Ramanujan Mathematical Society Lecture Notes Series, Number 2: The Riemann Zeta Function and Related Themes. S. D. Adhikari *et al.* (eds). Ramanujan Mathematical Society, Department of Mathematics, University of Mysore, Mysore 570 006. 2007.

The legacy of the genius of Srinivasa Ramanujan in number theory lives on in India, with a series of eminent contributors to number theory. K. Ramachandra is rightly viewed in the community as a leading light of the second half of the twentieth century in the area of analytic number theory. A conference was held at the National Institute of Advanced Studies, Bangalore, during 13–15 December 2003, in which Ramachandra was felicitated in the context of his having turned 70 during that year. The participants included many leading mathematicians from around the world.

This volume represents proceedings of the conference, published by the Ramanujan Mathematical Society, co-sponsored by IMSc, Chennai. The proceedings contain papers on a broad spectrum of topics in contemporary number theory. The volume also contains an enlightening exposition of Ramachandra's contributions to transcendental number theory, by Michel Waldschmidt. In what follows, I will describe some highlights of the results presented in the volume. For this purpose, the contents are classified into five parts.

1. Prime number theory and Zeta-function theory: An important problem in prime number theory is to study the asymptotic of the quantity $\pi(x) - \pi(x-y)$ with $y \le x$. Here Zaccagnini proves: If $x^{-(5/6)-\varepsilon(x)} \le \theta \le 1$, $0 \le \varepsilon(x) \le 1/6$, $\varepsilon(x) \to 0$ as $x \to \infty$, then

$$I(x,\theta) := \int_x^{2x} \left| \pi(t) - \pi(t - \theta t - \frac{\theta t}{\log t} \right| \mathrm{d}t$$

$$<< \frac{x^3\theta^2}{(\log x)^2}\left(\varepsilon(x) + \frac{\log\log x}{\log x}\right)^2.$$

In particular, for he θ in the above range, the interval $[t - \theta t, t]$ contains $\sim\theta t(\log t)^{-1}$ primes for almost all integers $t \in [x, 2x]$. The interesting inverse question, whether a bound for $I(x, \theta)$ implies something like zero-density theorem or a zero-free region for $\zeta(s)$, is also answered affirmatively.

Let $s = \sigma + it$. For σ sufficiently large, the Lerch zeta-function is defined by

$$\phi(\xi, a, s) = \sum_{n=0}^{\infty} \frac{\mathrm{e}^{2\pi i \xi n}}{(n+a)^s},$$

where $0 < a \le 1$. Balasubramanian, Kanemitsu and Tsukada establish the functional equation of ϕ and the mean-square of ϕ with respect to a, using the difference equation techniques.

An important Ω-theorem of $\zeta(s)$ on the line 1 has been proved by A. Granville and Soundararajan. They establish that: There are arbitrarily large values of t for which the inequality

$$|\zeta(1+it)| \ge \mathrm{e}^{\gamma}(\log_2 t + \log_3 t - \log_4 + O(1))$$

holds, where $\log_j u$ means the jth iterated logarithm. This improves upon a result of Levinson. Large moments of short Euler products

$$|\zeta(s,y) := \prod_{p \le y} (1 - p^{-s})^{-1}$$

on the line 1 have been established and extensions of these results to Dirichlet L-functions are also discussed.

Power moments of $\zeta(s)$ on the critical line play a significant role in the theory. In this connection, A. Ivić proves the upper bounds for the quantity

$$J_k(t,G) = \frac{1}{\sqrt{\pi}G} \int_{-\infty}^{\infty} \left| \zeta\left(\frac{1}{2} + it + iu\right) \right|^{2k} \times \mathrm{e}^{-(\frac{u}{G})^2} \mathrm{d}u.$$

where $t \asymp T$, $T^{\varepsilon} \le G << T$ and $k \in \mathbb{N}$.

2. Theory of partitions: A partition $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)$ is said to be an N-parity partition if $\lambda_i \equiv i \pmod N$, $i = 1, 2, \ldots, m$. This generalizes the notion of odd–even partitions and thereby Adiga, Anitha and Han have obtained a generalization of the Rogers–Ramanujan partition identity.

Ideas from probabilistic number theory are useful in studying the measures on partitions of integers. In an article, Jogesh Babu discusses the connection between Ewens sampling formula in population genetics and the partitions of an integer generated by random permutations. Functional limit theory for partial sum processes induced by Ewens sampling is also reviewed by him.

