Role of mathematics in chemistry

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ALTHOUGH chemistry was practised from the dawn of civilization as the discipline to create materials, including the extraction of metals, it was more of a craft of artisans than a subject of enquiry into the fundamentals. Physics, in contrast, evolved very rapidly as an 'exact' science, in particular in the hands of Archimedes, Galileo and Newton, where the fundamental laws of motion got quantified and predictions were precise enough to distinguish between the rival conceptual frameworks. The major reason why chemistry developed into an exact science relatively late is that the underlying laws of binding and transformations of chemical substances have their basis in the quantum behaviour of the constituents of matter. The behaviour of chemical substances, as isolated species or in bulk – which dominates our world of senses – are, however, only indirectly related to their microscopic constitution and this has remained a problematic ontological issue which deterred an intellectually satisfying and integrated quantitative conceptual framework for chemistry. Moreover, chemistry as a discipline enjoys a degree of autonomy in the sense that the desirable goals of a chemist (control of emergent chemical behaviour, designing molecules with specific chemistry, monitoring chemical transformations into well-defined channels) are determined by questions and aesthetics of chemical nature. In this sense, chemistry is more 'complex' than physics.

I look upon the less complex science as one which engenders the fundamental basis of a science which lies one tier higher in complexity. To borrow the terminology from biology we may say that the fundamental basis of a science are genotypes, while the emergent properties arising out of the genotypical laws are phenotypes. I want to argue that chemistry is the simplest science of complexity since the fundamental physical laws are its genotypes and the emergent chemical expressions are the phenotypes. Chemistry is thus compatible with physical laws but not reducible to them. The really interesting problems in chemistry seem to remain fully unresolved in terms of understanding from physical principles because scientists have not come to grips in discerning the pattern, structure and interconversions displayed by molecules from the fundamentals of subatomic physics. This is despite the fact that we understand the quantum and statistical mechanical laws of physics well enough but it is neither unique nor trivial to pose questions of chemical nature in terms of physical laws. The complexity of chemistry has even an underlying extra degree of freedom in the sense that the superstructure of chemical functions is to some extent insensitive to the physical laws underpinning them. Results from a more quantitative formulation from a more fundamental basis often lead to qualitatively similar but quantitatively different conclusions, so that certain empirical generalizations can well describe chemistry and even lead to an illuminating understanding, quite independent of the underlying laws of the substratum. Obviously, by 'understanding' we mean assessing the relative importance of the various processes reflected in some conceptual constructs which act together to shape the phenomena of interest. Models emerge when we tie up understanding and quantitative descriptions of the conceptual constructs and weave a story out of it. Stories are complete or convincing to various extents depending on the mix of understanding and quantification. Another appealing simile is sculpting. Much is removed but much remains also for the pattern to emerge. Recognizing what to remove yet emphasizing the essence in all its splendour is an orphic endeavour of sorts, involving inspiration, metaphor, symbolic representation and innovative analogy. The role of mathematics in chemistry must satisfy this polysemiotic and polymimetic richness.

We thus distinguish quantum molecular physics as somewhat distinct from theoretical chemistry when we want to discuss the role of mathematics in chemistry. An appreciation of this difference is often not made, leading either to a perception that brute force computation or even empirical quantitative simulation would lead to understanding chemical significance or to the dismissive attitude of the experimental chemists that theoretical chemistry fails to provide predictive answers of chemical significance, when in fact they are probably pointing out the limitations of computational molecular physics. When I talk about the role of mathematics in chemistry, I have in mind an evolving, conceptually integrated and many layered theoretical framework of molecular science, which subsumes both molecular/materials physics and chemistry and chemical biology as subdisciplines. This is a never ending saga but the stories become more and more complex as we begin to see more intricate patterns and can relate them more and more to the physical laws underpinning them.

