A new precision improvement in zenith tropospheric delay estimation by GPS

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Accurate GPS-derived ZTD (zenith tropospheric delay) plays a key role in near real-time weather forecasting, especially in improving the precision of Numerical Weather Prediction (NWP) models. However, ZTD is usually estimated under the assumption that all the GPS measurements, carrier phases or pseudo-ranges, have the same accuracy. These assumptions are unrealistic, which will inevitably degrade the precision of ZTD estimation. This communication aims to further improve the precision of ZTD estimation by stochastic modelling. The results show that the precision of GPS-derived ZTD can be obviously improved using a suitable stochastic model for GPS measurements. The stochastic model using satellite elevation angle-based cosine function is better than other investigated stochastic models. This improvement of ZTD estimation is certainly critical for reliable NWP and other tropospheric delay corrections.

Keywords: GPS, precision, stochastic model, zenith tropospheric delay.

THE GPS signal propagating through the neutral atmosphere is delayed by variation of refraction index due to temperature, pressure and water content, which results in lengthening of the ray-path, usually referred to as the ‘tropospheric delay’. This delay is an important error source for GPS positioning, which contributes a bias in height of several centimetres even when simultaneously recorded meteorological data are used in tropospheric models[1,2].

Today, GPS has widely been used to determine the total ray-path tropospheric delay[1,3–5]. The corresponding zenith tropospheric delay (ZTD) can be obtained from the ray-path delay through mapping functions[6], which can be transformed into the precipitable water vapour (PWV)[2,7–9]. In comparison with traditional techniques, such as radiosondes and WVR (water vapour radiometer), the GPS technique has advantages, such as its low cost, all-weather operability and high accuracy[9,10]. Therefore, GPS-derived ZTD plays a key role in near real-time weather forecasting, especially improving the precision of Numerical Weather Prediction (NWP) models. In addition, GPS-derived ZTD provides a new high-resolution tool for use in atmospheric sciences. Therefore, precise GPS-derived ZTD is valuable and beneficial.

ZTD estimation is traditionally estimated using the least squares (LS) principle. In order to employ the LS method for ZTD estimation, both the functional and stochastic models of GPS measurements need to be properly defined. The functional model, also called the mathematical model, describes the mathematical relationships between GPS measurements and unknown parameters, e.g. baseline components and ZTD. The stochastic model describes the statistical properties of the measurements, which are mainly defined by an appropriate covariance matrix indicating the uncertainty of, and the correlations between, the measurements[11,12]. Over the past two decades, the functional models for GPS measurements have been investigated in considerable detail. However, accurate stochastic modelling for GPS measurements is still both a controversial topic and a difficult task to implement in practice[13].

In the current stochastic models for use in estimating ZTD with GPS, it is usually assumed that all GPS measurements have the same variance. The time-invariant covariance matrix of the double-differenced (DD) measurements is then constructed using the error propagation law. Such assumptions are unrealistic, as GPS measurement errors are dominated by the systematic errors caused by multipath, ionosphere, and orbit effects, which are quite different for each satellite. Therefore, measurements obtained from different satellites cannot have the same accuracy due to varying noise levels[13,14]. This assumption will possibly degrade the ZTD estimation by GPS.

This communication aims to improve the ZTD estimation by stochastic modelling. First we describe the data processing method and strategy. Stochastic modelling methods and results are presented next.

The linear observation equations of GPS can be expressed as:

$$L = Ax + v, \tag{1}$$

where L is a vector of the observed-minus-computed DD carrier phase values ($O–C$), A is the design matrix, x is the unknown parameters (including ZTD, ambiguity, baseline, etc.) and v is the residual vector. Using the LS method, the unknown parameters and the accuracy indicator can be obtained, namely

$$\begin{aligned} \hat{x} &= (A^T(C_x)^{-1}A)^{-1}A^T(C_x)^{-1}L, \\ v &= L - A\hat{x}, \qquad (2) \\ \hat{\sigma}^2 &= \frac{v^T(C_x)^{-1}v}{n}, \end{aligned}$$

where n is the degree of freedom. It is easy to see that the estimation of unknown parameters x and its accuracy indicator is dependent on the stochastic model (the covariance matrix for the DD GPS measurements). Any mis-specifications of the stochastic model will result in unreliable

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unknown parameter estimations. In our initial studies here, the models to be considered are the ones that can be easily implemented in the GPS software package. Details of variations in the stochastic model are discussed below.

