

## BOOK REVIEWS

**Modern Algebra and the Rise of Mathematical Structures.** Leo Corry. Birkhauser Verlag, PO Box 133, CH-4010 Basel, Switzerland. 2004. 2nd revised edition. 431 pp. Price: EUR 65.

‘How many things by season seasoned are to their right praise and true perfection!’

exclaims the Bard and quite aptly too. Subjects require their time to take shape and evolve. For instance, algebra, which began as a mere servant to the number system, with the operations  $+$ ,  $\times$  and their inversions, has come a long way to become an impressive doctrine over the years. Its growth has indeed been phenomenal, thanks to the various brilliant mathematicians who moulded it.

In the book under review, the author deals with a particular aspect of the growth of algebra and gives an account of some of its developments, beginning with Dedekind’s theory of ideals and ending with category theory (a brainchild of Eilenberg–MacLane during the mid twentieth century). The author mentions in his preface that his original intention was to give only an account of the history of category theory. However, he decided later to lead up to it by discussing some of the contributions of mathematicians like Dedekind, Hilbert, Emmy Noether, Steinitz, Fraenkel and others to algebra. What started in the nineteenth century as a tool in dealing with polynomial equations (the great names of Lagrange, Abel, Ruffini, Galois, ... naturally come to one’s mind in this connection), finally emerged as an autonomous structural subject in 1930 in the pioneering work, *Moderne Algebra*, of Van der Waerden (this book itself was heavily influenced by the ideas of Emmy Noether and Emil Artin). This account occupies the first part of the book under review. The second part deals with the influence of the earlier period on the work of Oysten Ore, Bourbaki and finally on the work of Eilenberg–MacLane on category theory. This, in brief, is a summary of the contents of this book.

Apart from providing a historical account described above, the author enunciates and tries to uphold a philosophical tenet relating to the general growth of algebra (more generally of mathematics) as a structure. While doing so, he postulates two separate domains of discourse of the

subject, namely ‘the body of knowledge’ and ‘the image of knowledge’. He claims that this image is possible in mathematics in view of the ‘reflexive’ nature of the subject. (By the reflexive character of mathematics, the author means ‘the possibility of formulating and proving meta-statements about the discipline of mathematics, from within the body of mathematical knowledge’.) This reflexive nature is nothing special to mathematics and is equally valid for other scientific disciplines too. The heavy weather that the author makes, throughout the book, harping on the twofold distinction between the body and image of knowledge, appears artificial, and somewhat forced.

The author’s predilection with his philosophical tenet apart, one could go through the book as a piece of history. The reviewer discovered a number of mathematical inaccuracies, some grammatically and semantically queer constructions and a fair amount of typographical errors. Listed below are a few instances of the mathematical inaccuracies, which are especially mentioned, since they might present a wrong notion to the uninitiated.

Unique factorization is said to hold for all ideals ‘with given multiplicities’ (p. 97).

‘Polynomial’ should be replaced by ‘irreducible polynomial’ (p. 99, line 3 from top).

One should be careful to distinguish between the zero and the non-zero prime ideals.

There is a mention of non-existence of distributivity and zero element for a group (p. 188).

There is a mention of a construction of a field of quotients out of two rings (p. 191).

The definition of algebraic closure of a field is garbled and wrong as stated (p. 195).

‘Irrationally equivalent’ should be replaced by ‘birationally equivalent’ (p. 256).

The author seems to believe that objects in a category must have elements (p. 342).

The author seems to imply that a morphism in a category that is both monic and epic is an isomorphism, which is generally false (p. 343).

In footnote 6 on p. 347, the author talks about the group of characters of a topological space. He also seems to believe that *any* group is isomorphic to its character group.

In the text, the forgetful functor from **Groups** to **Sets** goes in the wrong direction (p. 396).

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**The Tests of Time – Readings in the Development of Physical Theory.** L. M. Dolling, A. F. Gianelli and G. N. Statile (eds). Princeton University Press, 41, William Street, Princeton, NJ 08540, USA. 2003. 716 pp. Price: US\$ 36.95.

There are theories and theories, all of them constructed with the purpose of explaining the physical world around us. There have been *ad hoc* theories that describe a limited set of data, models to fit observations or, on the other hand, superstructures that support entire areas of phenomena possessing prodigious powers of explanation and prediction. While some theories, like sand castles built on the seashore, have been swept away by the turning tide, others have proved to be monumental edifices that have stood the test of time. It is the latter that form the focal point of the present volume. It traces the evolution of five such theories by presenting excerpts from the writings of the pioneers who created them. These are the heliocentric theory, the electromagnetic theory, theory of relativity, quantum theory, and the big bang theory of the Universe. All these represent revelations and revolutions in our understanding of nature. In the course of time, some of them may have to be modified or may even be superseded by radically different theories. Nonetheless, the basic tenets of these theories are bound to be retained despite new developments that may occur in the future. In this sense, they will never become extinct.

