Rescaled range fractal analysis of a seismogram for identification of signals from an earthquake

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The recognition of seismic signals from the background random noise and identification of various phases in a seismogram are studied from the variation in fractal dimension, *D*, obtained from Hurst's *R/S* rescaled range analysis of seismograms. A rapid change in fractal dimension occurs with change in seismic attributes in a seismogram, although the nature of change varies from trace to trace with varying signal-to-noise ratio and with varying frequencies. The approach is based on the fact that noise has higher fractal dimension than the seismic signals. This method, when applied to the earthquakes in Bhuj and Koyna region of peninsular India, explains the origin of the complexities of the observed waveform, interpreted in terms of degree of heterogeneities of the lithosphere from the variance fractal dimension. The above findings are supported by previous studies of frequency dependence of seismic wave attenuation in the region.

THE accurate detection of arrival times of various seismic phases in a seismogram is of prime importance in seismology. This is particularly the case with seismic refraction and tomographic studies, where travel times of the first arrival are used to determine the seismic velocity structure of the medium. In local earthquake analysis, we get many reflected phases and dominating coda waves that characterize the structural heterogeneities of the medium. The detection of first-arrival seismic data reduces to the problem of differentiating the signal from the background random noise. Several methods both in time[1–4] and frequency domain[5,6] for locating a first motion in a seismogram have been published. The time-domain standard STA/LTA procedure[3] makes use of one or more characteristics functions (CFs), which are new time series different from the original data stream. The changes in CF indicate the presence of phase arrivals. The algorithm is based on the ratio of the current value of the characteristic functions to an ambient value and the event is declared if their ratio exceeds some threshold value. The ambient value of the CF is a long-term average (LTA) of the signal calculated using a low-pass digital recursive filter. The current value of the CF is based on the short-term average (STA), which is usually computed over a few samples to ensure a quick response to any changes in the CF. Stewart[2] used a modified data envelope based on the derivative of the data, where slope changes are emphasized. Anderson[4] looked for peaks between zero-crossing and compared them with the LTA. An event is declared when the peak exceeds a multiple of the LTA and the associated zero-crossings are sufficiently separated. Similarly, the method of Shensa[5] is based on the power spectral density (PSD). Mele[6] described a different approach in frequency domain of the signal; the average power spectral densities within eight overlapping frequency bands in the range 0.2–8.0 Hz constitute the current values to be compared with the LTAs in the same frequency bands. Nath and Dewangan[7] proposed a method for detection of seismic reflections from seismic attributes, with special emphasis on thin-bed delineation through fractal analysis. Joswig[8] used a pattern-matching scheme based on the PSD as a function of time. Ervin *et al.*[9] and Ramananantoandro and Bernitsas[10] have described algorithms for automatic picking of refraction first-arrival times that rely on the comparison of the trace with its immediate neighbours. Detectors and filters based on the eigenvalues and eigenvectors of the three-components variance matrix have also been constructed[11,15].

Here, I have introduced the *R/S* rescaled range analysis by Hurst *et al.*[16] for computing fractal dimension of the waveform recorded by a seismic station, to characterize a seismogram, in differentiating the signal from its noise level. The approach is based on the fact that noise is recognized to have higher fractal dimension than the seismic signals. Thus, the fractal dimensions may be used as a quantitative measure of the degree of heterogeneity of the underlying medium. Dimri[17] has carried out the multifractal and wavelet approach to study the aftershock sequence of Bhuj earthquake and computed fractal dimension, $D = 2.26$ from seismic *b*-value. The present study shows a variation in D values estimated from *R/S* analysis by Hurst[16]; and this need not be equal or even positively correlated with one another, as fractal dimensions obtained by different methods generally reflect different aspects of scale invariance.

Data

The data used in this study are one aftershock of Bhuj earthquake recorded at station Rapar (23.571°N, 70.408°E)

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in its epicentral region and one local earthquake recorded at station Maneri (17.3415°N, 73.7953°E) at Koyna in peninsular India. Stations Rapar and Maneri are part of NGRI local networks deployed in Bhuj and Koyna regions respectively. The data are recorded at sampling frequency of 100 Hz. To know the effect of background noise on the value of D of the waveform, I have studied both the raw data and their filtered versions in suitable frequency range (1–10 Hz for local earthquakes). The seismograms are corrected for instrument response before analysis so as to get the ground velocity.