In a commonly used stochastic model, it is usually assumed that all the carrier phases or pseudo-ranges have the same variance (σ^2) and are statistically independent. Therefore, the observations ϕ are treated as independent and uncorrelated, and the covariance matrix of the observations ϕ can be formulated as:

$$\mathrm{Cov}(\phi) = \sigma^2 I, \tag{3}$$

where I is the unit matrix. Through the error propagation law, the time-invariant covariance matrix (called the stochastic model) of the DD measurements can be obtained.

$$C_x = \sigma^2 \begin{bmatrix} 4 & 2 & \cdots & 2 \\ 2 & 4 & \cdots & 2 \\ \vdots & \vdots & \ddots & 2 \\ 1 & 2 & \cdots & 4 \end{bmatrix}. \tag{4}$$

This is a standard stochastic model for DD measurements, which is easy to implement in practice. However, this simplified stochastic model may contain some misspecifications and thus could result in unreliable ZTD estimates.

As the distances between GPS stations in a network are different, the baseline length-dependent variances for GPS measurements are defined with the following function[15]:

$$\sigma^2 = \alpha^2 + \beta^2 {*} \mathrm{Distance}^2, \tag{5}$$

where $\alpha = 9$ mm and $\beta = 0.1$ mm/km. This formula describes the relative qualities of GPS measurements from different GPS baselines in a network. The baseline components may be treated as observations with defined uncertainty to improve the geometry of the network solutions for ZTD estimation.

GPS measurement errors are dominated by systematic errors, such as signal-to-noise ratio, atmospheric delay and multi-path errors, which may be closely connected with the satellite elevation angles. The effects of these error sources are different for each satellite. Therefore, GPS measurements from different satellites may not have the same accuracy. In order to model the variances of GPS measurements from different satellites, a function of satellite elevation angle is used to describe the variances of raw GPS measurements in practice, namely:

$$\sigma^2_{\phi_r^s(i)} = a^2 + b^2 * f^2(\mathrm{elev}_r^j(i)), \tag{6}$$

where a and b are constants and $f(\mathrm{elev}_r^j(i))$ is the function of satellite elevation angle at epoch i. Given the variances of one-way GPS measurements, the covariance matrix for DD measurements is derived using the error propagation law.

$$C_{L(i)} = a^2 \cdot T_{ai} + b^2 \cdot T_{bi}, \tag{7}$$

where T_{ai} is the same as eq. (4), and

$$T_{bi} = \begin{bmatrix} f_{1i} + f_{2i} & f_{1i} & \cdots & f_{1i} \\ f_{1i} & f_{1i} + f_{3i} & \cdots & f_{1i} \\ \vdots & \vdots & \ddots & \vdots \\ f_{1i} & f_{1i} & \cdots & f_{1i} + f_{ni} \end{bmatrix}, \tag{8}$$

and $f_{ji} = f(\mathrm{elev}_j^i(i)) + (\mathrm{elev}_j^i(i))$, $j = 1, 2, \ldots, n$.

Because of the complexity of unknown factors in stochastic modelling, the functional relationship between the accuracy of GPS measurements and satellite elevation angles can only be approximately expressed. The sine and cosine functions of satellite elevation angles are often used for this purpose. In the GAMIT software package, the sine function of the satellite elevation angle is currently used to calculate the accuracy of the one-way GPS measurements[15]:

$$\sigma^2_{\phi_r^s(i)} = a^2 + b^2 / \sin^2(\mathrm{elev}_r^j(i)), \tag{9}$$

where $a = 4.3$ mm and $b = 3$ mm. The function is a good approximation to tropospheric sensitivity. In addition, the cosine function of the satellite elevation angle is also used to calculate the accuracy of the one-way GPS measurements[16–18], expressed as:

$$\sigma^2_{\phi_r^s(i)} = a^2 + b^2 / \cos^2(\mathrm{elev}_r^j(i)). \tag{10}$$

For this study, the GAMIT software package has been modified to include the functions as one of the options for stochastic modelling, and other unusual used functions of the satellite elevation angle are not discussed here.

