Mathematics of the 19th Century. A. N. Kolmogorov and A. P. Yushkevich (eds). Birkhäuser Verlag, P. O. Box 133, CH-4010, Basel, Switzerland. 2001. 328 pp. Price: sFr 128/DM 170.

The book under review, edited by mathematicians A. N. Kolmogorov and A. P. Yushkevich, is part of a series of volumes on the History of Mathematics. Three volumes covered the history prior to the nineteenth century and four volumes, out of which three have appeared, were assigned for the nineteenth century. The present volume deals with developments in mathematical logic, algebra, number theory and probability theory.

The nineteenth century was one of dramatic developments in mathematics, especially in algebra and number theory. Pre-nineteenth century algebra was mainly concerned with the solution of polynomial equations. Methods of solution of the solution of the quadratic, the cubic and the quartic equations were known, and efforts were on to solve the quintic equation. Solvability here means solution by radicals using only rational operations and roots. The fundamental theorem of algebra (FTA) had been proposed and algebraic proofs, notably by Euler, Lagrange and Laplace existed. However, all the algebraic proofs were incomplete, since they assumed the existence of roots and went on to assign properties to them. Gauss gave four proofs of the theorem in his lifetime. His second proof (1816) is almost totally algebraic and is considered by many to be the first rigorous proof of the FTA. Abel (1824–1826) gave the first clear proof of the unsolvability of the general quintic equation. Galois (1831) then showed that a polynomial equation of arbitrary degree is solvable, iff its Galois group (group of permutations of the roots which leave rational functions of the roots unaltered) is solvable. The word group was first used by Galois though Langrange, Van der Monde, Gauss, etc. had used group concepts in some way or the other. The notion of groups is one of the most important unifying ideas in mathematics as it draws together a wide range of structures in which the idea of a combination or product exists. Number theory, geometry and the theory of equations, all have had a decisive influence on group theory. Cayley was the first to consider abstract groups and also the first to give a definition of the group equivalent to modern definitions. Further great discoveries in group theory are associated with the name of Camille Jordan whose treatise also served as a textbook on group theory as well as Galois theory. In the latter part of the nineteenth century, ideas and methods that came into being in the first third of the century became current in mathematics. The main ones were: the idea of group and its invariants, the notions of field, module, ring and ideal, and the notions and apparatus of linear algebra. Algebra, thus made a transition from the old algebra of the solution of equations to modern abstract algebra.

Number theory (along with geometry) is the oldest branch of mathematics. The Greeks, Indians and Chinese had all made significant advances in number theory – the fundamental theorem of arithmetic, the theorem on the infinitude of primes, the Chinese remainder theorem, solution of Pell's equation, had all been discovered by them. Further significant progress was made in the seventeenth and eighteenth centuries, notably by Fermat, Euler, Lagrange and Legendre. Despite this, number theory was considered merely a sophisticated pastime for intelligent minds, and not a branch of mathematics to be studied seriously. The nineteenth century changed this radically. This was mainly because tools and methods from other disciplines such as algebra, geometry and analysis began to be extensively used in throwing light on number theory. An important milestone was the publication in 1804 of Gauss' *Disquisitiones Arithmeticae*, when Gauss was 24 years old. The first four sections of this book essentially tighten the work of the eighteenth century. The concept of congruences is introduced for the first time and two proofs of the quadratic reciprocity theorem are given. Gauss went on to give eight proofs of this theorem which he called the gem of arithmetic. The fifth and sixth sections deal with binary quadratic forms (expressions of the type $ax^2 + 2bxy + cy^2$), higher-order congruences and expand the notion of integer to include Gaussian integers (of the type $a + b\sqrt{-1}$, where a and b are integers). These integers satisfy all the properties that rational integers do. Further generalization by Dirichlet, Kummer, Eisenstein, Hermite and Kronecker in the 1840s and 1850s yielded the concept of an algebraic integer. Algebraic integers are roots of monic polynomials with integer (rational) coefficients.

Algebraic number theory was successful in throwing light on ordinary number theory. At the same time it also developed a life of its own, its concepts being used in the 20th century in the abstract theories of rings, fields and vector spaces. Another development contributing to major advances in number theory was the use of techniques from other areas, notably analysis in solving problems in number theory. Dirichlet, for example, proved the difficult theorem that every infinite arithmetic progression of the type $a + bn$ where a and b are coprime, has an infinity of primes using analysis. He also was the first to prove Bertrand's postulate, which states that there exists a prime between the integers n and $2n - 2$. Some of the other important developments were in the proof of asymptotic theorems, such as the prime number theorem which states that the limit of the number of primes less than n as n tends to infinity is $n/\log n$. The century also saw the birth of the theory of transcendental numbers – numbers that cannot be expressed as roots of any polynomial with rational coefficients.

Mathematical logic developed in the nineteenth century, primarily in the form of logical algebra. The 'formal logic' of A. De Morgan and Boolean algebra were some of the important developments of this century. Probability theory, similarly, made major advances, such as in the development of asymptotic theorems – various forms of the law of large numbers, the central limit theorem, etc. The theory of errors was constructed and the fields of population and mathematical statistics were born.