A proof of a revised conjecture of Andrews involving the functions $A_{\lambda,k,a}(n)$ and $B_{\lambda,k,a}(n)$ (which stand for the number of partitions of satisfying certain sets of conditions that we shall not go into here) is presented by Chandrashekara, Padmavathamma and Raghavendra.

For $n > 1$, define

$$\prod_{n \ge 2} (1 + \mathrm{e}(n)n^{-s}) = 1 + \sum_{n=2}^{\infty} f(n)n^{-s},$$

where $\mathrm{e}(n)$ is subject to some specified restrictions. The class $\mathcal{C}$ of such functions $f(n)$ includes besides others, the function $q^*(n)$ defined as the number of product partitions of n into distinct parts. A general asymptotic result has been obtained for the sum $\sum_{n \le x} q^*(n)$ under certain conditions by Kátai and Subbarao.

3. Multiplicative and elementary number theory: Let $q \ge 2$ be a fixed integer. The q-ary expansion of n is defined by

$$n = \sum_{\nu=2}^{\infty} \varepsilon_{\nu}(n) q^{\nu},$$

where $\varepsilon_{\nu}(n) \in \{0, 1, 2, \ldots, q-1\}$. A function g: $\mathbb{W} \to \mathbb{C}$ is said to be q-multiplicative if $g(0) = 1$,

$$g(n) = \prod_{j=0}^{\infty} g(\varepsilon_j(n) q^j)$$

for $n \in \mathbb{N}$. Define the sum digit function $\alpha_q(n)$ to be $\sum \varepsilon_j(n)$. Currently it is not known whether there exist infinite sequences of primes $\{p_j\}$, $\{q_j\}$ such that $\alpha_2(p_j)$ is even and $\alpha_2(q_j)$ is odd for $j = 1, 2, \ldots$. Kátai and Subbarao prove that there exists a constant $c > 1$ with the following property: Let g be a q-multiplicative function, $|g(n)| = 1$ and $B > 0$ is a constant. Assume that there exists a sequence of integers $N_1 < N_2 < \cdots$ and a sequence of integers $k_{\nu} \in [1, B \log N_{\nu}]$ and α_{ν} such that

$$g(\pi) = \begin{cases} \alpha_{\nu} & \text{if } \pi \in [q^{N_{\nu}}, 4q^{N_{\nu}}], \\ 1 & \text{if } \pi \in \mathcal{P}_k, \end{cases}$$

where $\mathcal{P}_k$ is the set of square-free numbers with exactly k-prime factors. Then, there exists some integer $r \in [1, c]$ such that $g^r(nq) = 1$, $(n \in \mathbb{N})$.

It is believed that

$$\overline{\lim_{n\to\infty}} (\log\log n)^{-(k+1)} \sum_{d|n} \frac{(\log d)^k}{d} = c_k \mathrm{e}^{\gamma},$$

for all positive integers k, where c_k is a positive constant and γ is the Euler's constant. This has been established when $k = 2$ or $k = 3$ with specific constants $c_2 = 3/2$ and $c_3 = 17/6$ by Sitaramaiah and Subbarao.

4. Algebraic, Transcendental number theory and diophantine equations: Let K be a number field with O_K – the ring of integers. Chahal and Ram Murty show that: If $f \in O_K[x]$ takes on irreducible values infinitely often, then either f itself is irreducible or $f = l(x)p(x)$ where $l(x)$ is a linear factor and $p(x)$ is an irreducible polynomial. Some examples of the second possibility are also exhibited.

Let f be an arithmetic function, periodic in q defined by $f(n) = \pm 1$, if n is not a multiple of q and 0 otherwise. Consider the sum

$$S = \sum_{n=1}^{\infty} \frac{f(n)}{n}.$$

With certain finite number of exceptional values of q, the non-vanishing of S has been established by Saradha. It is also shown in the same article that $S \neq 0$ whenever $2\phi(q) \geq q(1 - \frac{1}{h})$, where

$$h = \max_{1 \leq i \leq r} p_i^{\alpha_i} \text{ and } q = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_r^{\alpha_r}.$$

In an expository article by Waldschmidt, a theorem of Ramachandra is applied to algebraically additive functions, new consequences of this theorem are dealt in detail. For example, let E be an elliptic curve which is defined over the field of algebraic numbers and let Γ be a finitely generated subgroup of algebraic points on E. The question whether Γ is dense in $E(\mathbb{C})$ for the complex topology being dealt with, and other contributions of Ramachandra to transcendental number theory are also discussed in detail.

