This boundary theory is called the hologram.

The word, holography, is taken from optics. Its meaning in gravity and string theory, as we will see below, is similar in spirit, but not identical. In particular, in the present context, the hologram stands for an equivalent predictive description in its own right.

At first glance holography seems like a completely crazy idea. It is at odds with our intuition which tells us that there should be many more degrees of freedom in four space–time dimensions as compared to three. How then, one might ask, can two descriptions, one in four space–time dimensions and the other in three space–time dimensions really be equivalent?

Physicists use the notion of entropy, S, to quantify the number of degrees of freedom. Entropy is defined to be

$$S = \log(N), \tag{1}$$

where N is the number of states accessible to a system. In our usual experience, entropy is an extensive variable, scaling like the volume of the system (note 1).

As an example, consider a gas of volume V full of photons at a temperature T. Elementary statistical mechanics tells us that the entropy is given by

$$S = c_1 V T^3. \qquad (2)$$

Notice that entropy scales like the volume not like the area (note 2).

Now, the real issue in front of us is this: Take a region (in 3-dimensional space) of size R, is the maximum entropy one can put in this region, proportional to the volume, R^3 or to the area R^2? If the answer goes like the volume it seems very unlikely that holography is in fact correct.

It turns out that in a wide range of circumstances, where gravity can be neglected, the entropy does scale like the volume, in accord with our intuition. We saw this above in our example of photons. But it is much more generally true as well. For example, we can replace the photons with non-relativistic atoms. Or, keep the photons, but increase the temperature to a few million electron volts (MeV) at which point the box contains electrons and positrons as well. Or go up even further in temperature, to a few hundred million electron volts, so that there are other particles like quarks and gluons, besides photons and electrons and positrons in the box. In all these situations, as long as gravity can be neglected, the entropy continues to scale like the volume.

The question we are really interested in settling though, as mentioned above, is one of the maximum bound on entropy for a fixed volume. And it turns out that in maximizing the entropy in this manner, gravity always, eventually, gets important. In fact, as we will see below, black holes get important. And once this happens, the behaviour of the entropy changes very significantly.

So, we need to gain a better understanding of black holes next.

Black holes and entropy

Black holes

The correct setting for understanding black holes is Einstein's General Theory of Relativity. In broad terms, this theory says that matter–energy curves space–time and this curvature is what we call gravitation.

In a black hole the effects of the curvature are so intense that nothing, not even light, can escape its gravitational pull and reach an outside observer. As a result, a black hole is completely dark to the outside world.

More accurately, surrounding a black hole is a surface called a horizon (see Figure 1). A photon which starts from outside the horizon, can make its way to a distant observer. But a photon, which starts from within the surface, cannot make its way out, regardless of its energy. It can come towards the surface for a little while, but eventually the pull of gravity overwhelms it and the photon falls back into the black hole (note 3). In other words, the horizon is the critical surface which divides the region of space–time which can communicate with a distant observer from the region which cannot.

For a non-rotating black hole the horizon is spherical, with radius R_{H}, and an area

$$A_{\mathrm{H}} = 4\pi R_{\mathrm{H}}^2. \qquad (3)$$

The horizon, especially its area, will play an important role in the subsequent discussion.

For a black hole of mass M the radius of the horizon is

$$R_{\mathrm{H}} = \frac{2G_N M}{c^2}. \qquad (4)$$

Here, G_N is the gravitational constant, and, c is speed of light. For a solar mass black hole, $M = 10^{33}$ g,

$$R_{\mathrm{H}} \quad 3\ \text{km}. \qquad (5)$$

This works out to a density of about 10^{16} g/cc!

In fact black holes are the densest objects in the universe. Consider a star of mass M undergoing gravitational collapse. As long as its radius is bigger than R_{H}, light from its surface can reach a distant observer, but once its radius is smaller than R_{H} this is no longer true and a black hole forms. Thus black holes are the endpoint of gravitational collapse, denser than white dwarfs or even neutron stars.