Obviously, one cannot and should not try to read a tome of some seven hundred pages from cover to cover at a stretch. Furthermore, familiarity with the basic elements of the subjects dealt with is essential for the true appreciation of the

material contained in the volume, a prerequisite that is easily satisfied. All of us know to varying degrees the theories underlying our knowledge of the physical world. But the beauty and the importance of the book lie in the fact that it offers glimpses into the original ideas and the writings of the thinkers who forged and fashioned those theories. Take for instance the heliocentric model, which even a child would have learnt in school. Unfortunately, we are hardly ever introduced to the excitement and the overwhelming significance of the passage from the geocentric model to the heliocentric theory, from Ptolemy to Copernicus, from appearance to reality. The volume helps us in following this great transition, which literally defined the word *revolution*, by listening to the giants themselves, the likes of Aristotle, Ptolemy, Copernicus, Tycho Brahe, Galileo and Kepler. Short readings these glimpses are, but they offer penetrating bursts of originality. Thus the stage is set for the entrance of the greatest scientist of all time, Isaac Newton. His *Principia* explained practically everything that was known at his time. And it heralded new methods and directions that were to become the bases for all theories to follow. There are extraordinary ideas contained in the *Principia*, which, unfortunately, are not reflected in the excerpt reproduced in the volume. This is but a minor shortcoming.

Then comes the electromagnetic theory described through the writings of a number of authors who portray the historical development as well as the final formulation by Faraday and Maxwell. Whereas Newtonian gravity ushered in the notion of universal laws, electromagnetic theory brought to the fore the idea of unification of two apparently independent forces, namely the electric and the

magnetic. These two basic tenets, the universality of the fundamental laws of nature and the unification of forces, have become the guiding principles of modern theoretical approaches to the foundations of physics. Electromagnetism, its inconsistencies rather than its triumphs as it was interpreted in the nineteenth century, led Einstein to formulate his special theory of relativity enshrined in his 1905 paper titled 'On the Electrodynamics of Moving Bodies', with no mention of relativity at all! Once again, Einstein was troubled by the restrictive nature of the special theory, which was confined to inertial observers. A decade of relentless efforts, contemplation at the deepest level and the introduction of curved space-time, not as a mere arena for the physical phenomena to enact their drama, but as one of the characters identified with gravitation actively participating in it, led to the general theory of relativity, a creation of supreme genius. General relativity, a theory of gravitation radically different from that of Newton, has shown that there are more things in heaven and earth than had been dreamt of before. It is a joy to read the contributions of various outstanding physicists to the two theories of relativity, foremost among them being the creator of relativity himself.

'God does not play dice', declared Einstein although, ironically enough, he was one of those who framed the rules of the game. Reproduced in the section on quantum theory is the famous debate between Einstein and Niels Bohr on the epistemological problems inherent to the theory. Gathered round the two clashing Titans are the other greats like Planck, Schrödinger, Dirac and so on with their own ideas.

J. J. Thomson, the discover of the electron, hailed the general theory of relativity

as 'the greatest achievement in the history of human thought'. No wonder then that the theory has become the basis for the description of the grandest entity in nature, namely the Universe itself. We can glimpse here the shining stars of modern cosmology like Hubble, De Sitter and Lemaitre. Also included in this section are articles by Alan Guth and Paul Steinhardt on inflationary Universe and by Stephen Hawking on the big bang singularity. It is doubtful whether these qualify to be considered as examples of ideas that have stood the test of time.

The introduction to each section of the book is concise but clear. So are the brief biographical sketches of the authors. The philosophical introduction at the beginning of the book should be valuable to those who are seriously interested in the history and philosophy of science. These are additional bonuses that come with this excellent anthology. An anthology that can be savoured for enjoyment and studied for rare insights.

Bogged down in the details and intricacies of our own undertakings, we often forget and sometime are not even aware of the origins and the foundations of our scientific legacy. It is a deeply rewarding experience, exhilarating and even inspiring, to feel those early beginnings and follow the subsequent developments. What better way is there to gain such an experience than listening to some of the profound voices from the past reaching across vast distances of space and time!

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