Geometry: Our Cultural Heritage. Audun Holme. Springer-Verlag, GmbH & Co. KG, D-69126 Heidelberg, Germany. 2002. 378 pp. Price not mentioned.

As the author states in the introduction, the book under review is not intended to be a historical account of geometry, though the title may suggest it to be so. It has two parts, entitled 'A cultural heritage' and 'Introduction to geometry', and while the second part deals with certain modern themes in geometry, the first part is stated to be aimed at seeking out the roots of these themes.

There is a growing feeling that mathematics should be taught using a historical cultural perspective, consciously as a pedagogical tool. In this context several books have come out (e.g. *The Historical Development of the Calculus* by C. H. Edwards, *Mathematics and its History* by J. Stillwell, etc.) presenting a blend of mathematical topics along with their history. The present book belongs to the same genre, though it is somewhat different in structure.

It is of course no surprise that the endeavour of exploring the roots leads the author to Greek geometry of the yore. The author gives a delightful account of the achievements of that period of ancient history, with the enthusiasm of a bard. Starting with the geometry of Thales and Pythagoras the author traces in chapter 3 the developments such as the discovery of irrational numbers, the attempts at the famous classical problems of squaring the circle, doubling the cube, and trisecting angles, study of Platonic solids, etc., interspersed with various anecdotes. The presentation gives good insight into the concepts as they evolved.

Here is a tidbit from this chapter that feminist readers (not necessarily female) may relish. According to some accounts Pythagoras had a wife, named Theano, who was also a mathematician. She is the earliest known woman mathematician and is likely to have been the discoverer of irrational numbers. Their three daughters also belonged to the Pythagorean school, and two of them are credited with several works.

The account of various attempts at the classical trinity of problems should be useful to students and teachers alike. Though the problems are widely known, it is often not realized that the Greeks also had interesting 'solutions' to them, if the condition to use only (unmarked) straight-edge and compass is relaxed.

The story of geometry in the later Helenistic era, covered in chapter 4, is also fascinating. The great names of Euclid, Archimedes and Apollonius belong to this period. Various works of these and other less-known geometers are described, giving inspiring background details. It is interesting to learn, for instance, that Archimedes first found his favourite formula on the ratio of volumes of a cylinder and the inscribed sphere experimentally, using the principle of equilibrium. Of course he did not consider the verification a proof of the statement, but maintained that it facilitated finding a mathematical proof.

The author also devotes some pages, chapters 1 and 2, to the geometry of more ancient cultures. The treatment, however, is sketchy and restricted largely to Egyptian and Babylonian cultures. While discussing the Pythagoras theorem the author mentions that it is also contained in the Sulvasutras; however his discussion on what the 'typical argument there' would be, seems faulty and alien to the Sulvasutras.

Chapter 5 is the story of the re-awakening, following the so-called dark ages; some history through the dark ages is also sketched. In the 15th and 16th centuries powerful new concepts and techniques were added to Greek geometry. Desargues and Pascal ushered in the basic ideas of projective geometry. Descartes introduced algebraic techniques into geometry in a decisive way, via coordinate systems, thus revolutionizing the field. After discussing these developments, the author describes some basic linear algebra which brings strength to the method.

The 'Introduction to geometry' in the second part deals with axiomatic non-Euclidean geometry, projective geometry and some basic algebraic geometry. Modern geometry has branched off into a multitude of streams and of course one cannot expect to get even a flavour of many of them in a normal-sized book, but it would have been nice to see along with the above topics some others reflecting the impact of calculus on geometry. Except for this, the selection of topics is interesting.

Availing of the context of non-Euclidean geometries the author introduces the reader, in chapter 8, to various foundational issues in mathematics, that are not generally dealt with in similar sources. The idea of axiomatic theories and models for them is developed, and employed in the following chapters (9 and 10), where projective geometries and non-Euclidean geometries are discussed. The exposition in these chapters would be illuminating especially for a beginner. In chapter 12 the author presents simple algebraic proofs of the theorems of Desargues, Pappus and Pascal, illustrating the flexibility of algebraic techniques. Curves of higher degree in the affine and projective planes are discussed in chapters 13 and 14. Various basic notions such as singularities, tangency, asymptotes, projective equivalence, etc. have been presented in an instructive manner.

In chapter 16 the author reverts to the classical problems and presents proofs of the impossibility of trisecting arbitrary angles, doubling the cube and squaring the circle, by constructions involving only ruler and compass; (for the last one of course the author depends upon Lindemann's theorem on transcendence of π, which is recalled without proof). This is a topic not readily found in books, and the detailed treatment given here should be welcomed. The basic theory of field extensions, etc. involved in this is covered in chapter 15. The book concludes with a brief introduction to fractals and catastrophy theory in the last two chapters.

The book is profusely illustrated with figures (disappointingly, there are no photographs of mathematicians or mathematical artefacts though!). There are a good number of references listed at the end, including many to websites, and referred to in the text. However, at quite a few places one is still left wondering about the source of the historical statements made. I also feel that the division of the book into the two parts is artificial. A structure with a more homogeneous blending would have been more desirable. Among other drawbacks is the somewhat poor sentence construction at many places, mainly in the early part of the book. There are also many typographical errors. However these shortcomings are relatively minor, and the book is indeed a good addition on the scene.

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