The genotypes of chemistry are embedded in quantum mechanics, equilibrium and non-equilibrium statistical mechanics, and diffusion behaviour in fluids. The phenotypes are the molecules displaying myriad chemical properties in isolation and in transformation. The chemical concepts like bonds, lone pair, aromaticity, electronegetivity, reso-

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nance, functional groups, etc. evolved from the attempts of chemists to grapple with the diverse behaviour via these empirical constructs to systematize chemical behaviour.

The unreasonable effectiveness of mathematics in the natural sciences, in particular physics – as noted by Wigner, emphasizes the striking synergy of mathematics and physics. Examples abound spanning several centuries which tell us how old mathematics have been found to be tailor-made to promote progress in physics, how new mathematics got evolved to quantify new-found laws and how demands of the physical world have led to creation of new mathematics. Starting from the theory of curves, the development of calculus in quantifying laws of motions, and the use of non-Euclidean geometry to understand gravitation to the more recent examples of modular and elliptic curves and complex manifolds, and Calabi-Yau spaces are some striking examples of this symbiosis. Another versatile tool which has been finding more and more use in physics is compression of information using the idea of algorithmic complexity. On an immediate level this leads to optimal representation of a model with a given ontological content; as a long-term perspective it offers the possibility of modelling complex sciences through the irreducible genotype components – augmenting them with the initial and boundary conditions that are extraneous to the fundamental laws and thus of leap-frogging from a less complex science to a more complex form.

As of now, mathematics of classical era of physics has permeated the whole of chemistry as a quantifying tool of simpler chemical processes and for providing a microscopic understanding of chemical transformation from first principles. This has several levels of sophistication: (a) simple quantification of empirical data (empirical models) (b) statistical chemical data analysis, quantitative analytical chemistry, emerging into cheminformatics and chemical data analysis (c) molecular similarity described in terms of target phenotypes leading to quantitative structure–activity correlation (d) quantitative simulation models: reducing *ab-initio* theories to computational chemistry via parametrization and (e) simulation via modellization on a quantum mechanical scenario (path-integral molecular dynamics). Evolution of powerful computers has been providing the chemists with quantitative answers for predicting reactive intermediates, branching ratios of various products or to understand the relative stability in their ground, excited, or ionized states. Methods of molecular electronic structure theory are being routinely used to predict theories via black-box programmes. Studying the properties of metal alloys, complex fluids and materials of various types have also matured as robust computational discipline. These are the triumphs of quantum and statistical molecular physics. On a simpler substrata, simulations based on empirical fits or extracting quantum mechanical many-body potentials have led to successful predictions of reaction processes in solvated species, of the dynamics of the conformational changes and to ion migration through membranes. These developments, along with the emergence of cheminformatics have resulted in a discipline which is turning out to be very important as fundamental inputs to structural biology and bioinformatics. Control of spatio-temporal patterns via nonlinear systems equations, theoretical electrochemistry on random and ultrametric surfaces, behaviour of solvated species in critical and sub-critical conditions are also some of the important quantitative developments where mathematical modellings have led to fundamental insight into reacting diffusive systems, electrode processes and behaviour of solvated matter under phase transitions. They embody a vast corpus of chemical activity at a quantitative level, which lies at the interface of molecular physics and theoretical chemistry. This part of the story-telling entails first principles formulations leading to phenotypes from genotypes but only for the simplest of the chemical reality. Of far greater importance are of course the desiderata transcending the border of physics into the autonomous theoretical constructs of chemistry – in short, generating chemical theories.