Method

Observed seismograms contain noise, which are random time series. Some properties of such series can be studied by the Hurst component H. Since the introduction of fractals by Mandelbrot[18], the analysis of fractal dimensions has been widely used in several applications. This analysis is suitable for application to seismic data because of the ability of the fractal dimension to differentiate noise from signal patterns within the data. There exist several different methods to measure the fractal dimension of the waveform like divider method[19], box-counting method, etc. Here, rescaled range analysis by Hurst[16] is used in computing fractal dimension of the waveform by determining the R/S value within windows of different sizes of the waveform. The R/S for the given sampling of the random series is asymptotically given by a power law of the form:

$$\frac{R(\tau)}{S(\tau)} \propto \tau^{H}, \tag{1}$$

where $R(\tau)$ is the range which is the difference between the minimum and maximum 'accumulated' values or cumulative sum of $X(t, \tau)$ at discrete integer valued time t over a length of time sampling τ, S is the standard deviation estimated from the observed values $X(t)$, and H is the Hurst exponent. The value of $H = 0.5$ corresponds to the normal distribution sampling; other values correspond to the various degrees of correlation, which can be interpreted in terms of persistent coefficient. The ranges $0 < H < 0.5$ and $0.5 < H < 1.0$ of Hurst exponent characterize anti-persistence and persistence behaviours respectively. R and S are defined as:

$$R(\tau) = \max_{1 \le t \le \tau} X(t,\tau) - \min_{1 \le t \le \tau} X(t,\tau), \tag{2}$$

where t is the discrete time accepting integer values and τ is a length of the sampling (also called as time lag).

$$S(\tau) = \left(\frac{1}{\tau}\sum_{t=1}^{\tau}\left[\xi(t) - \langle\xi\rangle_{\tau}\right]^{2}\right)^{\frac{1}{2}}, \tag{3}$$

where

$$\langle\xi\rangle_{\tau} = \frac{1}{\tau}\sum_{t=1}^{\tau}\xi(t), \tag{4}$$

and for the cumulative average value, we get

$$\begin{aligned} X(t,\tau) &= \sum_{u=1}^{t}\left[\xi(u) - \langle\xi\rangle_{\tau}\right] \\ &= t\langle\xi\rangle_{t} - t\langle\xi\rangle_{\tau} = t\left[\langle\xi\rangle_{t} - \langle\xi\rangle_{\tau}\right]. \end{aligned} \tag{5}$$

Equation (1) is obtained by normalizing the range R on the standard deviation S of the data estimated from observed values $X(t)$ for the chosen sampling $\xi(t)$, and $< \xi >_{\tau}$ is its mean value on the segment containing τ samples. The Hurst exponent is related to fractal dimension by the following relation:

$$D = 2 - H, \tag{6}$$

which is obtained from the slope of the plot of (R/S) values against the window size τ in log-log space[20].

Results and discussion

The basis of the present first-arrival detection algorithm is that a change in H is expected when the seismogram encounters some changes in seismic attributes like amplitude, phase and frequency. This ultimately brings a change in D along the length of the seismogram. Figures 1 and 2 show the algorithm applied to seismograms recorded at Bhuj and Koyna region respectively. First the approximate region of the seismogram containing the first arrival is selected manually. Then, a window of suitable length is moved progressively along the seismogram through points A, B and C. The Hurst exponent, H, and the fractal dimension, D, in all windows A, B and C of the seismogram are calculated and are plotted at the location of the maximum time of the window. It gives rise to the variation in fractal dimension along the trace (Figure 1 *b*). The windows A, B and C yield estimates of $H = 0.157$ ($D = 1.843$), $H = 0.257$ ($D = 1.743$) and $H = 0.6508$ ($D = 1.349$) respectively. This can be explained in terms of persistent (or anti-persistent) behaviour. The result suggests that window A positioned with background noise shows non-correlated, anti-persistent (random) behaviour ($H < 0.5$), while window C within the part of the signal presents persistent and correlation behaviour characterized by $H > 0.5$. Figure 1 *c* shows that the real signal, in general, has low D value due to good correlation, whereas noise has higher fractal dimensions due to its poor correlation. The poor correlation may be due to lack of a consistent pattern of seismic phase arrivals in the data. However, the absolute value of the fractal dimension measured on different seismograms may vary depending on the signal-to-noise ratio, on the amplification of the signal, and on the sampling frequency. Also, determination of an optimum window length is important, as fractal analysis is sensitive to window length. The reason for change in both D and H values along the seismogram is not only the presence of the signal in the sampling considered, but slow variations of the correlated noise itself.