There is another way to understand this last point. Consider a region of space of radius R, and let us ask what is the maximum energy which can be stored in this region. This bound is achieved by a black hole. The mass of this black hole, in terms of the radius of the region R can be obtained by re-expressing eq. (4) and is given by

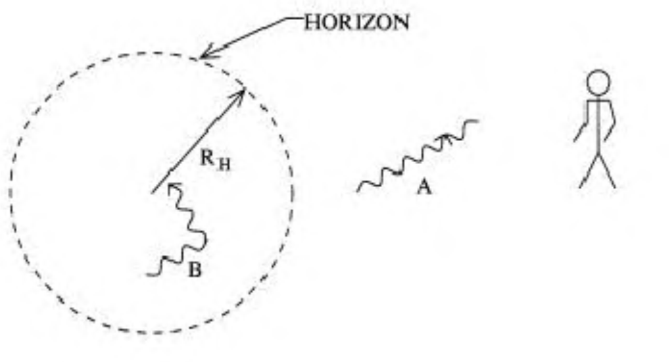

Figure 1. Photon A can reach distant observer, but not photon B.

$$M = \frac{c^2}{2G_N} R. \tag{6}$$

Any attempt to further increase the mass of the black hole M also increases its radius R, in accordance with eq. (6).

We started the discussion regarding entropy by mentioning the gas of photons. Let us see if we can apply some of the ideas mentioned above to it. Concretely we take a gas of photons in a box of size L with volume $V = L^3$, and at temperature T. We mentioned that the gas has entropy

$$S = c_1 V T^3. \tag{7}$$

It also has energy,

$$E = c_2 V T^4. \tag{8}$$

Notice that if the volume is fixed, the temperature must be increased to increase entropy This also increases the energy density. As a result, eventually, a black hole forms. Setting the size of the region, R, in (eq. (6)) to be of order L, and using the relation $E = Mc^2$, we see from eq. (8) that this happens when the temperature reaches a value, T_c, (note 4) such that

$$L^3 T_c^4 \sim \frac{c^2}{2G_N} L. \tag{9}$$

Supressing the natural constants for the moment, we see that T_c scales with the length L as:

$$T_c \sim 1/\sqrt{L}. \tag{10}$$

In summary then, we have seen that maximizing the entropy while keeping the volume fixed, has led to the formation of a black hole as was mentioned earlier. To carry our discussion of the maximum bound on entropy further, we need to understand what, if any, is the entropy which can be attributed to the resulting black hole.

Black hole entropy

We are now ready to discuss some of the developments in the 1970s in black hole physics. Our story begins in 1971. That year Stephen Hawking proved an important theorem, which set into motion a chain of events that culminated in answering the question of black hole entropy.

The theorem Hawking proved is called the Area theorem of black hole physics. This says,

The area of a black hole horizon never decreases.

That is, in any process let A_{initial} be the initial area of a black hole's horizon and A_{final} be the final horizon area then

$$\Delta A = A_{\text{final}} - A_{\text{initial}} \geq 0. \tag{11}$$

If there is more than one black hole, a similar theorem holds where the area refers to the total area of all the horizons.

Consider, for example, a situation where radiation is impinging on a black hole, some of it gets reflected, the rest of the radiation falls in. Hawking's theorem says that area cannot decrease in this process. Or consider two black holes, which have a cataclysmic collision and merge. Now one measures the initial total area and the area of the resulting final black hole, once again the area cannot have decreased.

Jacob Beckenstein was a young graduate student at Princeton University in 1971. He was struck by the similarity between the area theorem and the second law of thermodynamics.

As we know, the second law of thermodynamics says that:

The entropy of a closed system never decreases.

That is,

$$\Delta S = S_{\text{final}} - S_{\text{initial}} \geq 0. \tag{12}$$

Further thought suggested to Beckenstein that there was more to this similarity. He argued that in fact a black hole must have an entropy proportional to its horizon area (Beckenstein, 1973).

