Samanta's planet placing

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It is shown in this note that placing of the five naked-eye planets in the Tychonic model of the solar system envisaged by Samanta Chandra Sekhar, as deciphered from his Siddhanta Darpana, is in agreement with the well-known Titius–Bode law and even with the more basic Kepler's third law of modern astronomy. This is a unique instance in the traditional Indian system and a significant contribution of Samanta. It is also shown that both in ancient Greek and Indian astronomy, epicycles for solar anomaly (Úghra phala) give correct relative distances of planets, which was not realized by the then astronomers because of their geocentric viewpoint. It is also shown here how Samanta's estimate of the astronomical unit came out to be of the right order of magnitude.

Planetary distances in Siddhantas

The sun, moon and the five naked-eye planets (Mercury, Venus, Mars, Jupiter and Saturn) have been observed and studied by astronomers through the ages in both the East and West. Systematic models of their motion have been worked out starting with Ptolemy in Greece and Aryabhata in India. Ptolemy, his followers in the West and most of the Indian astronomers subscribed to the geocentric hypothesis of the solar system. While giving the details of the planetary motion they were able do decide distances of the planets from the common centre of motion. Of course, the Greek and the Indian astronomers based their works on two completely different premises. The Greek school relied on observations for data, whereas the Indian scholars put forth hypotheses and preferred assumed elements to finally tally with the observations. Still, strikingly enough, both the schools of thought came out with similar models and results.

Development of the concept of gravitation in the way of Newton's and Kepler's laws helped the Western workers to decide the distances of planets from the sun with fair accuracy. But the Indian astronomers have landed with figures for planetary distances, which often differ from author to author and from the modern values by magnitudes. Table 1 gives the circumference of the sun and five naked-eye planets centred on the earth from three ancient sources, i.e. *Aryabhatiya*, *Surya Siddhanta* and *Siddhanta Siromani*[1–3]. Samanta Chandra Sekhar (1835–1904)[4–7] happens to be one of the great traditional Indian astronomers. We also cite in Table 1 data from his magnum opus, *Siddhanta Darpana*[8]. The data are in Yojanas; one Yojana being equal to 5 miles or 8 km. It is straightforward to get the distance of a planet from the common centre from the elementary relation between radius and circumference of a circle.

A line by way of clarification is in order here. The sun, which in modern astronomy is regarded as a star, has been considered as a planet in the traditional Indian system. Hence it has been placed under the column of planets in Table 1. Further, it may be noticed from Table 1 that data in the other three columns are less by about a factor of 10, compared to those of *Siddhanta Darpana* in many cases.

It is also a well-known fact that none of the traditional Indian astronomers, including Samanta Chandra Sekhar, could reach the correct estimate of the planetary distances, solely due to the unrealistic approach they followed. Of course, the rates of motion of the planets were of greater relevance to them, than their distances, since these rates only directly entered into the calculations of ephemerides that they were concerned with. Nevertheless, it is pertinent to analyse as to how far these authors were correct, at least in maintaining the scale of proportion. In deciding such proportion, we are directly led to the Bode's law of modern astronomy and Kepler's laws.

Johann Titius and later on Elert Bode advanced an empirical formula for planetary distance of the solar system during AD 1766–1772. The formula has proved to be a powerful tool not only to give the correct relative distances of the five naked-eye planets, but also to indicate the position of the asteroid belt and new planets like Uranus. It holds good for the first eight places in the solar planetary system

Table 1. Circumference of planetary orbits in Yojanas

Planet	*Aryabhatiya*	*Surya Siddhanta*	*Siddhanta Siromani*	*Siddhanta Darpana*
Sun	2887668	4331500	4331497	47800800
Mercury	695472	1043209	1043235	18456420
Venus	1776420	2664637	2664623	34655580
Mars	5431296	8146909	8147108	72303600
Jupiter	34250136	51375784	51376035	245832000
Saturn	85114488	127668255	127671803	441237600

Table 2. Planetary distances in astronomical unit

Planet	Distance according to Bode's law	Actual distance
Mercury	0.4	0.387
Venus	0.7	0.732
Earth	1.0	1.0
Mars	1.6	1.524
Asteroid belt	2.8	2.68
Jupiter	5.2	5.203
Saturn	10.0	9.539
Uranus	19.6	19.19
Neptune	38.8	30.1
Pluto	77.2	39.5

