thousands of years. This is relevant especially for the Lonar crater, which is estimated to be 50,000 years old. The same will be true for the Barringer Crater.

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Optimal ordering policies of inventory model for deteriorating items having generalized Pareto lifetime

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In this communication, we develop and analyse an inventory model with the assumption that the lifetime of the commodity is random and follows a generalized Pareto distribution. It is also assumed that the demand is a function of stock and the money value is subject to inflation. Using the differential equations, the instantaneous state of inventory is derived. With suitable cost consideration, the total cost function is obtained. Minimizing total cost function, the optimal ordering quantity and cycle length are obtained. This model is useful in practical situations arising at places like the food and vegetable markets, oil industry and photochemical industry.

Keywords: Generalized Pareto distribution, inflation, perishable models, stock-dependent demand.

RECENTLY, much emphasis has been given in developing inventory models for deteriorating items with random lifetime. Several workers have reviewed the inventory models for deteriorating items[1–3]. In the study of inventory models for deteriorating items, the lifetime of the commodity plays a dominant role. Several researchers have studied the inventory models with exponential lifetime Tadikamalla[4] developed inventory models with gamma distribution for deterioration. Inventory model with Weibull distribution for the lifetime of a commodity has also been studied[5–7]. Nirupama Devi[8] has studied the inventory model with the assumption that the lifetime of a commodity follows a two-component Weibull distribution. No serious attempt has been made to develop and analyse inventory models with generalized Pareto distribution, except the work of Srinivasa Rao *et al.*[9], who studied the models with the assumption that the demand is a function of selling price or is time-dependent. They assumed that the money value is fixed and remains constant (without inflation). However, in deteriorating items like food and vegetables, photographic films and electronics, when a price increase is anticipated, then a large amount of items may be purchased, but the money value may change during the planning period with an inflation rate and the demand is stock-dependent. Several researchers have examined the inflationary effect on an inventory policy. Buzacott[10] developed an approach of modelling inflation

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by assuming a constant inflation rate. Misra[11] proposed an inflation model for the economic order quantity (EOQ), in which the time value of money and different inflation rates were considered. Mangiamely *et al.*[12] have reviewed and classified the models. Brahmbhatt[13] developed an EOQ model under a variable inflation rate. Hwang and Sohn[14] developed a deterministic inventory model for items that deteriorate continuously and follow an exponential distribution when a price increase is anticipated. Gupta and Vrat[15] developed a multi-item inventory model with a resource constraint system under a variable inflation rate. They have introduced the stock-dependent phenomenon in modelling inventory systems assuming the consumption rate to be a function of the order quality. Baker and Urban[16], and Mandall and Phaujdhari[17] have assumed nonlinear functions of the on-hand inventory. These authors have not considered the perishability of the item and the possibility of shortages in developing the inventory models. Padmanabhan and Vrat[18] developed an EOQ model for items having stock-dependent demand and exponential decay. Datta and Pal[19] considered the demand rate a linear function of the on-hand inventory in developing the inventory models for deteriorating items.

In this communication we develop and analyse an inventory model with the assumption that the lifetime of a commodity is random and follows a generalized Pareto distribution and the demand is stock-dependent, having constant rate of inflation. The generalized Pareto distribution is extensively used in the analysis of extreme events, especially in reliability studies, when robustness is required against heavier time or lighter time alternatives to an exponential distribution. Using differential equations the instantaneous state of inventory is derived. With suitable cost consideration, the total cost function is obtained and minimized with respect to the ordering quantity and cycle length. The sensitivity of the model has also been studied.

The following assumptions and notations have been used: (i) Demand rate is known and constant; (ii) Replenishment is instantaneous; (iii) Lead time is zero. (iv) Shortages are not allowed; (v) The length of one cycle is T; (vi) Inflation rate is a constant, say k (Rs/unit time); (vii) The inventory holding cost per unit per unit time is h; (viii) Deteriorated item is lost, and (ix) Length of planning horizon is H.

The lifetime of a commodity is random and follows a generalized Pareto distribution having probability density function of the form

$$f(t)=\begin{cases}\dfrac{1}{a}\left(1-\dfrac{ct}{a}\right)^{\frac{1}{c}-1} & c\neq 0\\[2ex] \dfrac{1}{a}\mathrm{e}^{-\frac{t}{a}} & c\neq 0.\end{cases}$$

Then the instantaneous rate of deterioration $h(t)$ is

$$h(t)=\frac{f(t)}{\left(1-\int_0^t f(u)\,\mathrm{d}u\right)}=\frac{1}{a-ct}.$$

Further, demand rate λ is considered stock-dependent, i.e. it depends on the ordering size and is of the form $\lambda=\alpha+\beta Q$, where α, β are constants and positive, and Q the ordering quantity in one cycle.