The emerging frontier of theoretical chemistry already encompasses the use of algebraic topology to discern patterns in structure–activity correlation. The various notions of theoretical linguistics are also finding their place in the axiomatic formulation of chemistry, although they have not succeeded as yet in making useful predictions. The concept of virial fragments in identifying functional groups separated by surfaces of zero density gradient in a molecule is a very fruitful innovation where theoretical chemistry could morph a chemical concept out of physical laws which has a credible autonomous validity and predictability. The use of concept of homotopy and manifolds has led to a very concise understanding of the various classes of the excited states of potential hyperenergy surfaces and to enumerate all possible topologically distinct reaction pathways. Use of artificial intelligence has proved a powerful tool to prune stray pathways in predicting chemical reactions and this, coupled with the homotopy theory, will prove more and more useful in classifying and discerning patterns of chemical transformations. The methods of control theory have also been innovatively transcribed into the field of laser-control of chemical reactions and fundamental insight has already been obtained in fine tuning of bond-breaking and bond-making process and also for identifying transition states. Although we are still far away from monitoring and controlling channel-specific reactions, the activity in this field promises to be very significant and control and systems theory will enter more and more in reaction dynamics which will redefine the boundaries of molecular science.

The complexity of chemistry precludes having a single 'correct' analysis of the chemical entities expressed in a single adequate language. The world of chemistry is just too rich and diverse, requiring multifaceted representations which capture the various layers of chemical reality. In contrast to physics, chemistry requires and has naturally evolved a symbolic representation (a language?) that is

expressed in notations that are at once symbolic and iconic as in Chinese ideograms. In empirical chemistry, a chemist, like an artist, moves subconsciously between the phenotypes and genotypes, between the levels of potentiality and actuality of transformations or even between different levels of reality. A chemist may enquire about a molecular structure and function generated by the primary, the secondary and the tertiary conformations. On one hand a chemist may try to understand the reactivity of a molecule (which is a property of propensity) and may try to relate it to a real reaction (actuality) which demands the linguistic dichotomy of the symbolic and iconic components embodying the propensity and actuality. The effect of environment generating the concepts of the directionality of the bonds and lone pairs reflects another level of dichotomy where standard quantum mechanics, in both time-independent and dependent forms, seems ineffective and theories of subdynamics describing open systems appear as more natural tools. Chemistry as a craftsmanship has always been aware of the looseness and multilayered nature of the concepts of bonds and lone pairs, and has creatively used octet rules, double quartet rules and various other stereoselectivity criteria to design rational synthesis and to understand chirality of molecules and the powerful use of chiral induction and chiral catalysis. A quantitative phrasing of these concepts in terms of subdynamics, where the environment leads to the emergence of the molecular shapes, including emergence of chirality from achiral interactions via symmetry breaking, is yet to emerge in an unambiguous manner but the theory of subdynamics has already reached a state of development where more quantitative and predictive formulations are anticipated in the foreseeable future. The effect of relativity in molecules containing heavy atoms has already been quantitatively understood for simple molecules, although the qualitative concepts embodying these quantitative formulations have been rather slow to emerge. Such innovations are essential for understanding molecular aggregates with heavy atoms, clusters with heavy atoms and their fruitful use in molecular electronics and for molecular assemblages tailor-made for molecular collective computation.

Matter in extreme conditions such as plasma has already been quantitatively understood in terms of gas and ion reactions and more quantitative models for ion beam imaging and ion lithography are rapidly emerging. Another intriguing frontier is evolving where very cold atoms appear to generate aggregates in Bose–Einstein condensation, with the possibility of a crossover between Bose–Einstein condensation of atoms (which are bosons due to intra-atomic coupling of electronic and nuclear spins) to the BCS type of pairing condensates via magnetic monitoring. These are giant aggregates (condensates) which are not molecules in the conventional sense, thus creating new forms of matter. This opens up the exotic field of ultra-cold chemical aggregates.

It seems eminently possible that mathematical tools like pattern recognition, artificial intelligence, collective computing, algebraic topology and fuzzy logic will make major forays into chemistry and create an integrated platform where these tools will enmesh fruitfully with quantum mechanics and statistical mechanics. The entire field of molecular, materials and biomolecular science is in a state of flux – an ideal melting pot to brew new ideas. The tantalizing transition from quantum molecular physics to theoretical chemistry will happen sooner than later.