Table 3. Ratio of planetary distances

Planet	*Aryabhatiya*	*Surya Siddhanta*	*Siddhanta Siromani*	*Siddhanta Darpana*	Period in years
Sun	1.00	1.00	1.00	1.00	1.00
Mercury	0.24	0.24	0.24	0.3861	0.24
Venus	0.615	0.615	0.6143	0.725	0.615
Mars	1.88	1.88	1.88	1.5126	1.880
Jupiter	11.86	11.86	11.86	5.1028	11.862
Saturn	29.47	29.47	29.47	9.230	29.450

Table 4. Radii of orbit of planets

	Surya Siddhanta				*Siddhanta Darpana*	
Planet	Even quadrant	Odd quadrant	Ptolemy	Values of Western astronomers as in 1840s	Even quadrant	Odd quadrant
Sun	1.00	1.00	1.00	1.00	1.00	1.00
Mercury	0.3694	0.3667	0.3750	0.3817	0.386	0.388
Venus	0.7278	0.7222	0.7194	0.7233	0.725	0.727
Mars	1.5319	1.5513	1.5190	1.5237	1.5126	1.5184
Jupiter	5.1429	5.0000	5.2174	5.2028	5.1428	5.2173
Saturn	9.2308	9.000	9.2308	9.5389	9.230	9.4773

particularly and also fairly well in case of satellites of four major planets, i.e. Jupiter, Saturn, Uranus and Neptune. Though initially put forth as a numerical conjecture, theoretical derivation of the law has been reported based on scale invariance and disc model[9,10]. A compact formula for the law can be given for distance of the *n*th planet from the sun by,

$$r_n = 0.4 + 0.3 \times 2^{n-2}\theta(n-1),$$

where n runs from 1 to N, starting with Mercury in the solar system, and $\theta(n)$ is the step function with property $\theta(n) = 1$ for $n > 0$ and zero elsewhere. One may check the validity of the law from Table 2, where the actual distances of planets and the asteroid belt from the sun are given along with those predicted by the above formula.

Of course, more fundamental in this respect is Kepler's third law, derivable from Newton's law of gravitation, which is useful in checking planetary distances. According to the law, $r^3 \propto T^2$, where r is the average distance of a planet from the sun and T its time period of revolution. Table 3 presents the ratio of the planetary distances to the sun–earth distance, i.e. the astronomical unit and the ratios are expressed in the same unit as in different traditional works, including *Siddhanta Darpana* of Samanta Chandra Sekhar. We cite data from three prime sources, i.e. *Aryabhatiya* of Aryabhata[1], and *Surya Siddhanta*[2] and *Siddhanta Siromani* of Bhaskaracharya[3] for comparison. Table 3 also gives the time periods of the planets in terrestrial year.

It is easy to check that the planetary distances derived from *Siddhanta Darpana*, exactly match with those computed from the power law of distance and time period. It is striking to note that the distances of planets given by Samanta Chandra Sekhar exactly follow both Bode's law and Kepler's law, whereas none of the other three sources follow any of these laws, even though they give almost the same time period for the naked-eye planets[4]. Besides, the ratios in Samanta's data are almost close to the modern values. This is definitely an important original contribution of Chandra Sekhar.

To come back to why astronomers before him did not reach the correct distances, it is worth mentioning here that they obtained the distance by assuming equal linear speed for all planets in their respective orbits. This is not the case with Samanta's figure, because his estimates are based on the Tychonic model, in which only the sun and moon move round the earth and other planets move round the sun.

Úîghra epicycle and planetary distances

Burgess[2] presents the radii of planetary orbits as in *Surya Siddhanta* and compares them with Ptolemy's values and those of Western astronomers in 1840s. The data seem to follow Bode's law (Table 4). The corresponding data from *Siddhanta Darpana* are also given in Table 4.

It is to be noted that the values given here are not the linear radii of the orbits of planets. They are the angular radii of the Úîgra epicycle (epicycle of conjunction) compared with the solar radius; when the deferent circumference is taken as 360°. For the interior planets, one takes the ratio of the Úîghra circumference with respect to that of the deferent and for the exterior planets, the reverse has been taken. For a simple check, we cite the *Surya Siddhanta*[2] and *Siddhanta Darpana*[8] values of the circumference of the Úîghra epicycle at the ends of the even and odd quadrants separately in Table 5.

This can be understood in the Tychonic model as follows.