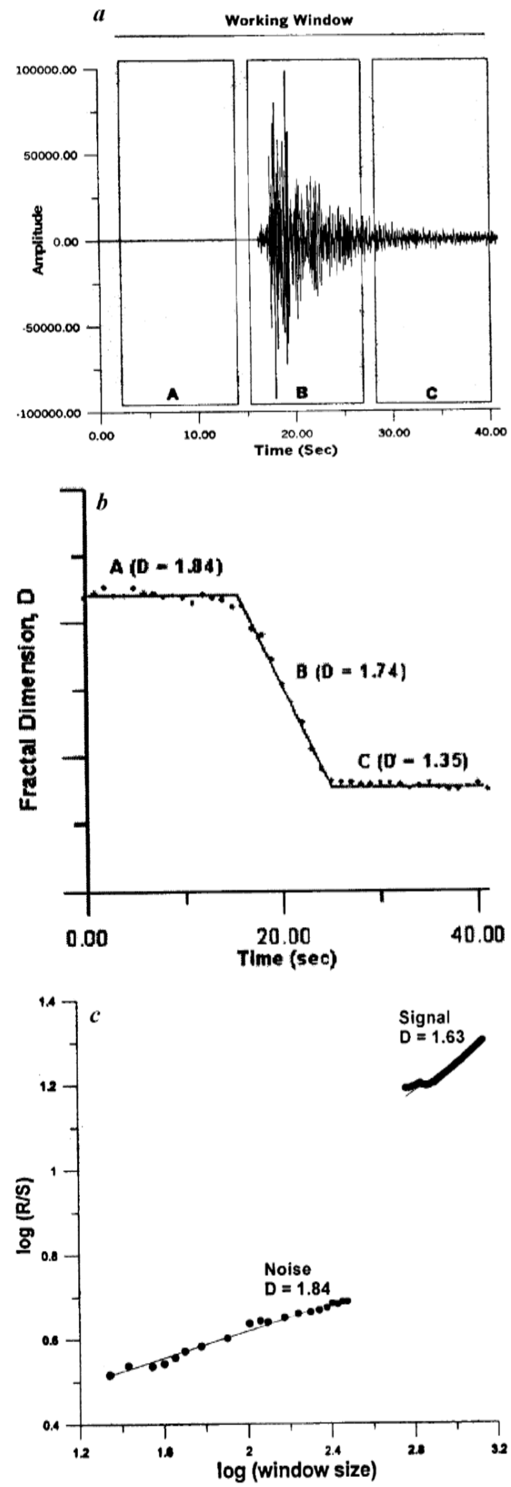

Figure 1. Schematic illustration showing the variation in fractal dimension D along the seismogram recorded at station Rapar (23.571°N, 70.408°E) and Bhuj (15 February 2000, 12 h, 7 min, 2 s). ***a***, The working window is manually selected to contain the first break and is moved progressively through windows A, B and C along the trace. ***b***, Variation in the value D from windows A to C along the trace. ***c***, Plot of log(R/S) versus log(τ) both for typical background noise and seismic signal. The lines identify the range of the best fit by which D is calculated.