When Beckenstein's paper first appeared, it met with widespread disapproval from most experts.

The experts said, for example,

> if a black hole has an entropy, it must also have a temperature and it must radiate like a black body does at this temperature.
> This is impossible.

Beckentein replied (this was actually part of his paper of 1973),

> if a black hole does not have an entropy the second law of thermodynamics will be violated. We could throw some matter into a black hole and lose entropy.

The controversy was settled in 1973, by Hawking. He found that black holes do radiate, just like a black body, and calculated the temperature of black hole radiation. As a result, he could also calculate the entropy of a black hole. It is proportional to the horizon area.

Beckenstein was right after all.

The temperature Hawking found is

$$T_{\mathrm{H}} = \frac{\hbar c^2}{8\pi G_N M}. \tag{13}$$

Notice it depends on Planck's constant $\hbar$ and arises due to quantum effects. For a solar mass black hole, T_{H} 10^{-7} K. This is very, very, small. The classical approximation, which says that nothing can come out of a black hole is very good for such a black hole.

The resulting entropy can be calculated from the first law of thermodynamics (note 5). It turns out to be (note 6)

$$S_{\mathrm{BH}} = \frac{1}{4}\frac{A_{\mathrm{H}}}{l_{\mathrm{pl}}^2}, \tag{14}$$

where

$$l_{\mathrm{Pl}} = \sqrt{\frac{\hbar G}{c^3}} = 10^{-33} \text{ cm}. \tag{15}$$

The entropy of a solar mass black hole is 10^{76}. This is very, very big. For example, in comparison, the entropy of the sun is approximately, 10^{58}. The solar mass black hole has such a huge entropy because l_{pl} is very small compared to the horizon radius, and in fact much smaller than any distance scale we have probed so far in experiments.

With Hawking's discovery everything fell into place. It was shown that the second law of thermodynamics was valid, in many different circumstances, when one included the entropy of black holes as well.

Let us return to the box of photons. The last time we considered it (in eq. (9), eq. (10)) we concluded that as we increase the entropy in fixed volume V, eventually a black hole forms when

$$T_{\mathrm{c}} \sim 1/\sqrt{L}. \tag{16}$$

We can now phrase our worry in a very precise manner. The entropy at the outset of black hole formation goes like the volume, but after the black hole forms we have learned above that the entropy goes like the area. Is this consistent with the second law of thermodynamics which says that the entropy cannot decrease?

We know enough now to settle this question with a direct calculation. The entropy in the gas at the outset of black hole formation is

$$S_{\text{initial}} \sim L^3\, T_{\mathrm{c}}^3 \sim (L/l_{\mathrm{pl}})^{3/2}. \tag{17}$$

The entropy after forming the black hole is

$$S_{\text{final}} \sim (L/l_{\mathrm{pl}})^2. \tag{18}$$

For a box of size $L \gg l_{\mathrm{pl}}$ we see that there is no danger of violating the second law at all. For example, with $L = 1$ m, $l_{\mathrm{pl}} = 10^{-33}$ cm,

$$S_{\text{final}} - S_{\text{initial}} \sim 10^{70} \gg 1. \tag{19}$$

The entropy scales like the volume before gravitational effects became important, and like the area after a black hole forms. But since l_{pl} is so small, compared to any length scale in our everyday experience or probed in the laboratory, this crossover is not in conflict with the second law of thermodynamics.

Before going on, let us also mention that in the example above, for $L \sim 1$ m, the temperature at which the black hole forms is $T_{\mathrm{c}} \sim 10$ GeV. This is certainly a high temperature compared to those experienced in day-to-day laboratory settings but the laws of physics are known well enough at this temperature, for us to trust the calculation leading to eq. (17) – eq. (19).

