First Steps in Number Theory: A Primer on Divisibility. Shailesh Shirali. Universities Press (India) Ltd, 3-5-819, Hyderguda, Hyderabad 500 029. 2001. 161 pp. Price: Rs 215.

The Universities Press has to be congratulated for bringing out several popular books on science in general and mathematics in particular. The book under review is one more feather in its cap. The author is well-known in mathematical circles through his articles in *Resonance* and this book is written in the same spirit; elementary material presented in an interesting way.

Number theory as a branch of mathematics is the oldest and deals with properties of integers. The basic operations with integers are addition, subtraction, multiplication and division. While the first three lead again to an integer, the fourth one does not always do so. When an integer N (*dividend*) is divided by another integer m (*divisor*), there is a *quotient* q and a *remainder* r, so that $N = mq + r$ with $0 \le r < m$. When $r = 0$, we say that m is a divisor of N or N is divisible by m. And herein lies the beginnings of number theory. A positive integer N which is > 1 and has no divisors other than 1 and N is called a *prime number*. Examples of prime numbers are 2, 3, 5, 17, 29 . . . . Euclid proved that there are infinitely many primes. (The standard simple proof of this fact could have been included in chapter 1 for the benefit of the uninitiated.)

Chapter 2 introduces the basic notion of *congruences*. Two integers a and b are said to be *congruent* with respect to an integer m, written as $a \equiv b$ (*mod m*) if $a-b$ is divisible by m. In this case, they leave the same remainder when divided by m. This concept and its consequences on elementary operations can be used to devise tests of divisibility by certain integers. Though many of us have been taught how to test whether an integer is divisible by 2, 3, 4 or 9, we would hardly have stopped to think why it works and whether these tests can be extended to test divisibility by other integers. In chapters 3 and 4, the author develops a unified theoretical basis for devising such tests. For testing divisibility by a positive integer m, we first check if for some positive integer n, 10^n is divisible by m; i.e. $10^n \equiv 0$ (*mod m*). This works for $m = 2$, 5 or 10 with $n = 1$ and $m = 4$, 20, 25, 50 or 100 with $n = 2$. Then it is easy to show that a number N is divisible by 2, 5 or 10 if and only if its last digit is so divisible. Similarly, a number N is divisible by 4, 20, 25, 50 or 100 if and only if the integer formed by the last two digits of N is so divisible. These are described in chapter 3 in a leisurely fashion.

When the above relation does not hold for any n, the next best thing we can hope for is whether $10^n \equiv \pm 1$ (*mod m*) for some n. This holds for $m = 3$, 9 or 11 with $n = 1$ and for $m = 7$ or 13 with $n = 3$. Based on this, divisibility tests for these values of m can be developed; these are presented in chapter 4. For instance, a number N is divisible by 3 or 9 if and only if the sum of all its digits is divisible by 3 or 9 respectively. However this approach may not work for all divisors. Even when it works, the test of divisibility can be so cumbersome that it would be simpler to carry out the usual division and see if the remainder is zero. A case in point is the test for divisibility by 17 (see p. 49). A different approach to tests of divisibility is developed in chapter 5. In this approach the test of divisibility by 17 turns out to be much simpler.

All the above tests depended on the fact that numbers are represented using base 10. Tests of divisibility when the numbers are represented in other bases are discussed in chapter 6. These six chapters form almost half of this book which appropriately is subtitled 'A Primer on Divisibility'.

The other half of this book is a nice introduction to several basic theorems in number theory. Chapter 7 in 15 sections presents these and should be of interest to mathematical Olympiad candidates and advanced readers. Among the topics covered are Wilson's theorem, Fermat's little theorem, tests for primality (primeness) of a given number and Pell's equation (which was also discussed in detail by Bhaskara in the 12th century). I take time out for some comments on another topic: Pythagorean triples (p. 93).

A Pythagorean triple (a, b, c) is an ordered set of three positive integers such that $a^2 + b^2 = c^2$. As is well-known, there is a right-angled triangle with c as the length of the hypotenuse and a and b as the lengths of the two sides which include the right angle. Without loss of generality we may take a and b to have no common factors and a as even. In this case (a, b, c) is called a Primitive Pythagorean Triple (PPT). One method of constructing such PPTs is given in p. 94. It is mentioned that this procedure does not exhaust all possible PPTs. However, with a little effort, the following general method of constructing *all* PPTs could have been given. (This is taken from Hardy, G. H. and Wright, E. M.: *An Introduction to the Theory of Numbers* – Theorem 225.) Let x and y be two positive integers of opposite parity without common factors such that $x > y$. Taking $a = 2xy$, $b = x^2 - y^2$ and $c = x^2 + y^2$, we get (a, b, c) as a PPT and indeed all PPTs arise in this way. The method given in p. 94 corresponds to the case $y = 1$. The author proves the following interesting property of PPTs: If (a, b, c) is a PPT, then 60 divides the product abc.

Chapter 8 lists thirty-five problems which the (advanced) reader is invited to solve. Many of these are of mathematical Olympiad standard and make use of the properties of congruences. Solutions to all these problems are also given, making this book a completely self-contained introduction to number theory.

This book is a must for mathematical Olympiad aspirants; laypersons will find the material interesting and within their reach. Thus this book is highly recommended for everyone with some inclination towards mathematics.

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Explorations in Mathematics. A. A. Hattangadi. Universities Press (India) Ltd, 3-5-819 Hyderguda, Hyderabad 500 029. 2001. 222 pp. Price: Rs 195.