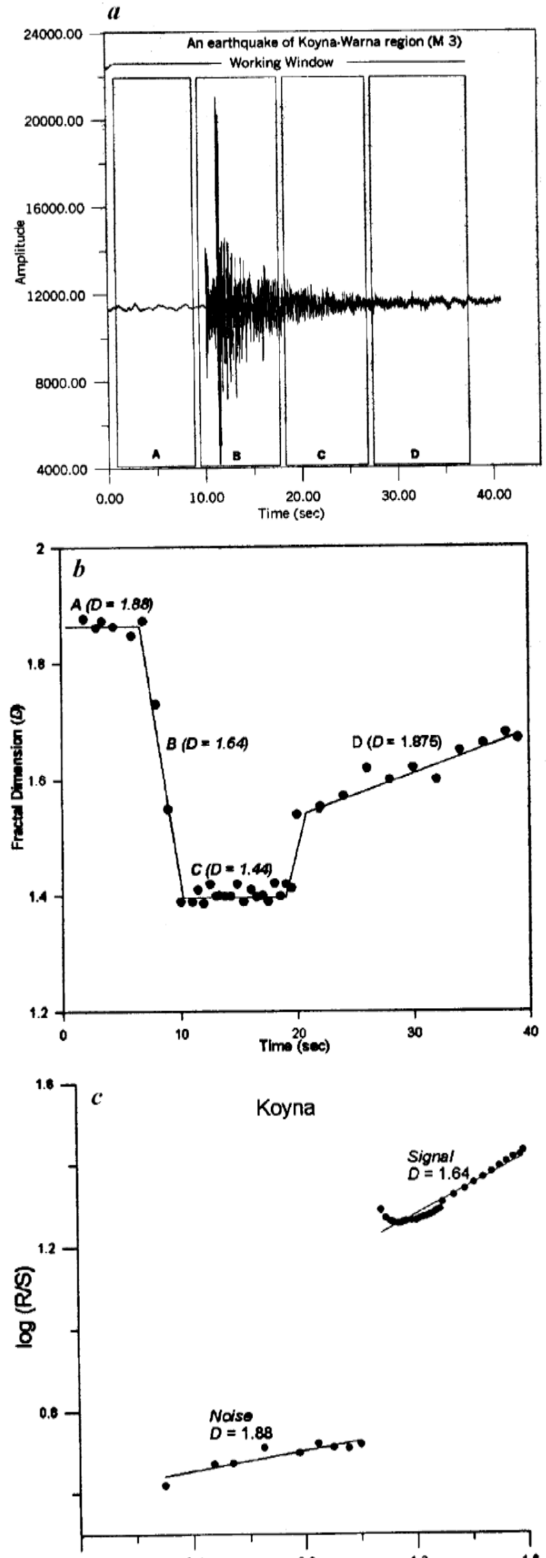

Figure 2. Same as Figure 1, but for the seismogram recorded at Maneri (17.3415°N, 73.79.7953°E), Koyna region (23 September 2002, 11 h, 38 min, 28 s).

The effect of non-stationary sampling of the signals composed of segments of differing spectral characteristics may also contribute to the observed change. The relationship between the spectral amplitudes in different frequencies allows, for the fractal detector, a complete analysis in a broad frequency range of seismic waves. The results of the present study, when combined with the detailed study of Withers *et al.*[21] to all kinds of seismic phases, ultimately infer its good sensitivity to even weak signals. Moreover, the fractal method has the ability to differentiate phases within a single event. The results of this analysis can be applied to a set of phenomena related with differentiation of similar signals of different nature. This study demonstrates that the relevant information on fractal and discrete properties of the system are contained in the seismic signals. The variation from anti-persistent to persistent behaviour in Hurst exponent along the length of the seismogram is consistent with the lack of a consistent pattern of seismic phase arrivals in the data, as mentioned earlier. The results of the same analysis applied to an earthquake recorded at Koyna region are shown in Figure 2 *a–c*. The results are also consistent with those shown for Bhuj earthquakes; the value of D is almost constant before the window reaches the first arrival along the seismogram (Figure 2 *a*). When it reaches the first arrival time, the value of D decreases quite rapidly and again assumes a near constant value in coda area depending on the strength of the signal present. Thus, the first onset on the seismogram brings a rapid decrease in D values on the fractal dimension profile along the seismogram. In order to access the effectiveness of the method, the pattern of the fractal dimension has been analysed for noisy (Figure 3 *a*) record of the seismic wave. There is a decrease in value of the D over the portion of the seismogram dominant in the signal, which is evident from Figure 3 *b*. The changes in phases inferred from the nature of fractal dimension may also describe the detailed structure of the medium. The fractal dimensions obtained by different methods generally reflect different aspects of the scale-invariance, and need not be equal to or even positively correlated with one another. For example, the capacity dimension D_0 estimated by box-counting methods, measures the space-filling properties of a fracture set with respect to changes in grid-scale[22]. The correlation dimension, D_c defined by Grassberger and Procaccia[23], measures the spacing or clustering properties of a set of points, and has also been applied to earthquake epicentres[24,25].

