Muon anomalous magnetic moment and 'new physics'

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On 8 February 2001, scientists at the Brookhaven National Laboratory in Upton, New York, in collaboration with researchers from 11 institutions from the US, Russia, Japan and Germany announced a precision measurement of the anomalous magnetic moment $(g-2)$ of the muon[1]. By itself, this would not have been of momentous consequence if it were not for the fact that the value deviates substantially from that predicted by the so-called Standard Model of Particle Physics. Before we go into the details of the results and the consequences of the measurement, let us summarize briefly some basic ideas about the anomalous magnetic moment of particles.

The proportionality constant between the magnetic moment $\vec{\mu}$ and spin $\vec{s}$ of a particle is given by the gyromagnetic ratio g

$$\vec{\mu} = g\left(\frac{e}{2m}\right)\vec{s}, \qquad (1)$$

which for the muon can be written as

$$\mu_\mu = (1+a_\mu)\frac{e\hbar}{2m_\mu}, \text{ where } a_\mu = \frac{(g-2)}{2}. \qquad (2)$$

The Dirac equation, which is the relativistic generalization of the Schrödinger equation for spin-half particles, predicts $g = 2$ which is a consequence of the minimal coupling of the photon to the electron or muon, $(\bar{\psi}\gamma_\mu\psi A^\mu)$. However quantum electrodynamics (QED) predicts that

$$G = 2 + O(\alpha) + \ldots,$$

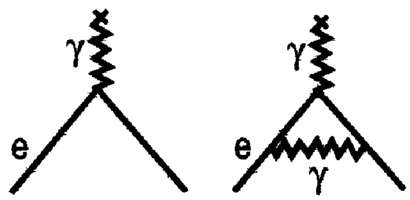
where α is the electromagnetic fine structure constant. This correction comes from a term generated at higher order in QED $(\sigma_{\mu\nu}F^{\mu\nu})$ that is not present in the Lagrangian and gives essentially a spin-orbit coupling of the spin to the magnetic field $\vec{\sigma}\cdot\vec{B}$. This is the anomalous magnetic moment, the deviation from the relativistic quantum mechanical value of 2 and comes from diagrams of the kind shown in Figure 1.

The electron magnetic moment has been measured to a few parts per billion and is described by QED to $O((\alpha/\pi)^4)$, and is presently limited only by our knowledge of the fine structure constant. In fact, the electron magnetic moment measurement is one of those used to determine the value of α – the others being the measurements of the quantum Hall effect and AC Josephson junction.

Today QED is the most stringently tested and most dramatically successful of all physical theories. Thus *any new particle that couples to e or* μ *will produce a correction to* $g-2$*. Since the QED prediction is so precise, it allows us to severely constrain the coupling strength of these new particles.* Although a_μ is not competitive in precision with a_e, it is much more sensitive to electro-weak loop effects as well as 'new physics' which give contributions which are more sensitive by a factor $(m_\mu/m_e)^2$, i.e. an enhancement of 4×10^4 in sensitivity. This is the reason muons rather than electrons are used in this experiment.

We briefly describe next the experimental set-up. Longitudinally polarized μ^+ at 3.09 GeV/c (we will see later the reason for choosing this energy) from a secondary beamline is injected into a storage ring 14.2 m in diameter with a homogeneous perpendicular magnetic field of 1.45 T. An electric quadrupole field is used for vertical focusing. The cyclotron frequency is given by the well-known result

$$\omega_c = \frac{eB}{mc},$$



(*a*) (*b*)

Figure 1. QED corrections to $(g-2)$.

which is also the frequency of rotation of the momentum vector. The Larmor spin precession frequency is

$$\omega_s = g\left(\frac{eB}{2mc}\right) = (1+a_\mu)\frac{eB}{mc}.$$

Note that for $g = 2$, these two frequencies are equal! This means that the muon maintains its initial polarization vector with respect to the momentum vector. For any $g > 2$, the spin turns faster than the momentum vector. For relativistic muons, these formulae change appropriately and one can write down the 'difference angular frequency' ω_a between the spin precession frequency and cyclotron frequency

$$\vec{\omega}_a \equiv \vec{\omega}_c - \vec{\omega}_s = -\frac{e}{m}\left[a_\mu\vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)(\vec{\beta}\times\vec{E})\right]. \qquad (3)$$

Note that the above result is true for relativistic muons. The dependence of ω_a on the electric field can now be eliminated by storing muons with 'magic' $\gamma(\mu) = 29.3$, which eliminates the second term above and corresponds to a muon energy of 3.09 GeV/c. A measurement of ω_a and B now determines a_μ.

