# Spatial analysis of the frequency–magnitude distribution of aftershock activity of the December 2004 tsunamigenic Sumatra earthquake

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**The study of aftershocks provides constraints on the fault dimension and the physical properties of materials within a seismogenic volume. To understand the tectonic setting, mode of faulting and other parameters which control the behaviour of the aftershock sequence, it is useful to study the *b*-value and the Omori decay of any tectonic setting. We have analysed the frequency–magnitude distribution and other parameters using modified Omori's law describing the aftershock activity of the Sumatra earthquake of 26 December 2004. Data from the NEIC catalogue for aftershocks of magnitude larger than 4.0 were used in this study. The rupture length of 1300 km has been divided into four different regions, which includes the main shock and the first major aftershock of magnitude 7.5. The *b*-values estimated for these four regions were found to vary from 0.9 to 1.2. We interpret these variations in terms of the rupture propagation of the main shock and the inhomogeneities within the aftershock region.**

**Keywords:** Aftershocks, frequency–magnitude distribution, rupture propagation, Sumatra earthquake.

THE $M_w$ 9.3 Sumatra earthquake of 26 December 2004 is the second largest instrumentally recorded earthquake. It had ruptured the entire length of the Andaman trench and the co-seismic rupture extended 1300 km, with the aftershock activity observed all along the rupture zone[1,2] (Figure 1). The aftershock activity occurred along the entire plate interface extending from the Simeulue Island in the south through the Andaman and Nicobar Islands. The seismicity patterns of the aftershocks have been divided into four regions and these sub-regions were chosen on the basis of rupture propagation scenario as proposed by Lay *et al.*[1] (Figure 1). The width of the mega-thrust in the Sumatra region[3] is about 200 km and decreases to 160–170 km in the Andaman and Nicobar islands in the north. Several studies have been carried out in order to understand the rupture process of this earthquake. Notable amongst them are those by Lay *et al.*[1] and Ammon *et al.*[2].

The intense aftershock activity which is still continuing today provides information about earthquake nucleation and the physical properties of the materials in the fault zone[4–6]. The tectonic setting and the mode of faulting control these aftershock sequences and their analysis yields information about the spatial variation of the aftershocks, *b*-value and the decay rate of the aftershock sequence. Spatial variations of *b*-values using the aftershocks have been extensively studied for various tectonic regimes such as the subducting slab[7] and fault zones[8]. In this communication we present a summary of the results of the modified Omori law and analysis of the spatial variation of the *b*-value along the entire rupture length of the mega thrust associated with the Sumatra earthquake.

The Gutenberg–Richter law that gives the relationship between the frequency of earthquake occurrence and the magnitude[9] is given by:

$$\log_{10}(N) = a - bM,$$

where $N$ is the cumulative number of the events having magnitude larger than or equal to $M$, $a$ and $b$ are constants. The *b*-value has been observed to vary spatially as well as temporally and is dependent on the stress regime, tectonic character of the region, material heterogeneity, and variation in the confining pressure and perturbations in temperature[10–12]. Generally, the global average value of

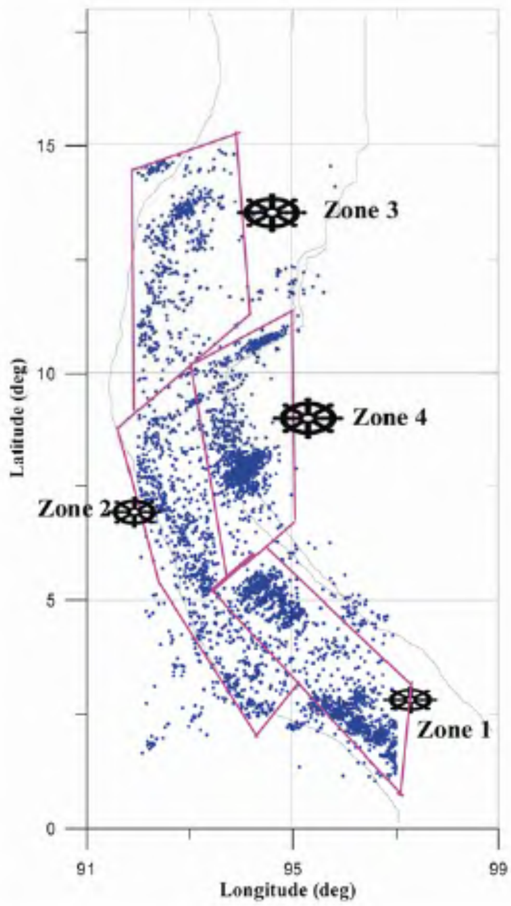


**Figure 1.** Location of the aftershocks of the Great Sumatra earthquake of 26 December 2004 (USGS data from December 2004 to December 2006) (* shows the main shock; the boxes define the four zones shown by the arrows and solid line shows the subduction front).