The test datasets used were from the Australian IGS GPS network and Eastern Asian IGS GPS network. The GAMIT software was used for data processing with tightly constrained positions of the IGS stations (horizontal and vertical coordinates were assigned a standard derivation of 0.005 and 0.010 m respectively). IGS precise orbits were used. Some parameters used in data processing were: cut-off elevation: 15 degrees, and GPS data sampling interval: 30 s. ZTD estimations for these stations were obtained at an interval of 2 h. The performance of the above four stochastic modelling methods for GPS measurements was evaluated: (i) Standard GPS processing method with a simplified stochastic model; (ii) Baseline length-dependent weighting; (iii) Satellite elevation angle-based sine function, and (iv) Satellite elevation angle-based cosine function.

Figure 1 *a* shows the standard deviations of ZTD estimations at the DARW station in Australia using different stochastic models. The standard deviations from method D are smallest, while those from method B are largest. The effect on the ZTD estimations is small (Figure 1 *b*). Figure 2 is the comparison at the USUD station, Japan,

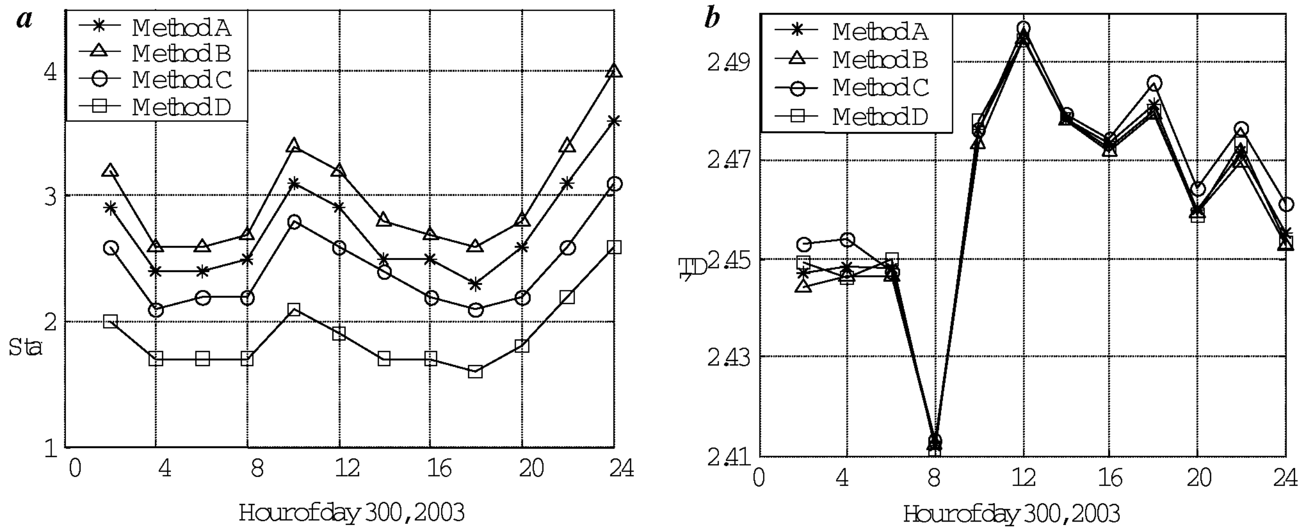

Figure 1. Comparison of standard deviation of ZTD estimation (*a*) and ZTD estimate on (*b*) at DARW station, Australia.