We now turn to the theoretical aspect of the contribution $g-2$. The Standard Model value of a_μ comes from all corrections from within the standard model of particle physics and can be written as

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{hadronic}) + a_\mu(\text{weak}), \qquad (4)$$

and as we have already stated, any discrepancy between the measured value and above value will indicate new physics. Since the electron $g-2$ agrees with the QED value to within a few parts per billion, it is used to determine α, which in turn gives us the value of the first term[2].

$$a_\mu(\text{QED}) = 116584706(3) \times 10^{-11} (0.025 \text{ ppm}). \quad (5)$$

The electro-weak contributions come from ref. 3 and are shown in Figure 2. Two loop corrections are significant and on including these and a value of m_H 150 GeV (the Higgs mass) one gets

$$a_\mu^{\text{EW}} = 151(4) \times 10^{-11}. \quad (6)$$

One should mention here that higher order leading logs are large and different theoretical techniques from standard perturbation theory, like all order resumation of these large logs have to be employed to obtain reliable results. The above error is quoted taking these factors into account.

Finally we come to the hadronic corrections. Starting at $O(\alpha^2)$, hadronic loop effects contribute to a_μ via vacuum polarization diagrams (see Figure 3). Unlike the QED and weak contributions which are well-described perturbatively, the hadronic contribution cannot be described purely by perturbative QCD but must be determined in part by using data from the cross-section for $e^+e^- \to$ hadrons in conjunction with other theoretical techniques like using a dispersion integral. The contribution from these diagrams has a long and very complex history which we cannot go into here. We refer the reader to ref. 4. This contribution provides the largest uncertainty in the theoretical evaluation of a_μ.

Higher order hadronic contributions come from three loop hadronic vacuum polarization diagrams (Figure 3) as well as hadronic light-by-light scattering (Figure 4). All these put together comprise the full hadronic contribution. A typical value quoted in the literature is

$$a_\mu^{\text{hadronic}} = 6739(67) \times 10^{-11}. \quad (7)$$

The complete Standard Model Prediction for a_μ is then

$$a_i^{\text{SM}} = a_i^{\text{QED}} + a_\mu^{\text{hadronic}} + a_\mu^{\text{EW}} = 116591596(67) \times 10^{-11}. \quad (8)$$

Notice, as expected, that the error within brackets (67) is essentially just that carried over from the hadronic contribution.

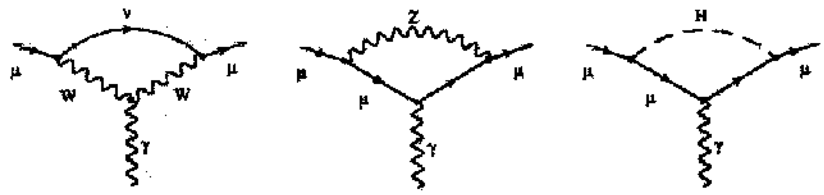


Figure 2. Electro-weak corrections to $(g-2)$.

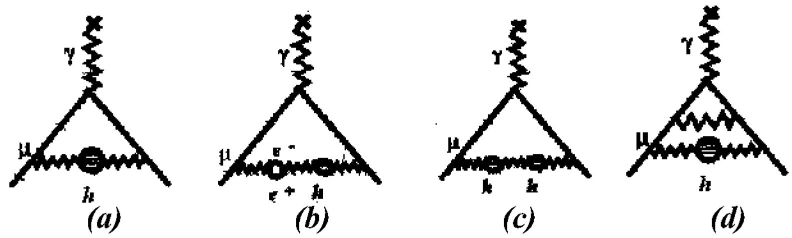


Figure 3. Higher order hadronic contributions.