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the $b$ parameter for different crustal volume rocks and different tectonic regimes is 1.

The rate of decay of the aftershock sequence is empirically described with time and is given by the modified Omori law[13]:

$$N(t) = k/(t + c)^p,$$

where $N(t)$ is the frequency of aftershocks per unit time, and $k$, $c$ and $p$ are constants. $N(t)$ was calculated following the Ogata[14] approach for fitting the cumulative number of aftershocks as a function of time after the main shock. The characteristic parameter $p$ may be related to the structural heterogeneity, stress and temperature in the crust. The $k$ value is dependent on the total number of events in the sequence and $c$ on the rate of activity in the earliest part of the sequence.

In this study we have used about 4000 aftershocks of magnitude ($M_b$) greater than 4.0 and the earthquake hypocentral parameters, including the body wave magnitude have been obtained from the NEIC database. The epicentral parameters in the NEIC catalogue were determined using teleseismic stations operated by various networks. These aftershocks span over a period of two years since the occurrence of the 26 December 2004 event, whose temporal variations are shown in Figure 2. The subduction front seismicity which has continued for two years since the occurrence of the $M$ 9.3 event is related to the fault plane that slipped during the event. In the present case the mega-thrust earthquake has definitely changed the state of stress and hence, the aftershocks continued to occur for a longer period of time manifesting the process of relaxing stress concentrations introduced by the rupture of the main shock. An active debate exists whether the occurrence of the aftershocks can be explained in terms of asperities and barriers concept[15]. A few other studies[16,17] explain that the occurrence of aftershocks is governed by processes like viscoelastic and poroelastic processes.

An analysis was carried out to ascertain the minimum magnitude of the background seismicity within the entire rupture zone prior to the occurrence of the 26 December 2004 earthquake. It was observed that earthquakes of

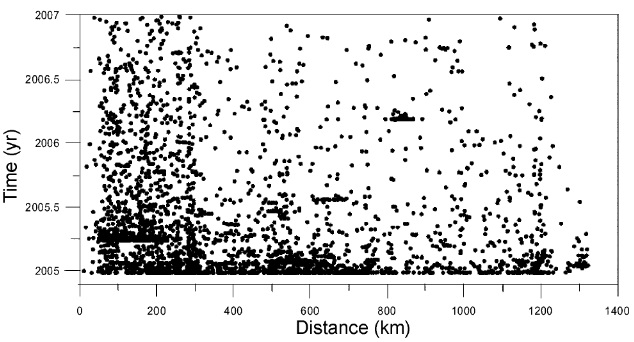


**Figure 2.** Space and time variation of aftershock activity in the region (from 26 December 2004 to December 2006 with the origin time and location as the origin).

magnitude ≥4 define the background seismicity of this region since the earthquake locations are determined using global stations. Hence, $M_c = 4$ (magnitude of completeness) was considered in our $b$-value analysis, but this choice of $M_c$ does not preclude the occurrence of smaller magnitude earthquakes.

Analysis of the aftershock activity reveals that the earthquakes are all confined to the top 50 km. However, events with 33 km focal depth are the model-assumed depths, which are not properly resolved, as these are determined using stations located at regional and teleseismic distances. Since the catalogue depths are found to have errors, the depth dependence of the $b$-value was not attempted. The analysis of frequency of the aftershock activity shows that about 3000 aftershocks of $M$ 4–5 occurred during the last two years, of which 1800 earthquakes occurred during the first two months.