The cost of placing an order at time t is $A(t)$, which is time-dependent and is of the form $A(t)=A_0\mathrm{e}^{kt}$, where A_0 is the cost of placing an order at time zero.

The cost of one unit at time t is $C(t)$, which is also a function of time, i.e. $C(t)=C_0\mathrm{e}^{kt}$, where C_0 is the cost price of one unit at time zero.

Let $I(t)$ be the inventory level of the system at time t $(0\leq t\leq T)$. Then the differential equation of the instantaneous state of $I(t)$ over the cycle length T is

$$\frac{\mathrm{d}}{\mathrm{d}t}I(t)+I(t)h(t)=-\lambda,\quad 0\leq t\leq T \tag{1}$$

with initial condition $I(T)=0$.

Solving the differential equation the on-hand inventory at time t is obtained as

$$I(t)=\frac{\lambda}{1-c}[(a-cT)^{1-1/c}(a-ct)^{1/c}-(a-ct)],\quad 0\leq t\leq T. \tag{2}$$

Let the horizon (H) consist of m cycles of length T, where m is an integer. For the number of replenishments to be made during the period H, since T is a constant interval of time between replenishments, we can assume $H=mT$.

Since the total system cost during the planning period H is the sum of the unit cost, inventory holding cost and replenishment cost, the total system cost can be expressed as

$$K(T,Q)=C+C_\mathrm{h}+C_\mathrm{r}, \tag{3}$$

where C is the cost of the units, C_h the inventory holding cost and C_r the replenishment cost in the interval $(0, H)$.

Cost of the units in $(0, H)$ is obtained as

$$\begin{aligned}C&=Q\left[C(0)+C(T)+C(2T)+\cdots+C(m-1)T\right]\\&=QC_0\left[\frac{\mathrm{e}^{kH}-1}{\mathrm{e}^{kT}-1}\right].\end{aligned} \tag{4}$$

Inventory holding cost in $(0, H)$ is obtained as

$$C_\mathrm{h}=h\sum_{n=0}^{m-1}C(nT)\left[\int_0^T I(nT+t)\mathrm{d}t\right]. \tag{5}$$

From eq. (2), $I(nT + t)$ can be written as

$$I(nT+t)=\frac{\lambda}{1-c}[(a-cT)^{1-1/c}(a-ct)^{1/c}-(a-ct)]. \quad (6)$$

Substituting eq. (6) in eq. (5), the inventory holding cost C_h is obtained as

$$C_h = h\sum_{n=0}^{m-1} C(nT) \times \left[\int_0^T \frac{\lambda}{1-c}[(a-cT)^{1-1/c}(a-ct)^{1/c}-(a-ct)]\mathrm{d}t\right].$$

On simplification

$$C_h = \lambda h C_0\left[\frac{\mathrm{e}^{kH}-1}{\mathrm{e}^{kT}-1}\right] \times \left[\int_0^T \frac{1}{1-c}[(a-cT)^{1-1/c}(a-ct)^{1/c}-(a-ct)]\mathrm{d}t\right]. \quad (7)$$

Replenishment cost in $(0, H)$ is obtained as

$$C_r = A(0) + A(T) + A(2T) + \cdots + A[(m-1)T] = A_0\left[\frac{\mathrm{e}^{kH}-1}{\mathrm{e}^{kT}-1}\right]. \quad (8)$$

Substituting eqs (3), (7) and (8) in eq. (3), the total system cost over $(0, H)$ can be obtained as

$$K(T,Q)=\left[\frac{\mathrm{e}^{kH}-1}{\mathrm{e}^{kT}-1}\right]\left\{A_0+QC_0+\lambda hC_0 \times \left[\int_0^T \frac{1}{1-c}[(a-cT)^{1-1/c}(a-ct)^{1/c}-(a-ct)]\mathrm{d}t\right]\right\}. \quad (9)$$

Substituting the quadratic approximation of $\mathrm{e}^{kT} = 1 + kT + (kT)^2/2$ in eq. (9), the total system cost is obtained as:

$$K(T,Q)=\left[\frac{\mathrm{e}^{kH}-1}{kT+\frac{(kT)^2}{2}}\right]\left\{A_0+QC_0+\frac{\lambda hC_0}{1-c} \times \left[\int_0^T [(a-cT)^{1-1/c}(a-ct)^{1/c}-(a-ct)]\mathrm{d}t\right]\right\}. \quad (10)$$