Let us now summarize what we have learnt so far:

- The maximum entropy which can be stored in a region of size R, in three spatial dimensions, goes like the area, R^2.
- The state which maximizes this entropy is a black hole, with a horizon radius equal to R.
- The maximum entropy is

$$S_{\mathrm{BH}} = \frac{1}{4}\frac{A_{\mathrm{H}}}{l_{\mathrm{pl}}^2}, \tag{20}$$

- In view of these lessons, holography does begin to seem more plausible. After all what holography really says is that there should be an alternate description in terms of the 'genuine' degrees of freedom. This is the hologram.

One final comment. The result that a black hole's entropy is proportional to its area is very general. It is true in space–times with dimensionality different from four too (note 7). As a result, the principle of holography can be applied to these cases as well. In the discussion in string theory that follows, the string theory will live in a space–time with five large dimensions and the hologram in four large dimensions.

Holography in string theory

The Maldacena conjecture

In the interests of brevity and at the grave risk of providing a distorted historical account, we will now jump from the 1970s to the late 1990s in our discussion.

Three important contributions in the intervening years however, must at least be mentioned. First, Kip Thorne and his group found that from the viewpoint of an observer located outside the horizon, the behaviour of a black hole could be conveniently understood in terms of a membrane located close to the black hole horizon. Various physical properties like electrical conductivity could be attributed to this membrane (note 8). G. 't Hooft argued that if the laws of quantum mechanics held for the observer outside the horizon, the Beckenstein Hawking entropy must correspond to physical degrees of freedom at the horizon. He further emphasized the generality of this result arguing in particular that any three-dimensional region must have a two-dimensional image. Finally, L. Susskind discussed the importance of these ideas in the context of string theory, emphasizing, in particular, that string theory was the natural setting in which to look for a holographic description and discussing some of the consequences for the theory.

We now move forward to the late 1990s. In 1997, Juan Maldacena, shortly after finishing his Ph D, made a remarkable conjecture:

> String theory in a particular curved ten-dimensional space–time is equivalent to a Gauge theory in flat four dimensional space–time.

The curved space–time is called: five-dimensional Anti de Sitter space × the five sphere ($AdS_5 \times S^5$). The gauge theory is an $SU(N)$ gauge theory with a lot of supersymmetry.

Let us try to understand the curved space–time, five-dimensional Anti de Sitter space × the five sphere, better. The five-sphere is a five-dimensional generalization of the two-sphere. It is a very symmetrical space with a constant positive curvature. Anti de Sitter space is a *space–time* with constant negative curvature. The full space–time has ten dimensions, five of these lie along the five-sphere and the remaining directions are along five-dimensional Anti de Sitter space. One can think of this as follows: at each point in the five-dimensional Anti de Sitter space, sits a five-sphere. This is similar to a cylinder which can also be described as a Line × a Circle (Figure 2).

Now what does this conjecture have to do with holography? In a nutshell, the conjecture is saying that the gauge theory is the hologram of the string theory. At first glance, this does not seem to be true, because the string theory, which contains gravity, lives in ten dimensions while the gauge theory lives in four space–time dimensions. But in fact, the five-sphere in Anti de Sitter Space $\times S^5$ has a very small radius. So there are five large dimensions of the AdS_5 in which gravity and all the other modes of string theory live. Furthermore, five-dimensional AdS space–time has a boundary which is four-dimensional and the gauge theory lives on this boundary. So we see that the conjectured equivalence between the two descriptions is in fact a statement of holography.

We have depicted five-dimensional Anti de Sitter space pictorially in Figure 3, with the four-dimensional boundary in black. The extra direction present in the AdS_5 will be sometimes referred to in the radial direction below. Also sometimes in the discussion below, we will refer to string theory in the $AdS_5 \times S^5$ space–time as the gravity description, to draw attention to one of its more important features, in contrast, the hologram will be referred to as the gauge theory description.