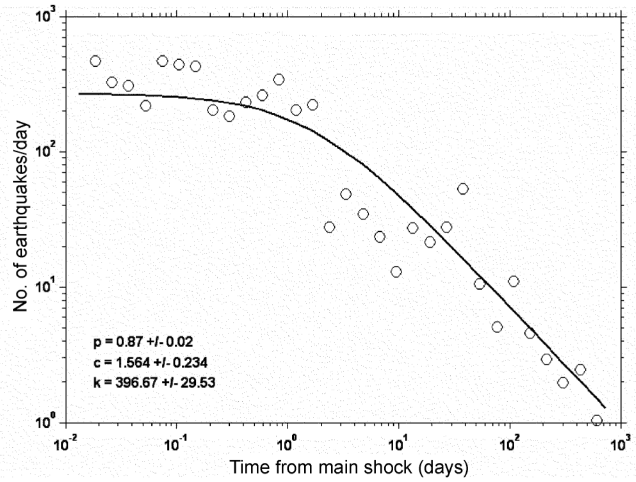


**Figure 3.** The $p$-value estimates of the aftershocks.

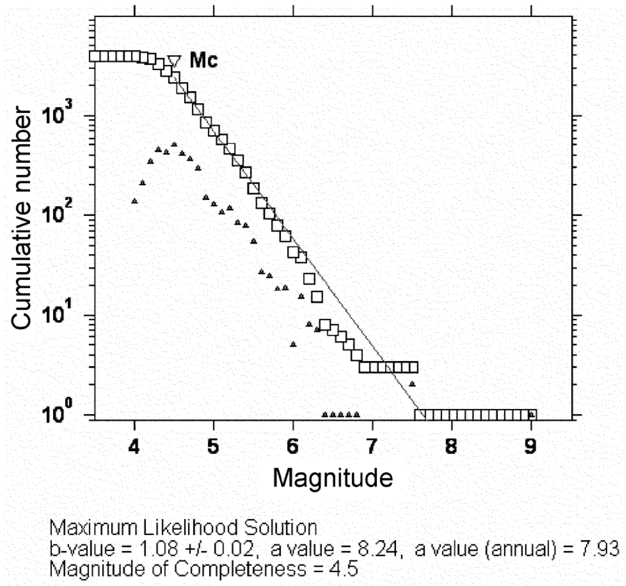


**Figure 4.** The frequency–magnitude curve ($b$-value) of the entire rupture zone.

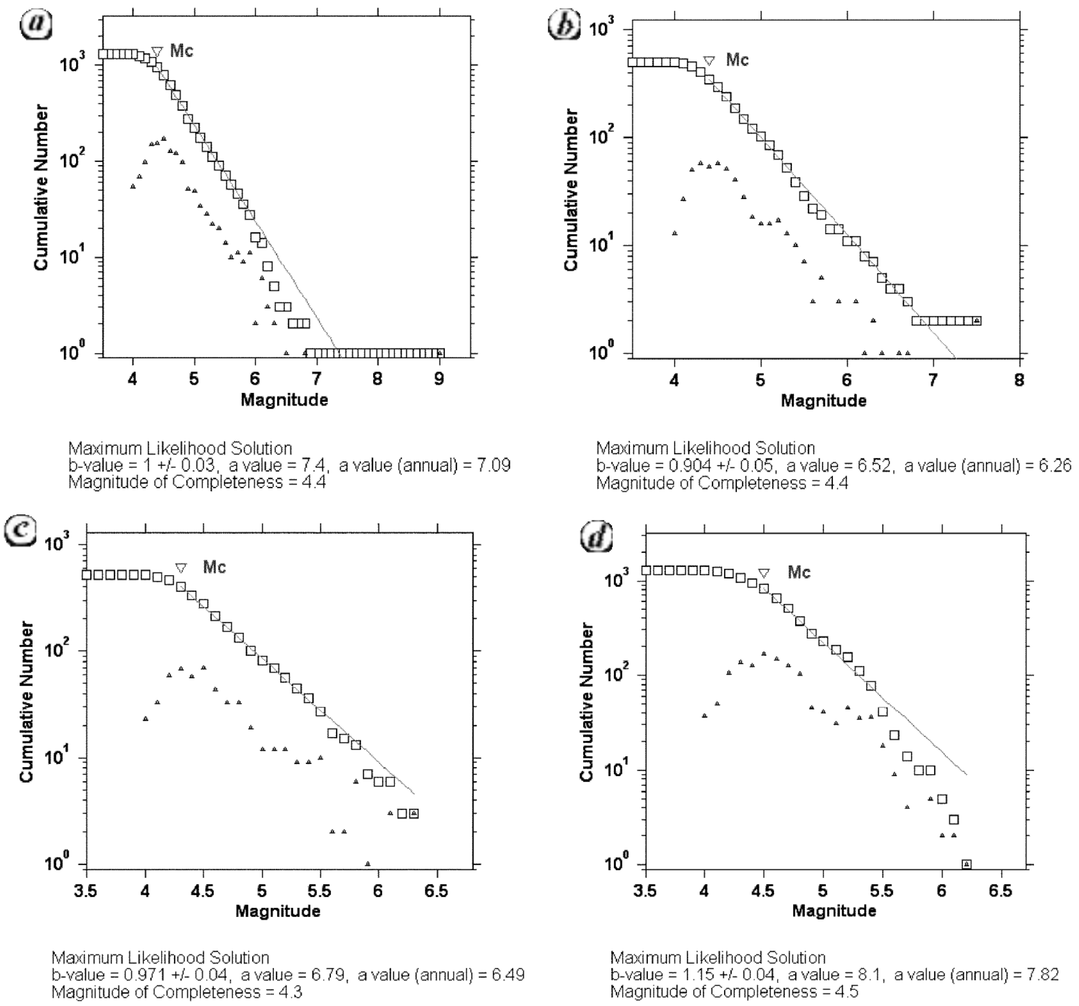


**Figure 5.** The frequency–magnitude (*b*-value estimate) for different blocks. ***a***, Zone-1; ***b***, Zone-2; ***c***, Zone-3; ***d***, Zone-4.

The *p* and *b* values have been estimated using ZMAP software package[18]. The values of *p*, *c* and *k* were obtained using the maximum likelihood method, while the decay rate of the aftershock distribution was modelled by the modified Omori law. Figure 3 shows the rate of decay aftershocks modelled by the Omori relation. The decay of activity in the first two months did not follow the Omori law. The value of *p* for the Sumatra earthquake was 0.87 and the *c* value was 1.564 (Figure 3).

Using the Guttenberg–Richter relation, *b*-value for the aftershocks coincident with the entire rupture was estimated to be 1.08 (Figure 4) and the corresponding *a*-value was 8.24.

In order to investigate the spatial variation of the *b*-value, the subduction front seismicity was divided into three zones (Figure 1) and the Andaman spreading ridge seismicity into a separate zone. Zone 1 represents the Sumatra segment which comprises of the main shock. Zone 2 is the Nicobar segment comprising two largest aftershocks of $M \geq 7$ and zone 3 represents the Andaman segment which coincides with the northern part of the rupture.

The Harvard CMT solutions for the above three zones were distinctly characterized by thrust-type earthquakes. Zone 4 is the back arc-spreading centre and the Harvard CMT solutions for the aftershocks in this zone are mainly normal and strike slip. Further, zone 4 is characterized by one of the most energetic swarm activities in the world[2,3] to be recorded till date, with about 600 earthquakes of *M* 4–6 having occurred during 26–31 January 2005. Figure 5 *a–d* shows the spatial variation of