For any integer $\nu > 1$, let $P(\nu)$ and $\omega(\nu)$ denote the greatest prime factor of ν and the number of distinct prime divisors of ν respectively. Shorey proves that the equation

$$n(n + d)\cdots(n + (j - 1)d)(n + (j + 1)d) \\ (n + (k - 1)d) = by^2,$$

where $0 < j < k$, $d > 1$, $P(b) < k$ and b is square-free, does not hold whenever $k \in \{6, 7, 8\}$.

5. Combinatorial number theory: Let $d \geq 1$ and $n \geq 2$ be given integers. Define $s(n, d)$ to be the least positive integer t such that any given sequence $a_1, a_2, \ldots, a_t$ (not necessarily distinct) in $\mathbb{Z}^d$, has a subsequence $a_{i_1}, a_{i_2}, \ldots, a_{i_n}$ satisfying $a_{i_1} + a_{i_2} + \cdots a_{i_n} \equiv 0 \pmod{n}$. Erdös, Ginzburg and Ziv have proved that $s(n, 1) = 2n - 1$. Reiher has shown that $s(n, 2) = 4n - 3$. For other values of $d \geq 3$, the exact value of $s(n, d)$ is unknown. Adhikari and Rath discuss the recent result of Reiher and some generalizations of this problem. Gao and Thangadurai address a non-abelian version of this problem.

On the whole, the proceedings contain high quality research articles spreads over the vast and versatile interests of Ramachandra (it may be noted that in each of the above-mentioned areas of number theory Ramachandra has significant contributions) and hence I am sure that this volume would be a treasure to workers in the area, in the years to come.

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Children with Cerebral Palsy: A Manual for Therapists, Parents and Community Workers. Archie Hinchcliffe. SAGE Publications India Pvt Ltd, B1/11, Mohan Cooperative Industrial Area, Mathura Road, New Delhi 110 044. 2007. Second Edition. 257 pp. Price: Rs 495.

This book on cerebral palsy (CP) whose contents can be truly expected to motivate the users in their respective special areas of work concerning the disability, is highly and effectively communicative in terms of usage of language, style, format and presentation techniques, without any dilution of the technical, diagnostic, treatment and therapeutic approaches to the disability. In developing countries where there is a high correlation between poverty and incidence of CP and lack of medical facilities, the author's experiences in the Middle East and emphasis on holistic approach with a sound scientific base come across in the book as an empowering guide even to parents and community workers.

The book is divided into nine chapters. The salient features include vital analysis of transition from medical to social model of rehabilitation for persons with CP and the rationale to be proactively followed that is important for the community to mobilize resources to support persons with CP. But it is equally important that interventions based on time-tested expertise are essentially utilized if one has to maximize the potentials of children with CP and make them as independent as possible.

Chapter 1 provides a comprehensive overview of CP, a chapter to be read by every General Pracitioner. Chapters 2–4 spell out the 'last words in the *Bible*(!)', starting from accessing the CP child to observation and analysis, assessment and therapeutic approaches, including adaptation of aspects of speech and language therapy techniques, involving eating and swallowing, etc.

Chapters on observation and analysis, provide the appropriate and positive rehabilitation intervention services which would make the goals relevant to the CP child achievable. The author's approach in dealing with major operational components of observation and analysis in detail, if well read, understood and practised by the therapist would act as the most powerful intervention programming and transdisciplinary tool in the training of parents and community workers.

The emphasis on systematic functional assessment for children with CP with the backdrop of both neurodevelopmental theory and details of guidance provided, is a boon to the therapist who has to work in locations where access to multidisciplinary inputs is not available.

The illustrations based on the drawings can be viewed with clarity and even a parent who is not literate can easily follow the guidelines for seating, positioning, motor patterning with ease. Chapter 4 on contractures and deformities as a separate chapter helps focus on the preventable aspects of these conditions since the CP child is not born with these problems. The assessment techniques spelt out in detail regarding threatened and established contractures and technically sound therapeutic guidance have great contributory value to preventive aspects in the rehabilitation of a child with CP. Common deformities in CP are well illustrated and information on surgical intervention for CP is briefly but appropriately described in this context.