Before proceeding further, it is worth emphasizing just how remarkable the conjecture is: It says that string theory, in a particular background, is equivalent to a gauge theory. Now, String theories, which amongst other things contain gravity, are still quite mysterious. On the other hand, gauge theories are much better understood theoretically as well as experimentally. If true, the conjecture is a big step forward in understanding string theories. Much of the excitement in the field stems from this fact.

As we have been emphasizing above, the proposed equivalence is a conjecture. While there is no proof for

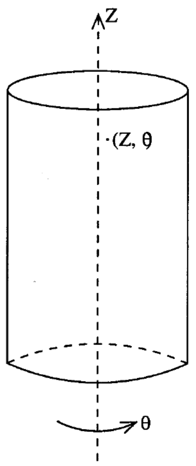

Figure 2. Any point on cylinder is labelled by position along the Z direction and angle θ.

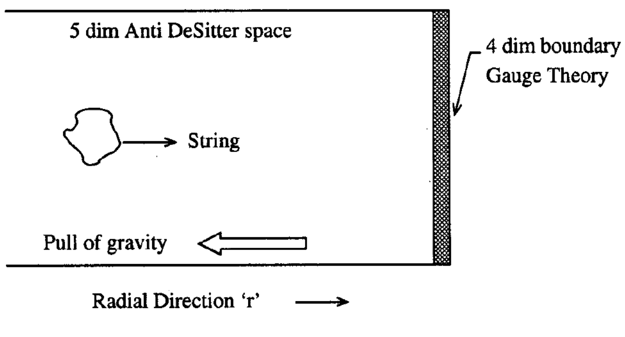

Figure 3. 5 dim. Anti De Sitter space.

it, there is mounting evidence in its support. In the next section we will describe some of the tests to which the conjecture has been put, all of which it has passed.

Tests of the conjecture

Parameter counting. If the two descriptions are equivalent, the number of parameters needed to specify each side must agree. Table 1 shows that both descriptions are specified by two parameters.

And in fact one can match these parameters in a very precise way.

Something quite interesting is revealed by this matching. It turns out that for a choice of parameters such that the string theory is tractable, the gauge theory is difficult to directly calculate with, since it is strongly coupled. And vice-versa.

This means, first of all that tests of equivalence will have to be devised with some cleverness.

More importantly it means that for some values of parameters, when the gauge theory is strongly coupled, it can be more conveniently thought of as a string theory.

The particular gauge theory involved in the conjecture is unlikely to be of much use, outside of string theory, since it has a lot of supersymmetry, etc. But the hope is, and considerable progress has already been made in this direction, that the conjecture can be extended to situations where the gauge symmetry has less or no supersymmetry. String theory would then be of use in understanding the strongly coupled behaviour of these gauge theories.

The potential connections with the study of the strong interactions and possibly other branches of physics as well, mentioned at the beginning of this article, all arise from this feature.

Symmetries. The symmetries in both descriptions must agree.

The gauge theory has $3+1$ dim. Poincare invariance. It has scale invariance. It has 32 supersymmetries. And so on.

It turns out that the string theory has all these symmetries too. In fact the full superconformal group of symmetries agree on both sides.

Table 1. Parameters for string and gauge theories

String theory	Gauge theory
g_s	g_{YM}
String coupling	Gauge coupling
Curvature	N
of AdS space	Rank of Gauge group

The operator, mode correspondence. The observables in the gauge theory are gauge invariant operators. The natural degrees of freedom in the string theory are the various fluctuating modes about the background geometry. These should be in one-to-one correspondence.

For a large class of special operators, $\hat{O}_i$, on the gauge theory side the corresponding modes on the gravity side ϕ have now been identified.

$$\hat{O}_i \leftrightarrow \phi. \tag{21}$$

Their symmetry properties match exactly. In fact they fall into the same representations of the superconformal group.

Observables. A precise form of the Maldacena conjecture has been formulated. This allows the correlation functions of gauge invariant operators in the gauge theory, to be calculated in the gravity description as well.