With computers so much a part of the educational scene today, a question asked quite frequently, in various circles, is about the implications of such technology to classroom-teaching. The question assumes particular relevance for mathematics teaching, when one becomes familiar with powerful symbol manipulation packages such as *Mathematica*, *Maple* and *Derive*. The book under review offers one possible way of incorporating computers into teaching. The author has chosen to work with the old and trusted language of *BASIC*, which while possessing a great many limitations, continues to be – at the school level – one of the simplest languages to learn. (For the purist, *PASCAL* with its modular structure would probably be the language of choice; there are many who feel that *BASIC* encourages a poor programming style.)

The focus of the book throughout is on numerical experimentation. (A more accurate title for the book might be *Computational Explorations in Mathematics*.) Starting with approaches to compute π, it goes on to explore the Fibonacci sequence, the golden ratio, Pythagorean triples, computations involving large numbers (for symbol manipulation packages this is no problem at all, they routinely churn out 500-digit numbers with barely a hiccup; but not so for a language such as *BASIC*), prime numbers, construction of calendars, biorhythm cycles, construction of tables of logarithms, the game of tic-tac-toe, base conversion (decimal to binary, octal, etc.), writing numbers in word-form (for writing cheques), linear Diophantine equations, the notorious '$3X + 1$' problem. Conway's enigmatic game of 'Life', and finally magic squares. The *BASIC* programs needed to do the investigations are all included.

Each chapter has a well-written preamble that introduces the topic, gives some sense of its history and current status, and places it in a setting; 'Magic Squares' has a particularly nice preamble. Proofs and mathematical analysis are for the most part avoided (though the author has not been able to resist from giving Euclid's proof for the infinitude of primes!); in each case the relevant theorems and formulae are simply presented as facts. Included here and there are puzzles that act as teasers; e.g. the one about bank deposits. In a few places the author poses a provocative question without letting out the answer; for example, he computes the partial sums of the reciprocals of the primes, then wonders whether the sums converge. In such cases he invites the reader to discover the facts by further exploration. In the chapter on Pythagorean triples, the snippet about the 'right-angled triangle in Tamil poetry' is unusual and interesting. And, of course, 'Life' is fascinating.

In a few areas the author could have included a bit more. For instance, in 'Diophantine Equations' he could have considered ways of solving Pell's equation too. In the chapter on 'Tic-tac-toe', he could have taken up the more challenging three-dimensional version of the game known as *QUBIC* (played on a $3 \times 3 \times 3$ cubical frame). In other chapters, themes could have been listed at the end for further exploration; for example, in the chapter on number bases, he could have posed questions like: 'For any positive integer n, let the sum

$$\left\lfloor \frac{n}{2} \right\rfloor + \left\lfloor \frac{n}{4} \right\rfloor + \left\lfloor \frac{n}{8} \right\rfloor + \left\lfloor \frac{n}{16} \right\rfloor + \ldots$$

be computed; what relationship does the sum bear to n?' Other such questions are readily envisaged.

A few minor errors have crept in. On p. 83 it is stated that Gauss proved the prime number theorem (this is the statement that as $x \to \infty$, the ratio

$$\frac{\text{number of primes between 1 and } x}{x/\ln x}$$

tends to 1). In fact, Gauss only conjectured the result (based on extensive empirical work); its proof came a full century later. On p. 62, following a reference to Fermat's Last Theorem, the statement that 'no one has been able to find a general proof' is contradicted a sentence later by the statement that the theorem was proved by Andrew Wiles in 1994. (The latter is of course the case.) On p. 207, a 4×4 magic square is displayed with Ramanujan's birth date (22, 12, 18, 87) forming the top row; it is described as having been given by Ramanujan himself, but this is doubtful. The magic squares studied by Ramanujan were of a very different kind. But on the whole the book is quite free of errors.

The question to ask is, who will use the book, and in what way? Ideally it could be used in mathematics/computer science clubs, at the middle/high-school level, where ideas for computational projects are often in short supply. With leading questions provided by a well-meaning teacher, a student could be started-off on promising trails. But at some stage the student would have to graduate to analysis and proof, and not be content only with empirical discovery.

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Optical Solitons: Theory and Experiments. Porsezian, K. and Kuriakose, V. C. (eds). Indian Academy of Sciences, Bangalore 560 080. 2001. 1164 pp. Price not mentioned.

The concept of soliton as a nonlinear, localized, solitary wave propagating without change in its properties is known in hydrodynamics since the 19th century. This property arises from the balance between dispersion and nonlinearity. These localized waves are stable against mutual collisions and retain their identities. This particle-like property justifies the name soliton analogous to the names phonon and photon representing vibrational wave quantum and optical wave quantum respectively. The invention of the laser in the 1960s opened up a new area of research known as quantum optics. This discovery stimulated the growth of new research fields such as nonlinear optics and nonlinear spectroscopy. Optics research using lasers comprises holography, optical information processing, optical communication and even optical computation. In particular, the nonlinear optics itself is of considerable interest both for theoreticians as well as for experimentalists because of its potential application in technology related to optical fibre communications. The concept of optical soliton and its potential use in fibre-optics communication of the future is the most important and interesting consequence of nonlinear optics. Indeed, the prospect of possible applications of solitons in telecommunications, pulse compression, optical switching, logic gates, etc. has prompted their experimental studies.

This book comprises articles reprinted from the special issue of *Pramana – Journal of Physics*, (November and December 2001, **57**, nos 5 and 6), devoted to 'Optical Solitons: Theory and Experiments'. Eminent scientists from India and abroad have contributed some rather readable articles discussing overall theoretical models and recent experimental studies of this exciting field of optical solitons. These articles cover a wide range of topics, such as temporal and spatial solitons, nonlinear optical materials, nonlinear Schrödinger equation and its higher order generalization, the concept of dark solitons, etc.

Solitons play a key role for lossless propagation through nonlinear optical fibres.