Thus the R/S method of fractal analysis is successful in identifying seismic events from its attributes on the basis of the change in the value of fractal dimension, along the seismogram. The method works well for a broad range of frequencies, including both body and surface waves. The method works quite well in identifying the events even with noisy waveforms. The real signal is differentiated from the background noise by its low fractal dimension value. The nature of change in fractal dimension values indicates the nature of seismic wave attenuation through the underlying medium in a wide frequency range and also explains the origin of the complexities of the observed waveform. This inference can be interpreted in terms of degree of heterogeneity of the lithosphere, which is beyond the scope of this article.

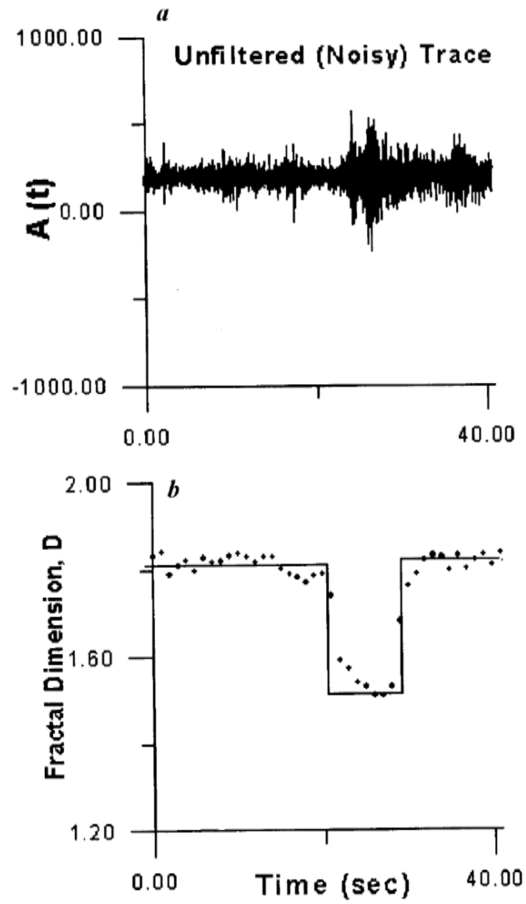

Figure 3. Behaviour of the pattern of fractal dimension in case of (***a***) noisy data and (***b***) its variation along the trace.

The advantages of the fractal approach over classical methods lie in detecting the surface-reflected and depth-reflected phases just form a single seismogram, without the need for array data and without constraint on velocity control. The efficacy of the method lies in pulling out the signal in the complex coda, which gives the nature of the heterogeneities of the medium. The efficiency of the method needs to be tested with gradually decreasing order of signal to noise and more importantly, in case of first arrival detection, with signal-to-noise ratio of one or less than one. Thus, the strength of the signal detection becomes more enhanced by the fractal approach than the conventional classical methods.

1. Vanderkulk, W., Rosen, F. and Lorenz, S., Large aperature seismic array signal processing study. IBM Final Report, ARPA Contract Number SD-296, 1965.