Two and three point functions have been calculated in both descriptions using this prescription for the special operators mentioned earlier. They agree. This is quite impressive.

The essential idea is that external sources coupling to operators in the gauge theory are related to boundary conditions for modes in the gravity theory.

Black holes and thermal states. Finally black holes in the gravity description must correspond to a thermal state in the gauge theory at the same temperature. This leads to a possible test: does the entropy of the black hole agree with that of the thermal state in the gauge theory?

The weakly coupled gauge theory, in volume V and temperature T, gives

$$S = \frac{2\pi^3}{3} N^2 T^3 V. \tag{22}$$

The coefficient can change in extrapolating to strong coupling, where the gravity calculation is reliable. But the N^2 dependence should be robust.

On the gravity side, one finds that the black hole has an entropy

$$S = \frac{\pi^2}{3} N^2 T^3 V. \tag{23}$$

We see that the N^2 dependence agrees, although the coefficient has changed.

There are other examples of holography that Maldacena also conjectured to be true. In one of these, the hologram is in $1+1$ dimensions. This case is, in some ways, more tractable, for e.g., in the comparison of entropy of black holes and corresponding boundary states.

Motivation for the conjecture

We have seen above that the conjecture passes some non-trivial tests.

Let us discuss now, at least briefly, the physical insights which led to Maldacena's remarkable conjecture.

For this purpose, we need to introduce the concept of a *D*3-brane. A *D*3-brane is an extended object in string theory, which stretches out in three out of the nine spatial dimensions. It has mass, charge – with respect to a gauge field, and it preserves a lot of supersymmetry. These properties make a *D*3-brane rather special.

The crucial insight was to take several *D*3 branes together and consider their dynamics at low-energies. It was already known that the low-energy dynamics can be described as a gauge theory.

At low-energies, though, a second description for *D*3-branes also becomes valid. This is because, many complications of string theory became irrelevant at low-energies and Einstein's theory of gravity, more accurately a ten-dimensional version of it with supersymmetry called supergravity, becomes an adequately good approximation. In this theory it turns out there is precisely one state with the rather special properties of the collection of *D*3 branes mentioned above, i.e. precisely one state with the same mass, charge, etc. This state is a particular solution in the theory, with a geometry, which, in the low-energy limit, is five-dimensional Anti de Sitter space $\times$ S^5. It is then reasonable to equate this state and all the fluctuations about it, with the gauge theory; since both describe the low-energy dynamics of the *D*3-branes. This gives rise to the conjecture.

The extra direction in the hologram

The discussion under the section 'What is holography', makes holography somewhat plausible, but it remains a counterintuitive idea nonetheless. One puzzling question for example is this: the gravity description has one extra dimension, how is information about this extra dimension stored in the boundary hologram?

This question has been addressed in a few different ways by now. The answer is quite revealing, and possibly profound.

One approach followed was operational in nature (note 9). The idea was to take a well-localized state in the gravity description and construct its image in the boundary hologram. Then by moving the state in the extra direction present in the gravity description, and asking how the image changes, one could learn how information about the extra direction is stored in the hologram.

The first step in carrying out this procedure was to develop some mathematical formalism which allows one to map a state in the gravity description to its image in the hologram. The essential idea is that any excitation in the bulk decays away towards the boundary, with a characteristic decay pattern. The state in the boundary can be constructed from this decay pattern. In effect, determining this mapping between states in the gravity description and the hologram, is like constructing a 'camera' which constructs the holographic image, from the state in the gravity description.

The next step was to choose some localized excitation in the gravity description, as a starting point. A particularly convenient choice is a state called a '*D*-instanton' (note 10). The detailed description of this state is not important for the present discussion. All that is relevant is that the state is well localized, in fact one can think of a *D*-instanton as being point-like within the approximations of the present discussion. This makes it particularly well suited for the question on hand.