Substituting $\lambda = \alpha + \beta Q$ in eq. (10), $K(T, Q)$ can be obtained as:

$$K(T,Q)=\left[\frac{\mathrm{e}^{k+1}-1}{kT+\frac{k^2T^2}{2}}\right]\left[A_0+QC_0+hC_0\left(\frac{\alpha+\beta Q}{1-c}\right) \times \int_0^t [(a-cT)^{1-1/c}-(a-ct)]\mathrm{d}t\right]. \quad (11)$$

The initial inventory after replenishment is

$$I(0) = Q = \frac{\lambda}{1-c}[a^{1/c}(a-cT)^{1-1/c}-a] = \lambda\left[\frac{a^{1/c}(a-cT)^{1-1/c}-a}{1-c}\right]. \quad (12)$$

Substituting $\lambda = \alpha + \beta Q$ in eq. (12) we get

$$Q=\frac{\alpha D}{(1-\beta D)}, \quad (13)$$

where $D = \left[\frac{a^{1/c}(a-cT)^{1-1/c}-a}{1-c}\right]$.

Substituting eq. (13) in eq. (11) and expanding and neglecting higher powers of $1/a$ in $(a - ct)^{1-1/c}$, the total system cost is obtained as

$$K(T)=\left[\frac{\mathrm{e}^{kH}-1}{kT+\frac{(kT)^2}{2}}\right] \times \left\{A_0+\frac{\alpha C_0\left[T+\frac{T^2}{2a}+\frac{(1+c)T^3}{6a^2}+\frac{(1+c)(1+2c)T^4}{24a^3}\right]}{1-\beta\left[T+\frac{T^2}{2a}+\frac{(1+c)T^3}{6a^2}+\frac{(1+c)(1+2c)T^4}{24a^3}\right]} + hC_0\alpha\left[\frac{T^2+\frac{T^3}{6a}+\frac{(1+2c)}{24a^2}T^4+\frac{(1+2c)(1+3c)}{120a^3}T^5}{1-\beta\left[T+\frac{T^2}{2a}+\frac{(1+c)T^3}{6a^2}+\frac{(1+c)(1+2c)T^4}{24a^3}\right]}\right]\right\}. \quad (14)$$

For obtaining the optimal cycle length of the system, differentiate eq. (14) with respect to T and equate to zero. This gives

Table 1. Optimal values of cycle length, total system cost and ordering quantity

a	*c*	*k*	A_0	α	*h*	*C*	β	*H*	*T**	*TC*	*Q**
105	0.3	0.8	5	170	0.2	1	1	1	0.16601	311.8449	33.8719
106									0.16602	311.842	33.8729
108									0.16603	311.837	33.8748
106	0.2	0.8	5	170	0.2	1	1	1	0.16602	311.84255	33.87291
	0.3								0.16602	311.84257	33.87290
	0.4								0.16602	311.84258	33.87288
105	0.3	0.81	5	170	0.2	1	1	1	0.16638	312.71	33.9622
		0.82							0.16675	313.58	34.05
		0.83							0.16712	314.44	34.143
105	0.3	0.8	6	170	0.2	1	1	1	0.17876	319.4399	37.042
			7						0.19011	326.507	39.9509
			8						0.20038	333.156	42.65304
105	0.3	0.8	5	171	0.2	1	1	1	0.16561	313.446	33.97318
				172					0.16522	315.0474	34.07417
				173					0.16483	316.647	34.1748
105	0.3	0.8	5	170	0.3	1	1	1	0.16207	314.037	32.91035
					0.4				0.15839	316.1722	32.0216
					0.5				0.1549	318.25	31.197
105	0.3	0.8	5	170	0.2	2	1	1	0.12404	578.256	24.089
						3			0.1039	838.051	19.725
						4			0.091	1094.4	17.1194
105	0.3	0.8	5	170	0.2	1	2	1	0.1034	356.302	22.1759
							3		0.0791	390.803	17.66
							4		0.065	420.593	15.115
105	–6	0.8	5	170	0.2	1	1	1	0.16601	311.8449	33.87259
106									0.16602	311.84173	33.87359
108									0.16603	311.83704	33.87548
105	–5	0.8	5	170	0.2	1	1	1	0.16601	311.84427	33.87248
	–4								0.16601	311.84441	33.87237
	–3								0.16601	311.84445	33.87226
105	–6	0.81	5	170	0.2	1	1	1	0.16638	312.7142	33.96289
		0.82							0.16675	313.5824	34.05355
		0.83							0.16712	314.44874	34.14458
105	–6	0.8	6	170	0.2	1	1	1	0.17876	319.4389	37.04302
			7						0.19012	326.5061	39.9519
			8						0.2003	333.155	42.65406
105	–6	0.8	5	171	0.2	1	1	1	0.16562	313.4456	33.9738
				172					0.16522	315.046	34.0748
				173					0.16483	316.64687	34.1755
105	–6	0.8	5	170	0.3	1	1	1	0.16207	314.0366	32.9109
					0.4				0.15839	316.1715	32.02218
					0.5				0.15495	318.25289	31.19777
105	–6	0.8	5	170	0.2	2	1	1	0.12404	578.255	24.089
						3			0.10392	838.05021	19.72611
						4			0.09143	1094.42	17.1146
105	–6	0.8	5	170	0.2	1	2	1	0.10341	356.30195	22.17604
							3		0.07795	390.802	17.66693
							4		0.06557	420.5929	15.11535

