Probability and Statistics. Echoes from *Resonance* Series. Mohan Delampady, T. Krishnan and S. Ramasubramanian (eds). Indian Academy of Sciences, Bangalore and Universities Press, 3-5-819 Hyderguda, Hyderabad 500 029. 2001. 201 pp. Price: Rs 190.

Probability theory had its birth in 1654 through a correspondence between Blaise Pascal and Pierre Fermat concerning a problem of points in a game of chance raised by a gambler, Chevalier de Mere. It was through this analysis that Pascal arrived at his algorithm for tabulating the binomial coefficients in the form

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1   1   1   1   1 . . . . . .
  ↗   ↗   ↗   ↗
1   2   3   4   5 . . . . . .
  ↗   ↗   ↗
1   3   6   10   15 . . . . . .
  ↗   ↗
1   4   10   20   35 . . . . . .
  ↗
1   5   15   35   70 . . . . . .
⋮   ⋮   ⋮    ⋮    ⋮
⋮   ⋮   ⋮    ⋮    ⋮
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where the binomial coefficients in the expansion of $(x + y)^n$ constitute the entries along the nth diagonal. The subject assumed the statistical character of inductive inference with the discovery by Jacob Bernoulli of the weak law of large numbers for an increasing number of binomial trials (called nowadays as Bernoulli trials) and its application to jurisprudence. His book *Ars conjectandi* (the art of inductive inference) was posthumously published in 1703. The next great development came from Abraham De Moivre in 1733, when he established that the distribution of the number of successes in n Bernoulli trials, when suitably normalized, approaches, as $n \to \infty$, the standard normal distribution with its bell-shaped density function

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}.$$

A proof of this result in full detail is presented in article 4 of this volume under review. This demonstration together with the advancements in analysis through the contributions of Newton, Taylor, Stirling and Euler brings out, rather remarkably, the close relationship among the twin magical numbers e and π of mathematics and the fast growing $n! = \Gamma(n + 1)$ in the following glorious identity:

$$\log \quad (n) = \left(n - \frac{1}{2}\right)\log n - n + \frac{1}{2}\log 2\pi + \sum_{k=1}^{\infty} \frac{B_{2k}}{2k(2k-1)n^{2k-1}},$$

where $\Gamma(n)$ denotes the gamma function and $\{B_k\}$ is the sequence of Bernoulli numbers determined by the power series expansion

$$\frac{x}{e^x - 1} = \sum_{k=0}^{\infty} \frac{B_k}{k!} x^k.$$

(One has $B_0 = 1$, $B_1 = -\frac{1}{2}$, $B_k = 0$ for odd $k \geq 3$, $B_4 = -\frac{1}{30}$, $B_6 = \frac{1}{42}, \cdots$).

As a corollary one obtains

$$n! = \sqrt{2\pi}\, e^{-n}\, n^{n+\frac{1}{2}} \times \exp\left(\frac{1}{12n} - \frac{1}{360n^3} + O\left(\frac{1}{n^5}\right)\right),$$

a refined version of Stirling's formula. This accounts for the appearance of the numbers e and π in several mathematical and statistical models.

Laplace, Gauss and some astronomers arrived at the same normal distribution through a theory of errors. This was later consolidated in the form of the central limit theorem of probability theory by Liapunov around the year 1900. According to this theorem, under very broad conditions, the distribution of a 'large number of independent 'small' random quantities can be safely approximated by the normal distribution. Around the same year, Karl Pearson discovered the famous χ^2-test by which the validity of theoretical statistical models could be tested on the basis of experimental data. The central limit theorem and the χ^2-test constituted the foundations of statistical practice for nearly a century till the advent of the modern computer revolution.

Around 1920, while analysing the patterns of sequences of vowels and consonants in Alexander Pushkin's great poetic work *Eugene Onegin*, the Russian probabilist Markov stumbled on the idea of a transition probability matrix and the notion of a Markov chain. And today, there is no branch of science which is not influenced by developments in the theory of Markov processes.

Einstein's 1905 paper on the theory of Brownian movement, Wiener's (1923) construction of the standard Brownian motion as a probability measure on the infinite dimensional space of continuous trajectories and Kolmogorov's (1933) famous little book *Foundations of Probability*, have strongly influenced the growth of probability theory as a major branch of modern mathematics.

Most of the developments indicated above do find their echoes in this collection of twenty-five articles compiled by the editors from the issues of *Resonance* during the years 1996–98. The volume opens with an essay on the axioms of probability theory, with an application of the binomial distribution to a problem in artillery and then proceeds to another on the relationship between 'randomness and probability'. There is a detailed discussion of the Buffon's needle problem. This, already suggests the possibility of evaluating π by dropping needles at random on ruled paper. Apart from its applications to statistics, modern probability theory has developed a new feature in the form of stochastic algorithms. Article 16 presents a delightful account of this aspect in the form of an algorithm called 'simulated annealing', where the travelling-salesman problem and the notion of a Markov chain have a fruitful meeting together. On the statistical side, some of the articles appear weak and some models do not appear convincing enough. There are essays on the connection between Markov chains and electrical networks, algorithmic complexity in the sense of Kolmogorov, contributions of Kolmogorov to probability and of Fisher to statistics and mathematical genetics. As proclaimed on the cover page, this little volume can, indeed, be used by students of mathematics and statistics to complement what they learn in their examination-oriented courses.

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