

BOOK REVIEWS

Einstein's Miraculous Year. John Stachel (ed.) Published by Scientia by arrangement with Princeton University Press, USA. 2001. 198 pp. Price: Rs 195.

It is significant that among the five path-breaking papers he wrote in 1905, Einstein regarded his paper on the quantum hypothesis as the 'most revolutionary'. In this paper he used ideas which he had developed over a long time and which became subjects of several of the other papers published later in the same year – ideas on statistical mechanics, relativity and Brownian motion. Einstein started thinking about the problem of black-body radiation soon after Planck's work and was never quite satisfied with Planck's derivation of the black-body formula. Applying the canonical ensemble and the resulting equipartition theorem to a collection of oscillators, he showed that one gets Rayleigh–Jeans law which agrees with Planck's distribution at low frequencies but cannot possibly be valid at high frequencies, since that would lead to an infinite total energy. He next showed that in the high frequency limit the entropy of monochromatic radiation has a form which is identical to an ordinary gas composed of statistically independent particles. This led him to the quantum hypothesis, viz. radiation at high frequencies is made of statistically independent indivisible quanta, each with energy proportional to the frequency. Matter and radiation can interact only through the exchange of these quanta. Everything then falls into place and phenomena related to Stokes' effect in fluorescence, ionization of gases by UV light and of course, the photoelectric effect follow after a few lines of algebra. In arriving at this conclusion he had used his intensive study of the foundations of statistical mechanics which he undertook in 1902–1904. At the same time he was thinking about the problem of Brownian motion and developing methods for calculating mean square fluctuations. He was also thinking of the principle of relativity, had abolished the ether, and had realized (though probably did not prove at that point) the equivalence of mass and energy. Light thus appeared to him as made of independent structures and he took the bold step of applying the canonical ensemble to radiation.

The above is an example of the kind of insight one gets by reading John

Stachel's valuable book *Einstein's Miraculous Year*. I had not read the paper on quantum hypothesis earlier and was therefore always under the impression that it was basically about the photoelectric effect. It was revealing to find out that the photoelectric effect was just one of the consequences of a line of profound thinking about the compatibility of radiation and thermodynamics.

Students of physics seldom read original papers once they become standard text-book material. In fact, it is very often hard to read these papers. The scientific language changes rapidly over the years, the mathematics sometimes changes and later expositions of great pieces of work often turn out to be better than the original papers. The five papers of Einstein reprinted in this book are striking exceptions to the general rule. All of these papers are very readable and it is a true revelation to read them.

Consider, for example, the paper on special relativity. Remarkably, this paper continues to be one of the best expositions of the fundamentals of the theory. It is in fact a thrilling experience to go through the logical steps which led Einstein to this seminal work. It has been generally known at the turn of the century that the presence of velocity-dependent forces and the wave theory of light implied that the principle of relativity has a basic conflict with the theory of electrodynamics. The commonly accepted resolution was Lorentz's microscopic theory which gave a special role to the 'ether' rest frame in which Maxwell's equations are valid. However, by 1904 Lorentz had invented the 'Lorentz transformations' for positions and time and for components of the electric and magnetic fields, so that Maxwell's equations are identical in all reference frames. This helped Lorentz to explain why the motion of the earth through the ether cannot be detected, as has been shown by the experiments of Michelson and Morley. Einstein took a completely new approach to the subject by taking the principle of relativity as a basic principle to be respected by all laws of physics. By his profound analysis of the meaning of simultaneity and the assumption of the constancy of the velocity of light, he could derive Lorentz transformations from a fundamental principle, independent of the specific model from which they first arose. All this is, of course, well known. It is nevertheless instructive

to read the paper and marvel at its clarity and depth.

The paper on Brownian motion is another example of clarity as well and it is interesting to realize how the generality of his approach to the problem of fluctuations was so intimately tied with his work on the quantum hypothesis.

What makes this book outstanding are the introductions written by Stachel to each of these papers. They are marvellous introductions which describe the main logical steps and provide the historical background in an integrated fashion. This provides valuable perspective. For example, the relationship of Einstein's work on relativity and quantum hypothesis to that of his predecessors is explained extremely well. I would specially mention the introduction which Stachel wrote for Einstein's paper on special relativity. One gets a rather clear idea about what was known before this paper and how Einstein's approach revolutionized our thinking. One also learns what Einstein did not contribute, e.g. the idea of integrated space-time which was the contribution of Minkowski. While the relation of Einstein's work with that of Lorentz is dealt in detail, one gets the impression that the same is not true about the relationship with the work of Poincaré. That would have certainly made the treatment more complete.

Finally, the book contains a rather thought-provoking preface by Roger Penrose. You might not agree with some of Penrose's own opinions, but it is nevertheless interesting to read them.

It is often thought that it is not necessary for scientists to study the history of science. In fact, most scientists do not. A look at this book will reveal how much one can gain by taking history seriously. The publication of an Indian edition of John Stachel's book is a very welcome step. I would consider this as essential reading for any serious physicist, mathematician and, of course, historian of science.

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Problems and Methods in Mathematical Physics. J. Elschner *et al.* Birkhäuser Verlag, P.O. Box 133, CH-4010, Basel, Switzerland. Vol. 12. 536 pp. Price: sFr198/DM 248.