In Figure 1 *a* for an interior planet, E is the earth, S is the sun on a different circle and P is the planet in its orbit, which serves as the Úîghra epicycle. Thus

$$\frac{r}{R} = \frac{SP}{ES} = \frac{\text{Radius of Úîghra epicycle}}{\text{Radius of the deferent}}$$

$$= \frac{\text{Circumference of Úîghra epicycle}}{\text{Circumference of deferent}}.$$

In Figure 1 *b* for an exterior planet, E is the earth, S is the sun in orbit around the earth and P is the planet in orbit around the sun, which serves as the deferent. Here the Ú îghra epicycle is the reflected motion of the sun, shown as Œ. Therefore we have,

$$\frac{r}{R}=\frac{SP}{ŒP}=\frac{\text{Radius of the deferent}}{\text{Radius of Úîghra epicycle}}$$

$$=\frac{\text{Circumference of deferent}}{\text{Circumference of Úîghra epicycle}}.$$

Earlier astronomers could not realize this on account of their geocentric view.

Samanta's estimate of astronomical unit

From Table 1, we see that Samanta's value of the astronomical unit comes out to be $4.78 \times 10^7 \times 8/2\pi$ km $= 6.09 \times 10^7$ km. This is of the right order of magnitude and only 4/10 of the true value of 14.96×10^7 km. It is worth presenting therefore, how Samanta arrived at this figure. It has been elaborately described by Ray[4].

It was in the context of calculation of eclipses where the horizontal parallaxes of the sun and moon enter and their numerical figures matter in the prediction of these events. But like all earlier traditional astronomers, Samanta did not know how to determine the distance of the sun. Ray narrates how Chandra Sekhar found a way to do so. The account reads, 'One day, thus dejected in mind, while he was coming home, he noticed an image of the sun, projected through a narrow aperture in a fence of palm leaves close to his house'. This gave Samanta the idea of estimating the sun's distance, only if its linear diameter was known. But he was sure that it was not correctly known either; sources differed with their data.

Ray continues 'These questions were not out of his mind for a single day through a long year'. While reading the *Atharvan Upanishad*, he was surprised to find that the sun's diameter was not 6500 Yojanas as given in the *Surya Siddhanta*, but 72000 Yojanas. And this led him to decide the distance using accurate estimate of the angular diameter and also, may be, independently by projection through the aperture.

Besides, Samanta specifically mentions at least in two places in *Siddhanta Darpana*, regarding the diameter of the Sun.

द्विसप्ततिसहस्त्र योजनमितार्कबिम्बायतिर्महापुरुषवाचयेत्यनुजगावथर्व्वा श्रुतिः।
मयैतदनुसारतो नयनगोचरर्क्षग्रहप्रमाणपरिधिग्रहादिकमक्श्मलं कल्प्यते।।

(*Siddhanta Darpana* Ch. 8, Sloka-12)

(72000 Yojanas is the spherical diameter of the sun, as per Seers' words in the *Atharva Veda*. In accordance with that I deduce without any flaw, the visual diameters of the asterisms and planets, their circumference and eclipses, etc. to agree with observation.)

न हि श्ब्दाखिलत्यागादनुमानं प्रशस्यते ।
विस्तारोऽङ्गीकृतो भानोस्तन्मया श्रुतिसम्मतः ।।
ब्रह्मविद्योपनिषदि प्रणवार्थनिरुपणे ।
द्वासप्ततिसहस्राणि योजनानीति भास्वतः ।।
श्रीमहापुरुषेणोक्ता विस्तृतिर्यान्तिमश्रुतै ।
दृक्सिद्धा सार्कसिद्धान्तभुव्यासश्च तथा मतः ।।

(*Siddhanta Darpana* Ch. 19, Sloka-48-50)

(The Vedic words cannot be overridden by hypothesis. Therefore, I have accepted the diameter of the sun given in the *Vedas*. While explaining the implication of '*pranava*', the *Brahmavidyopanishad* gives the diameter of the sun to be 72000 Yojanas. This measure given by the great seers is true to observation, and also true is the diameter of the earth given by *Surya siddhanta*.)

In these slokas, the author emphatically says that the figure taken from the *Brahmopanishad* of *Atharva Veda*, is also 'दृक्सिद्ध' (true by observation). This is because the value of parallax thus arrived used in the prediction of eclipses definitely agreed with his observations.

We have further traced the clue into the original source, *Brahmavidyopanishad15*. In this scripture, with the starting verses is given the annotation of *pranava*, i.e. Omkar. We quote the relevant lines only.