The authors have done a commendable job in trying to describe all the developments with a fair amount of detail. Out of necessity, however, there are gaps left in the treatment that the reader has to fill in herself. I, therefore, feel the book will be a bit too advanced for the lay reader. Here, it should be noted that the book is not a history of mathematicians, rather it is a history of the developments in mathematics. Lay readers could more profitably read a book such as E. T. Bell's classic *Men of Mathematics*. Undergraduate students majoring in mathematics or graduate students in mathematics who are interested in discovering how mathematics evolved will find this book interesting and enlightening. It will be a valuable reference material for students taking courses on the history of mathematics. All in all, a worthwhile addition to mathematical libraries.

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Mysterious Motions and other Intriguing Phenomena in Physics. G. S. Ranganath. Universities Press India Ltd, 3-5-819. Hyderguda, Hyderabad 500 029. 2001. 149 pp. Price: Rs 195.

Matter and motion are two of the most significant factors of the universe. Starting from macroscopic objects like the galaxies to microscopic entities like the atoms, everywhere the presence of matter and motion is seen. The motions exhibited by the material entities are the subject of man's quest in understanding. The entire domain of science in general and physics in particular, deals with the detailed study and understanding of the motions. A large variety of motions fall under the direct focus of the basic laws of physics. However in many situations, we come across motions which do not apparently fall under the normal laws of physics. They appear mysterious, somewhat paradoxical and often appear to be exceptions to the basic laws. In this remarkably lucid presentation, Ranganath clearly points out how the mysterious motions can be understood by a proper scientific application of the established postulates of physics.

The mysterious motions described in the book belong to a variety of situations. Why was solar probe *Ulysses* launched towards Jupiter, although the objective was for it to circle around the sun? Why do tides not occur when the moon is directly above or below the sea? What are chaotic motions? Are they repeatable or not repeatable? What are the unusual features in the flow of grains like sand grains? Why does the Tippe-top toy jump upside down while spinning? Similar questions come in a variety of other areas like elasticity, motions involving heat, light, oscillations and electrodynamics. Ranganath illustrates the mysterious motions encountered in all these areas and brings out the scientific principles involved.

The style of the book is clear and lucid. It is a reader-friendly approach that the author has employed with practically no mathematical equations, but still keeping faithful representations of scientific principles and their applications. The objective of the book is to attract the young students into many classical domains of physics. But, it does much more than that. It illustrates many physics principles with applications which are not often cited.

I would unhesitatingly recommend the book to all students of physics, whether undergraduate student or a research student and also to all practising physicists, whether physics teacher or researcher who wants to enjoy physics.

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Origin of Inertia – Extended Mach's Principle and Cosmological Consequences. Amitabha Ghosh. Affiliated East-West Press Pvt Ltd, 105 Nirmal Tower, 26 Barakhamba Road, New Delhi 110 001. 147 pp. Price: Rs 135.

The idea that 'inertial properties of an object are determined by the presence and distribution of mass–energy throughout all space' is one way of expressing the principle given by Ernst Mach, the Austrian philosopher–scientist. Einstein thought that the general theory of relativity (GTR) given by him in 1915 incorporated this principle. However, the first solution of the theory discovered by Schwarzschild in 1916, described an isolated massive object which seemed to violate Mach's principle. Wheeler used the Mach's principle as a boundary or initial condition to rule out single mass solutions, unless they are embedded in the rest of the universe. Many others, including Hoyle and Narlikar, have attempted to incorporate the Mach's principle in their theories.

Among various proposals, there is one due to D. W. Sciama, given in 1953, which adds another term to the well-known Newtonian expression for gravitational attraction between two massive objects. This additional term is proportional to the acceleration between the objects. When the interaction between one object and the rest of the universe is considered, we have to integrate over all the masses in the universe. The normal term vanishes as matter is distributed symmetrically all around, while the new term makes a contribution proportional to the acceleration (which breaks the symmetry). This acceleration is with respect to the mean rest frame of the universe. It gives rise to an expression similar to the second law of Newton. As the term is due to gravitational mass, its equivalence to the inertial mass term in the second law of Newton follows in a natural fashion. Sciama had also discussed the possibility of a term dependent on velocity, but did not pursue it. The author of the book under review, Amitabha Ghosh, postulates a velocity-dependent force in the gravitational interaction of two particles and discusses its consequences in some astrophysical and cosmological contexts. Ghosh who teaches at IIT Kanpur, has also served as the Director of IIT, Kharagpur.

The main effect of the new term introduced is to slow and stop motion and thus act as a drag force. The author seems to have a fascination for the Aristotilean idea that motion must decrease, and the object must come to a stop in course of time. The Galilean idea incorporated in the first law of Newton, postulating non-stop uniform velocity, is certainly non intutive and many students in schools have lots of problems getting used to the idea.

Before going into some of the details presented in the book, I have to express my unhappiness with total absence in the book of a discussion of broader frameworks and principles of physics like Lorentz invariance and general relativity. It is unclear how the modification of Newton's law of gravitation suggested in the book, will fit into the time-tested framework of physics. The postulated term is proportional to square of the velocity. One might have thought that this is the first post-Newtonian correction of the