This is a collection of articles dedicated to the memory of Siegfried Prösdorf. It is the Proceedings of the 11th TMP held at Chemnitz (Germany) from 25 to 28 March 1999. Published in the series 'Operator Theory, Advances and Applications'.

Siegfried Prösdorf has been called the most prolific mathematician that ever worked in the areas of integral equations and their numerical analysis. In more than 130 publications, among them five books and numerous survey articles, he dealt with problems in operator theory, in the theory and numerical analysis of integral equations, pseudodifferential equations and boundary value problems, in approximation theory and boundary element methods. It is only appropriate that the memorial volume should contain contributions in these areas.

It is difficult to compartmentalize these contributions in any one of the above areas. A broad classification would be as follows:

Singular integral operators and their approximate solution – Böttcher, Karlovich and Rabinovich study the algebra generated by Cauchy singular operators and the operator of complex conjugation on a weighted Lebesgue space. The method of approach is based on transforming the operator locally into Mellin pseudodifferential operators. Laurita and Mastroianni re-examine a quadrature method for Cauchy singular integral equations with constant coefficients. They construct a polynomial approximation of the system and give estimates in mean weighted norm of the error. Grigorieff and Sloan apply the quolocation and collocation method to index-zero singular integral equations with piece-wise continuous coefficients using continuous splines defined on a quasi-uniform mesh. Junghans and Mastroianni apply a Banach algebra technique to the investigation of the stability of collocation method with respect to the Chebyshev nodes of the second kind for an approximate solution of a Cauchy singular integral equation.

Wavelets – Bourgeois and Nicaise use the biorthogonal wavelet approximation method for the heat equation in its integral formulation with Dirichlet and Neumann boundary conditions. The unknown solutions of the integral equations belong to anisotropic Sobolev spaces and are approximated by the Galerkin method using an appropriate wavelet basis. Harbrecht and Schneider examine the implementation for the wavelet Galerkin scheme in two dimensions using biorthogonal wavelets. They develop an optimal algorithm for the computation. Rathsfeld considers the problem of a smooth boundary surface of a three-dimensional domain and the space of piece-wise linear functions defined over a uniform triangular grid. A wavelet basis is introduced which is a variant of the well-known three-point hierarchical basis. These wavelets can also be used for finite element methods. Prestin and Selig give a particular class of orthogonal trigonometric Schauder bases for $C_{2\pi}$ by periodic wave packet functions. These bases are of minimal growth of the polynomial degree. The corresponding approximation error is asymptotically optimal.

Riemann–Hilbert problem – Bergher and Dai study the solvability of the Riemann–Hilbert for a singular Venkua system. The number of continuous solutions is shown to depend not only on the index, but also on the location and type of the singularities. Efendiev and Wendland introduce orientable and non-orientable Riemann–Hilbert problems. The number of connected components in the two cases differ significantly. The degree of quasilinear Fredholm maps is used to prove global existence of solutions.

Pseudodifferential and other operators – Gorenfeld and Mianardi present three random walk models, discrete in space and time for the symmetric case of space-fractional diffusion processes. For properly scaled transition to vanishing space and time steps, these models converge to the corresponding time-parametrized stable probability distribution. Prösdorf and Yamamoto consider an ill-posed linear compact operator equation in a Banach space. They adopt a discretization of the Lavrent'ev regularization to reconstruct the solution. Gohberg and Krupnik establish a connection between the determinant of a polynomial operator pencil and the characteristic numbers of this pencil. They give three

examples from trace class and nuclear operators.

Problems arising out of physical phenomena – Plato investigates a parameter estimation problem with noisy data, which arises as an inverse problem in ground filtration. In appropriate Hilbert spaces, the problems can be formulated as a linear, non-compact, ill-posed problem with a model perturbation that can be estimated only at the solution of the problem. von Wolfersdorf studies the plane potential flow of an inviscid incompressible fluid around and through a circular cylinder of porous material. It reduces to a nonlinear boundary value problem of Poincaré type and leads to an infinite system of algebraic equations and a related nonlinear integral equation. Vainikko constructs a fast solver for the generalized airfoil equation, which is periodized with the help of cosine transforms. The fast solver is constructed on the basis of a fully discrete version of trigonometric collocation method with product integration. Kravchenko gives a new approach to studying Dirac equations with potentials and Maxwell's system. It is based on the possibility of reformulation of these equations in terms of complex quaternions. Natroshvili and Tediashvili study a direct mixed-type boundary value problem for a generalized Helmholtz equation, when in the sound-soft part the Dirichlet condition is given, while in the sound-hard part of the boundary, the Neumann condition is prescribed. Elschner and Schmidt examine problems of diffraction arising in optimal design of binary gratings. They obtain the form of the derivative of the reflection and transmission coefficients.

Approximation theory – Hackbusch and Khoromskij develop a class of matrices with improved data sparsity to approximate elliptic operators in one, two and three-dimensions. In BEM applications, this reduces the order of expansion. Grudsky establishes theorems about the representation of functions with given asymptotics of the argument in the neighbourhood of a discontinuity in the form of a Blaschke product. A theory of normal solvability for Toeplitz operators on the unit circle whose symbols have oscillating discontinuities, is constructed. Luther and Mastroianni show that the result regarding Fourier projections in the space of all 2π -