सूर्यमण्डलमध्येऽथ ह्यकारः शङ्खमध्यगः ।
उकारश्चन्द्रसंकाशस्तस्य मध्ये व्यवस्थितः ।।
मकारस्त्वग्निसंकाशो विधुमो विद्युतोपमः ।
तिस्त्रो मात्रास्तथा ज्ञेयाः सोमसुर्याग्निरुपिणः।।
शिखा तु दीपसंकाशा तस्मिन्नुपरि वर्तते ।
अर्धमात्रा तथा ज्ञेया प्रणवस्योपरि स्थिता ।।
पद्मसूत्रनिभा सूक्ष्मा शिखा सा दृश्यते परा ।
सा नाडी सूर्यसंकाशा सूर्यं भित्वा तथापरा ।।
द्विसप्ततिसहस्राणि नाडीं भित्वा च मूर्धनि ।
वरदः सर्वभूतानां सर्वं व्याप्यावतिष्ठति ।।

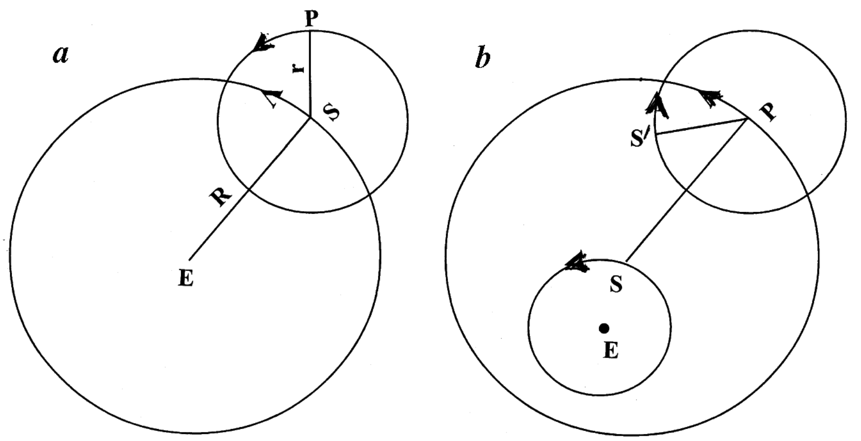

Figure 1. Úîghra epicycle for (***a***) inner planet and (***b***) outer planet.

Table 5. Circumference of Úîghra epicycle of planets in degrees

Planet	*Surya Siddhanta* Even quadrant	*Surya Siddhanta* Odd quadrant	*Siddhanta Darpana* Even quadrant	*Siddhanta Darpana* Odd quadrant
Sun	360	360	360	360
Mercury	133	132	139	140
Venus	262	260	261	262
Mars	235	232	238	237
Jupiter	70	72	70	69
Saturn	39	40	39	38

(On the solar sphere there lies the a-kar (अ कार) inside a conch. The u-kar (उ कार) exists inside that like the moon. The ma-kar (म कार) resembles the fire, blazing and dazzling. The three matras (मात्रा), i.e. components are to be visualized as sun, moon and fire. The flame rises as lamp, above that; and that is visualized as the ardha-matra (half component) above the pranava (i.e. om-kar). That flame appears as fine as lotus fibre and measures a nadi. It is as bright as the sun; and enters deep inside seventy two thousand nadis is the measure through to the top. That (pranava) is all pervading and bestows benedictions to the material world.

It is definitely an interesting picture of 'Omkar' structure compared with the visual image of the sun, i.e. with the photosphere and the outer flame. The half mâtra 'chandrabindu' is visualized as the flame with the dimension of a nâdî; whereas the solar disc from bottom to the terminal is 72000 nâdîs. Samanta has definitely taken the figure from here. But there surfaces a problem of units. Nâdî in the traditional Indian system is a unit of time, which is 1/60th of a solar day. But here a nâdî is no doubt taken as a measure of length. Samanta has reckoned a nâdî as a Yojana, although we find no such mention in ancient literature.

Once the linear diameter is fixed, it is easy to arrive at the distance either by the aperture projection, for which the formula is given in sloka 62–64, Ch. 19 of *Siddhanta Darpana*; or directly from the angular diameter of the sun, which Samanta decides to be 32′32″06‴. It is equal to 72000/2213, as given by Samanta in sloka-15, Ch. 08. The figure in the denominator, i.e. 2213 comes out of the relation,

$$\theta' = \frac{72000}{7608294} \times \frac{180 \times 60}{\pi} = \frac{72000}{7608294/(180 \times 60)/\pi} = \frac{72000}{2213}.$$

The distance of the sun is given as 7608294 Yojanas and θ′ is the angular diameter in min.

Conversely,

$$d = \frac{72000 \times 10800}{\theta' \times \pi}.$$

The accuracy up to which the angular diameter has been given could not have been attained by measurement, with the instruments Samanta was using for the purpose. Probably, with the assumed linear diameter of 72000 Yojanas for the sun, he has determined the distance by the method of projection through the aperture.

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