When the image of the *D*-instanton was constructed in the gauge theory, using the formalism mentioned above, it turned out to be a well-known classical solution in the gauge theory called an 'instanton'. Interestingly, the instanton in the gauge theory is not a point-like object at all. Instead it is a lump-like object with a characteristic size. Furthermore, it was found that, this size depends on the position of the *D*-instanton in the extra direction. As the *D*-instanton is moved away from the boundary, the size of the image gauge theory instanton grows (in length), as is shown in Figure 4.

The basic lesson of this investigation then, which also emerges from others studies, is that information about the extra dimension gets stored in the characteristic length scale of the holographic image.

Put another way, physicists use the word renormalization group flow, to describe how the behaviour of a theory changes as one goes to longer and longer length scales. Here one is finding that renormalization group flow in the gauge theory corresponds in the bulk to moving further and further away from the boundary.

The full significance of this finding is still unclear to us. One thing can be said: a well-known physical phenomenon called gravitational red-shift, underlies why longer length scales on the boundary are related in the bulk to positions away from the boundary.

To understand this, let us return to Figure 3. There is one important feature of the five-dimensional Anti de Sitter space which we have not mentioned so far, namely that the pull of gravity is away from the boundary along the radial direction.

This means that photons, with the same initial energy, which start further away from the boundary, need to work harder against the gravitational pull and so lose more of their energy by the time they get to the boundary (see Figure 5). This is why positions along the radial direction further away from the boundary correspond to lower energies in the gauge theory. This loss of energy for a photon, is an example of a

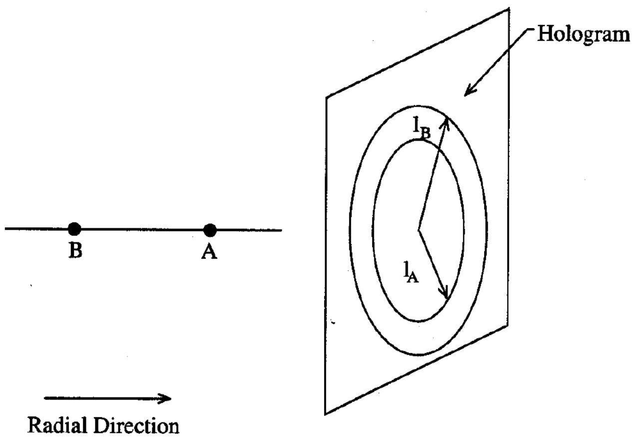

Figure 4. The image of pt A is spreadout over radius 1A. The image of pt B is spreadout over a bigger radius 1B.

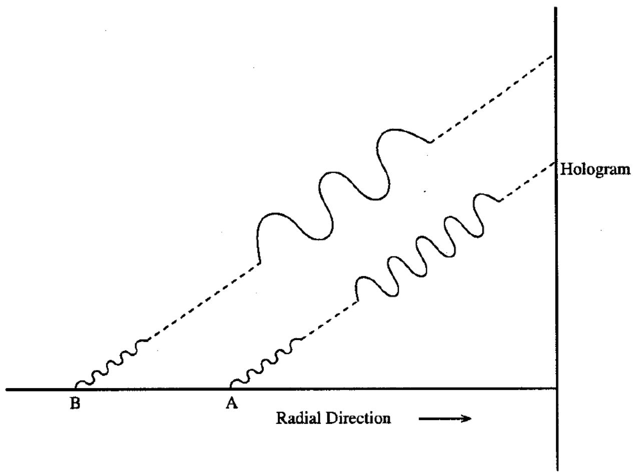

Figure 5. Photon B which starts further away arrives at the boundary with a bigger wavelength.

well-known phenomenon in gravitation, called the gravitational red-shift.

We can now summarize some of the main points of this section. The Maldacena Conjecture says that string theory, containing gravity, is equivalent to a gauge theory. It has passed some very non-trivial checks. The conjecture is clearly of great importance for string theory, gravity and black holes. It is also of interest in the study of strongly coupled gauge theories. Much more needs to be done before we understand it fully.