$$\alpha C_0\left[1+\frac{T}{a}+\frac{(1+c)T^2}{2a^2}+\frac{(1+c)(1+2c)T^3}{6a^3}\right]+hC_0\alpha$$

$$\times\left[\left[1-\beta\left[T+\frac{T^2}{2a}+\frac{(1+c)T^3}{6a^2}+\frac{(1+c)(1+2c)T^4}{24a^3}\right]\right]\right.$$

$$\times\left[T+\frac{T^2}{2a}+\frac{(1+2c)T^3}{6a^2}+\frac{(1+2c)(1+3c)T^4}{24a^3}\right]$$

$$+\beta\left[\frac{T^2}{2}+\frac{T^3}{6a}+\frac{(1+2c)T^4}{24a^2}+\frac{(1+2c)(1+3c)T^5}{120a^3}\right]$$

$$\times\left.\left[1+\frac{T}{a}+\frac{(1+c)T^2}{2a^2}+\frac{(1+c)(1+2c)T^3}{6a^3}\right]\right]-\frac{(k+k^2T)}{\left(kT+\frac{k^2T^2}{2}\right)}$$

$$\times\left[A_0\left[1-\beta\left[T+\frac{T^2}{2a}+\frac{(1+c)T^3}{6a^2}+\frac{(1+c)(1+2c)T^4}{24a^3}\right]\right]\right.$$

$$+\alpha C_0\left[T+\frac{T^2}{2a}+\frac{(1+c)T^3}{6a^2}+\frac{(1+c)(1+2c)T^4}{24a^3}\right]$$

$$+hC_0\alpha\left[\frac{T^2}{2}+\frac{T^3}{6a}+\frac{(1+2c)T^4}{24a^2}+\frac{(1+2c)(1+3c)T^5}{120a^3}\right]$$

$$\times\left.\left[1-\beta\left[T+\frac{T^2}{2a}+\frac{(1+c)T^3}{6a^2}+\frac{(1+c)(1+2c)T^4}{24a^3}\right]\right]\right]=0. \quad (15)$$

Solving eqs (13)–(15) iteratively using numerical methods for given values of a, c, k, A_0, h, α, β and C, we obtain the optimal values of Q^*, total system cost TC, and cycle length T^* (Table 1).

From Table 1 it is observed that when the scale parameter a increases, the optimal cycle length T^* and the optimal ordering quantity Q^* also increase, and the total system cost TC decreases, for other costs and parameters fixed. When the shape parameter c increases, TC increases and Q^* decreases, for other costs and parameters fixed. When the inflation rate k increases, T^*, Q^* and TC also increase, when other costs and parameters remain constant. When the cost of placing an order (A_0) increases, then T^*, Q^* and TC also increase, when other costs and parameters are fixed. When α increases, Q^* and TC increase and T^* decreases, for other costs and parameters fixed. When the holding cost h increases, T^* and Q^* decrease, while TC increase, for other costs and parameters fixed. When the cost of the unit C increases, T^* and Q^* decrease while TC increases, for other costs and parameters fixed. When β increases, T^* and Q^* decrease, while TC increases, for other costs and parameters fixed.

The model developed is useful for analysing situations at several inventory control systems arising at places like food and vegetable markets, oil and photochemical industries.

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