Combining this with the present experimental average

$$a_\mu^{\text{exp}}(\text{average}) = 116592023(151) \times 10^{-11}, \quad (9)$$

gives

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 427 \pm 165 \times 10^{-11}, \quad (10)$$

which is a difference of 2.6σ. Depending on which calculation of the hadronic contribution is taken, the deviation from the experimental value will vary between 2 and 2.6σ. Therefore there is no possibility of the discrepancy being explained from 'standard physics'. A graphical plot of the experimental values of a_μ along with the standard model value is shown in Figure 5. E821 in the figure refers to the present experiment.

All aspects of a_μ (theory) are being heavily investigated. However, a large discrepancy of this kind implies that some new physics is needed for explanation. In what follows we will try to summarize some of the recent attempts in this direction. We will also point out towards the end that due to some very recent changes in the theoretical value itself, talk of new physics as described below may be somewhat premature.

There have been a whole slew of attempts to explain the origin of this discrepancy. They range from supersymmetry to compact extra dimensions, compositeness, technicolour, anomalous W couplings, new gauge bosons, leptoquarks and radiative muon masses. There is not enough space to discuss all these attempts and we shall instead aim to describe a few of the more popular scenarios.

Some of the most favoured explanations are in terms of supersymmetric theories. Supersymmetry (SUSY) is a symmetry that unifies bosons and fermions so that a SUSY transformation converts a boson into a fermion and vice versa. Put differently, bosons and fermions are considered to be different states of the same particle. Thus for every bosonic particle there is a fermionic partner and vice versa. Supersymmetric contributions to a_μ arise due to the existence of these sparticles – the supersymmetric partners of the usual particles – more specifically from smuon-neutralino and sneutrino-chargino loops (see Figure 6). Depending on sparticle masses and mixing and other parameters, the contribution of a_μ^{SUSY} can span a broad range of values. The details are a bit technical, but the attraction of SUSY lies in the fact that in the limit of large tanβ (which is a measure of the SUSY-breaking or more specifically, the ratio of the vacuum expectation values of two Higgs doublets)

$$|a_\mu^{\text{SUSY}}| \cong 130 \times 10^{-11} \left(\frac{100 \text{GeV}}{\tilde{m}} \right)^2 \tan\beta, \quad (11)$$

where $\tilde{m}$ is a typical SUSY loop mass. This gives us (assuming the full discrepancy is saturated by SUSY effects), for a range of tanβ in the of $4 \sim 40$, $\tilde{m} \cong 100 - 450$ GeV. This is precisely the range where SUSY particles are often expected. In addition, this range of values of $\tilde{m}$ implies that a whole range of new SUSY particles would be discovered soon, either at the Fermilab Run II of the Tevatron or at the LHC. There is now a considerable amount of literature on SUSY contributions to $g - 2$ and a summary may be found in refs 5 and 6.

Another attractive scenario is that of radiative muon mass models. The relatively light masses of the muon and most other fundamental fermions has given rise to the suggestion that they all arise out of loop corrections from 'new physics' beyond the standard model. This picture also provides an elegant solution to the flavour mass hierarchy issue, i.e. why fermion masses are so much smaller than the electroweak scale of 250 GeV. The interesting aspect is that without going into details of models of mass generation (like chiral symmetry breaking), the additional contribution to a_μ is quite generally given by

$$a_\mu(\text{newphysics}) \cong C\frac{m_\mu^2}{M^2}.$$

Here M is the physical high mass scale associated with the new physics and C is the model-dependent number of $O(1)$.

Other scenarios include anomalous W boson properties such as anomalous W boson magnetic dipole and electric quadrupole moments, and the existence of new gauge bosons by expanding the $SU(3)_c \times SU(2)_L \times U(1)_Y$ to a larger gauge group. Finally the possibility of contributions from extra space–time dimensions to $(g - 2)$ has also been considered. It has been found however (for a review see ref. 6) that with the present limits on the size of extra dimensions, the effects of a_μ, particularly compared to SUSY effects is very small. More details and a large list of references may be found in refs 5 and 6.