2. Stewart, S., Real-time detection ;and location of local seismic events in central California. *Bull. Seismol. Soc. Am.*, 1977, **67**, 433–452.
3. Allen, R., Automatic earthquake recognition and timing for single traces. *Bull. Seismol. Soc. Am.*, 1978, **68**, 1521–1532.
4. Anderson, K. R., Automatic analysis of micro-earthquake network data. *Geoexploration,* 1978, **16**, 159–175.
5. Shensa, M., The deflection detector, its theory and evaluation on short-period seismic data. TR-77-03, Texas Instruments, Alexandria, Virginia, 1977.
6. Mele, F., Elaborazione automatica di segnali sismici presso l'Istituto Nazionale di Geofisica. In Ricerce nel campo della sismologia e geodinamica negli anni 1990–1991, Istituto Nazionale di Geofisica, Roma, (in Italian), 1993, pp. 21–61.
7. Nath, S. K. and Dewangan P., Detection of seismic reflections from seismic attributes through fractal analysis. *Geophys. Prospect.*, 2002, **50**, 341–360.
8. Joswig, M., Pattern recognition for earthquake detection. *Bull. Seismol. Soc. Am.*, 1990, **80**, 170–186.
9. Ervin, C. P., McGinnis, L. D., Otis, R. M. and Hall, M. L., Automated analysis of marine refraction data: a computer algorithm. *Geophysics*, 1983, **48**, 582–589.
10. Ramananantoandro, R. and Bernitsas, N., A computer algorithm for automatic picking of refraction first-arrival time. *Geoexploration*, 1987, **24**, 147–151.
11. Aster, R., Shearer, P. and Berger, J., Quantitative measurements of shear-wave polarizations at the Anza seismic network, Southern California: Implications for shear-wave splitting and earthquake predictions. *J. Geophys. Res.*, 1990, **95**, 12449–12473.
12. Magotra, N., Ahmed, N. and Chael, E., Seismic event detection and source location using single-station (three-component) data. *Bull. Seismol. Soc. Am.*, 1987, **77**, 958–971.
13. Magotra, N., Ahmed, N. and Chael, E., Single-station seismic event detection and location. *IEEE Trans. Geosci. Remote Sensing*, 1989, **27**, 15–23.
14. Montalbetti, J. and Kanasewich, E., Enhancement of teleseismic body phases with a polarization filter. *Geophys. J. R. Astron. Soc.*, 1970, **21**, 119–129.
15. Flinn, E., Signal analysis using rectilinearity and direction of particle motion. *Proc. IEEE*, 1965, **53**, 1874–1876.
16. Hurst, H. E., Black, R. and Sinaika, Y. M., *Long-term Storage in Reservoirs: An experimental Study*, Constable, London, 1965.
17. Dimri, V. P., Multifractal and wavelet approach to study the aftershock sequence of Bhuj earthquake, India. JSS02/01A/A06-006, abstr. vol., IUGG Sapporo, Japan, 30 June–11 July 2003.
18. Mandelbrot, B. B., How long is the coast of Britain? Statistical self-similarity and fractional dimension. *Science*, 1967, **156**, 636–638.
19. Feder, J., *Fractals*, Plenum Press, New York, 1989, p. 283.
20. Russ, J. C., *Fractal Surfaces* Plenum Press, NY, 1994.
21. Withers, M., Aster, R., Young, C., Beiriger, J., Harris, M., Moore, S. and Trujillo, J., A comparison of select trigger algorithms for automated global seismic phase and event detection. *Bull. Seismol. Soc. Am.*, 1998, **88**, 95–106.
22. Hirata, T., Fractal dimension of fault systems in Japan: Fractal structure in rock fracture geometry at various scales. *Pure Appl. Geophys.*, 1989, **131**, 157–170.
23. Grassberger, P. and Procaccia, I., Measuring the strangeness of strange attractors. *Physica D*, 1983, **9**, 227–230.
24. Kagan, Y. Y. and Knopoff, L., Spatial distribution of earthquakes: the two-point correlation function. *Geophys. J. R. Astron. Soc.*, 1980, **62**, 303–320.
25. Hirata, T., A correlation between the *b* value and the fractal dimension of earthquakes. *J. Geophys. Res.*, 1989, **94**, 7507–7514.

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