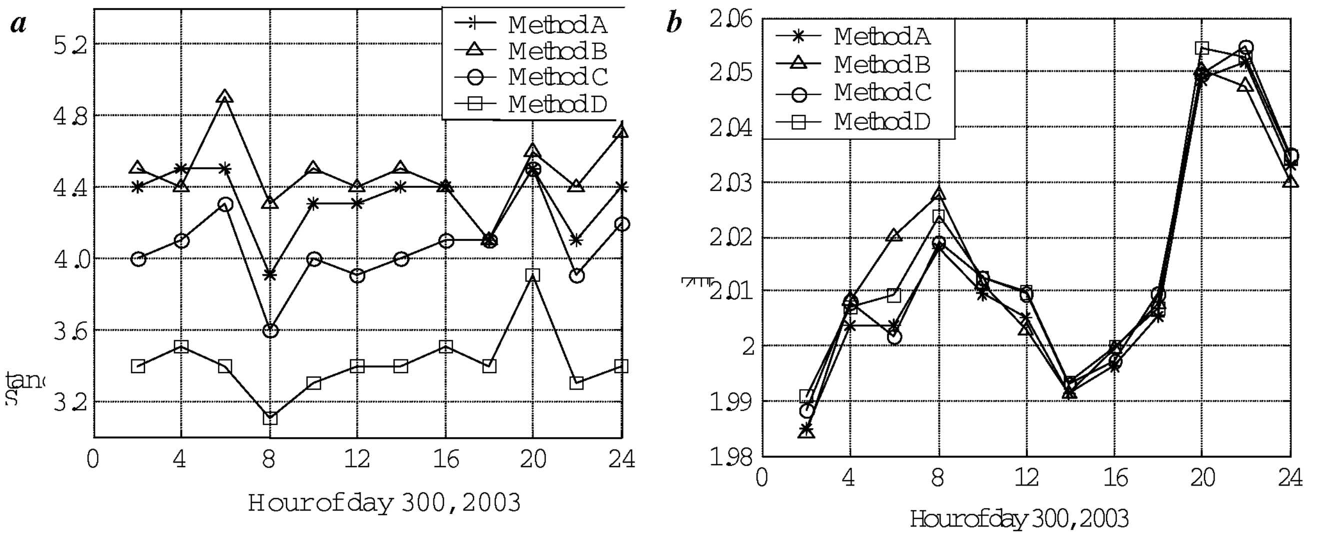

Figure 2. Comparison of standard deviation of ZTD estimations (*a*) and ZTD estimations (*b*) at the USUD station, Japan.

and the result is the same. The standard deviations of ZTD estimates based on method B are higher than those from method A. Method D is better than other tested stochastic models. Among all the tested stochastic modelling methods, method D has the best performance.

From above tests, the results show that misspecifications in the stochastic models will result in unreliable ZTD estimations. Using method D, the precisions of GPS-derived ZTD can be improved. IGS network data for days 325, 2003 and 151, 2004 were also tested, and conclusions are the same.

ZTD was usually estimated under the assumption that all GPS measurements have the same variance.

Our results have shown that misspecification in the stochastic models will result in unreliable ZTD estimations. Using the satellite elevation angle-based cosine function, the precision of GPS-derived ZTD estimations can be improved by 30% (Figures 1*a* and 2*a*). Therefore, the satellite elevation angle-based cosine function is proposed to be modified in the current popular scientific software, GAMIT.

This improvement of GPS-derived ZTD is certainly critical for reliable NWP applications and tropospheric research. This initial study has demonstrated that the stochastic model methods play an important role in the ZTD estimation process. Suitable stochastic modelling strategies for GPS measurements, baseline components (or the coordinates of GPS tracking stations) and ZTD parameters should be further investigated.