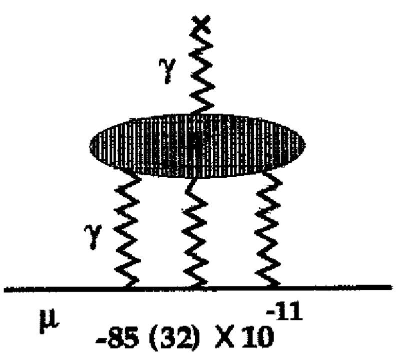


Figure 4. Hadronic light by light scattering.

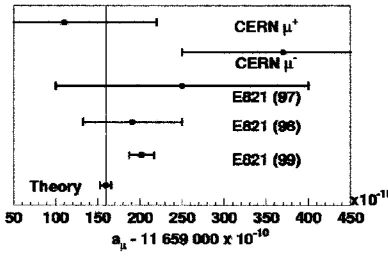


Figure 5. Various experimental determinations of $(g - 2)$.

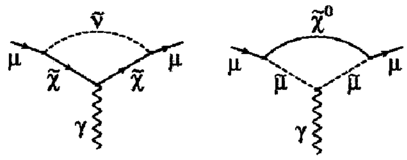


Figure 6. Typical SUSY contributions to $(g - 2)$.

It is important to stress here that these higher order QED calculations that are crucial for obtaining a reliable value of $g - 2$ are horrendously complicated and can only be done by using symbolic manipulation packages written specifically for such purposes, like FORM. Thus most of these results need to be checked by different groups before they can be considered reliable.

In the middle of the year 2001, renewed effort was made in recalculating the theoretical value of $(g - 2)$, concentrating in particular on the hadronic contribution. In November 2001, Knecht and Nyffler[7] recalculated one particular part of the hadronic light-by-light scattering contribution (known as the pion-pole contribution) and found it to be of the opposite sign from that calculated earlier by Kinoshita! This was followed by Kinoshita (along with Hayakawa)[8] redoing their earlier calculations and obtaining agreement with Knecht and Nyffler's value. Their earlier erroneous result had originated from an oversight of a feature of the algebraic manipulation program FORM mentioned above, and its definition of the Levi-Civita tensor $\varepsilon_{\mu\nu\rho\sigma}$. The net result of these changes in the value of the pion-pole contribution to the light-by-light scattering piece is that the older value

$$a_\mu^{L\,\text{by}\,L,\pi^0} = -5.7(0.3)\times 10^{-10},$$

is changed to

$$a_\mu^{L\,\text{by}\,L,\pi^0} = -5.8(1.0)\times 10^{-10},$$

(the Kinoshita value is 5.56). Thus the complete light-by-light scattering contribution changes from $-85(25) \times 10^{-11}$ to $89.6(15.4) \times 10^{-11}$. This *increases* the hadronic contribution appropriately, thereby reducing the discrepancy between theory and experiment to

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 25(16)\times 10^{-10}. \qquad (12)$$

which is a 1.6σ deviation – considerably reduced from the earlier 2.6σ discrepancy. Efforts are underway to check other parts of the hadronic contribution (though it is unlikely that there will be a substantial change in the value), but it is perhaps fair to say that there is no serious threat (for the moment!) to the Standard Model of High Energy Physics from the $(g - 2)$ experiment.

1. Brown, H. N., *Phys. Rev. Lett.*, 2001, **86**, 2227.
2. Kinoshita, T., *Rep. Prog. Phys.*, 1996, **59**, 1459.
3. For a summary, see Czarnecki, A. and Marciano, W. J., *Nucl. Phys. Proc. Suppl.*, 1999, **76**, 245–252 (hep-ph/9810512.
4. Marciano, W. J. and Lee Roberts, B., hep-ph/0105056.
5. Czarnecki, A. and Marciano, W. J., *Phys. Rev.*, 2001, **D64**, 013014 (hep-ph/0102122).
6. Pran Nath, Probe of SUSY and Extra Dimensions by the Brookhaven $g - 2$ Experiment, hep-ph/0105077.
7. Knecht, M. and Nyffler, A., Hadronic light-by-light corrections to the muon $g - 2$: the pion-pole contribution, hep-ph/0111058.
8. Hayakawa, M. and Kinoshita, T., Comment on the sign of the pseudoscalar pole contribution to the muon $g - 2$, hep-ph/0112102.

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