Summary and discussion

Holography, which had its origins in the study of black hole, is a deep general principle. Attempts to incorporate it into string theory, have already yielded a rich harvest of remarkable ideas.

The subject is in its infancy, especially within the context of string theory. Future directions, where one can hope for progress, include:

- Understanding holography in space–times other than Anti de Sitter space, e.g. flat space and cosmological space–times. This throws up new conceptual challenges. For example, the boundary of Anti de Sitter space is time-like, but the boundary of cosmological space–times is often space-like and for flat-space it is null. Extending holography to these cases, will surely be worthwhile.
- Understanding how space–time emerges from the holographic description better. One example of this sort of question was discussed earlier, where we sketched the role of the extra dimension present in the gravity description, in the hologram. A deeper understanding of this question is likely to radically alter our view of space–time.

- Understanding strongly coupled gauge theories with no supersymmetry.
  In this article we have seen one example where a strongly coupled gauge theory is more easily described as a string theory. There is already some progress in extending this to cases where the gauge theory has less or no supersymmetry and no scale invariance. It is hoped that this progress will allow us to understand asymptotically free gauge theories, like quantum chromodynamics, at least in some limits, as string theories.
- Another place where strongly coupled gauge theories might enter our description of nature is in the physics which underlies the standard model, especially the breaking of electro-weak gauge symmetry. Some progress has already been made in casting such scenarios in terms of an equivalent description involving gravity in one higher dimension. This could get really exciting once the next round of collider experiments begin.

Notes

1. In our definition the entropy is dimensionless. This is different from the usual definition which includes a dependence on the Boltzmann constant.
2. [2]c_1 is constant whose precise value will not be very relevant in the subsequent discussion.
3. Strictly speaking by the word horizon, I mean here, and in most of the rest of the article, the event horizon of the black hole. The experts will notice that when this description is over simplified, the horizon is really a null surface.
4. Actually, the constants c_1, c_2 also change as the temperature is varied, since the effective number of species of particles changes. This was discussed under 'what is holography' above, but we will neglect this complication here.
5. In fact, before the discovery of black-hole radiance, Bardeen, Carter and Hawking had shown that a version of the First Law of Thermodynamics in the presence of black holes is valid if black holes have an entropy proportional to their area.
6. The reader will notice that the entropy $S \sim 1/\hbar$, this dependence of the entropy is familiar from examples like photons in a box.
7. Of course what one means by area changes as the dimensionality of space–time changes. For example, a sphere of radius R in *n space* dimensions has an area going like R^{n-1}.
8. For more on this see, Thorne, K., Price, R. H. and Macdonald, D. A., *Black Holes and the Membrane Paradigm*, Yale University Press.
9. This discussion is based on 'Holographic Probes of Anti de Sitter Spacetimes', Balasubramanium, V., Lawrence, A., Kraus, P. and Trivedi, S. P., *Phys. Rev.*, 1999, **D59**, 104021.
10. The experts will recognize that we are working here in Euclidean space.

1. For an historical account of some of these development, see Kip Thorne, *Black Holes and Time Warps, Einstein's Outrageous Legacy*, W.W. Norton & Company, ISBN 0-393-03505-0.
2. For more about string theory in general terms, see Brian Greene, *The Elegant Universe*, Vintage Books, ISBN 0-375-70811-1(pbk.)
3. A popular account of holography can be found in Susskind, L., *Sci. Am.*, 1997, **276**, 52–57.
4. Also of popular interest is the article, in Taubes, G., *Science*, 1999, **285**, 512–517.
5. For a somewhat more technical discussion accessible to physicists, see 't Hooft, G., *Dimensional Reduction in Quantum Gravity*, Published in Salamfest 1993:0284-296; gr-qc/9310026.
6. Also of interest to physicists is the article in Susskind, L., *J. Math. Phys.*, 1995, **36**.