1. Fang, P., Bevis, M., Bock, Y., Gutman, S. and Wolfe, D., GPS meteorology: Reducing systematic errors in geodetic estimates for zenith delay. *Geophys. Res. Lett.*, 1998, **25**, 3583–3586.
2. Tregoning, P., Boers, R. and O'Brien, D., Accuracy of absolute precipitable water vapour estimates from GPS observations. *J. Geophys. Res.*, 1998, **103**, 701–710.
3. Van Hove, T. M., Alber, C. and Johnson, J. M., Atmospheric water vapour as noise and signal for global positioning system applications. In 6th International Technical Meeting of the Satellite Division of the U.S. Institute of Navigation, Salt Lake City, Utah, 22–24 September 1993, pp. 797–804.
4. Emardson, T. R., Elgered, G. and Johansson, J. M., Three months of continuous monitoring of atmospheric water vapour with a network of GPS receivers. *J. Geophys. Res.*, 1998, **103**, 1807–1820.
5. Zhang, Z. *et al.*, Parallel computing of a variational data assimilation model for GPS/MET observation using the bay-tracing method. *Adv. Atmos. Sci.*, 2004, **21**, 220–226.
6. Niell, A. E., Global mapping functions for the atmosphere delay at radio wavelengths. *J. Geophys. Res.*, 1996, **101**, 3227–3246.
7. Bevis, M. *et al.*, GPS meteorology: Mapping zenith wet delays onto precipitable water. *J. Appl. Meteorol.*, 1994, **33**, 379–386.
8. Duan, J. *et al.*, GPS meteorology: Direct estimation of the absolute value of precipitable water. *J. Appl. Meteorol.*, 1996, **35**, 830–838.
9. Manuel, H. *et al.*, A new strategy for real-time integrated water vapour determination in WADGOPS networks. *Geophys. Res. Lett.*, 2001, **28**, 3267–3270.
10. Vedel, H., Mogensen, K. S. and Huang, X., Calculation of zenith delay from meteorological data, comparison of NWP model, radiosondes and GPS delay. *Phys. Chem. Earth*, 2001, **26**, 497–502.
11. Rizos, C., *Principles and Practice of GPS Surveying Monograph 17*, School of Geomatic Engineering, University of New South Wales, 1997, p. 555.
12. Brunner, F. K., Hartinger, H. and Troyer, L., GPS signal diffraction modelling: The stochastic SIGMA-Δ model. *J. Geodesy*, 1999, **73**, 259–267.
13. Wang, J., Stewart, M. and Tsakiri, M., Stochastic modelling for static GPS baseline data processing. *J. Surv. Eng.*, 1998, **124**, 171–181.
14. Bona, P., Precision, cross correlation, and time correlation of GPS phase and code observations. *GPS Solut.*, 2000, **4**, 3–13.
15. King, R. W. and Bock, Y., Documentation for the GAMIT GPS analysis software, Massachusetts Institute of Technology, Cambridge, Mass. USA, 1999.
16. Jin, X. X. and de Jong, C. D., Relationship between satellite elevation and precision of GPS code observations. *J. Navig.*, 1996 **49**, 253–265.
17. Barnes, J. B., Ackroyd, N. and Cross, P., Stochastic modelling for very high precision real-time kinematic GPS in an engineering environment. In XXI International Congress of FIG, Brighton, UK, 19–25 July 1998.
18. Hugentobler, U., Schaer, S. and Fridez, P., Bernese GPS software version 4.2, Astronomical Institute, University of Bern, 2001.

ACKNOWLEDGEMENTS. We thank Drs Bob King and Peng Fang for their valuable discussions and help during this study.

Received 28 April 2005; accepted 14 June 2005

Efficient microwave-assisted hydrolysis of triolein and synthesis of bioester, bio-surfactant and glycerides using *Aspergillus carneus* lipase

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Microwave irradiations are known to alter the rate of chemical reactions. Often this results in enhanced reaction rates; higher yields, purity of products; more efficient and homogeneous distribution of energy and heating effects, and easier water elimination. The enzyme lipase (triacylglycerol ester hydrolase, EC 3.1.1.3) is a special class of esterase enzyme that acts on fats and oils and hydrolyses them in steps into the substituted glycerides and fatty acids, and finally on complete hydrolysis into glycerol and fatty acid. Hydrolysis of triolein using a fungus, *Aspergillus carneus* lipase has been carried out both under normal conditions and microwave irradiations of LOW 10 (175 W, 38–40°C) and HIGH 100 (800 W, 90°C). Analysis of the hydrolytic products on TLC plates showed the presence of triolein, diolein, monolein and oleic acid in 24 h under normal conditions, compared to microwave irradiations where it took 45–90 s under LOW power level and 15–30 s under HIGH power level for the production of monolein and oleic acid. However, complete hydrolysis of triolein took place in 160 s at LOW power and 75 s at HIGH power level. The synthesis of bioester, biosurfactant and glycerides could successfully be carried out rapidly within 30 s under both solvent-containing and solvent-free condition under HIGH power microwave irradiation.

Keywords: *Aspergillus carneus*, hydrolytic and synthesis reactions, lipase, microwaves.

MICROWAVES (0.3–300 GHz) fall between the infrared and radio frequency region of the electromagnetic spectrum[1]. In recent years, besides other uses, microwaves have gained considerable attention for their utilization as domestic ovens and other scientific applications[2]. Microwave irradiation is becoming an increasingly popular method of heating, replacing the classical one, because it proves to be a clean, cheap and convenient method[3]. Often, it affords higher yield and results in shorter reaction times. These reactions are especially appealing as they can be carried out in open vessels, thus avoiding the risk of development of high pressures. Gedye *et al.*[4] were the first to use the microwave oven in organic synthesis. Since then, this technique has evolved into a useful tool[5–9]. Currently, besides other uses in biological reactions, microwaves are being utilized to assist hydro-

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