the *b*-values in the four zones along the Sunda–Andaman arc. The *b*-value in zone-1 comprising the main shock was $1.0 \pm 0.03$, zone-2 was $0.90 \pm 0.05$ and zone-3 was $0.97 \pm 0.04$. The *a*-values for the three segments were similar, with values ranging from 6.5 to 7.4. Zone-4 representing the Andaman back arc-spreading centre had a *b*-value of $1.15 \pm 0.04$ and *a*-value of 8.1.

Using 4000 aftershocks ($Mc \geq 4.0$) of the $M_w$ 9.3 Sumatra earthquake of 26 December 2004, *p*- and the *b*-values have been estimated. The *p*-value estimated for the Sumatra aftershock sequence was 0.87. Deviation of the *p*-value from 1 could be related to the fault zone heterogeneity, local stress fields and temperature. The *p*-value varied either due to all of the above or any one of them. If the $p$-value is near the normal value of 1, it indicates slow attenuation of the aftershock sequence. In the case of the Sumatra earthquake, it was observed that the aftershock sequence attenuation was slow as the magnitude was large. The spatial variations of the $b$-values were analysed with the rupture zone being divided into three different zones and the back arc-spreading region as a separate zone. The objective of this study was to look for possible variations in $b$-values, as these would reflect the rupture process of the main shock and/or the crustal heterogeneity. The $b$-value estimates for the three individual segments which were classified based on the rupture characteristics[1] were not significant, as they varied between 0.9 and 1.0. The rupture processes is basically governed by the material properties. A mega-thrust earthquake of this magnitude has not only ruptured the regions which have not experienced any great earthquake since a long time, but has also ruptured regions of previous large earthquake rupture zones in the Andaman region volume, indicating that the fault zone characteristics did not possibly influence rupture propagation. The insignificant variation of the $b$-value amongst the three rupture segments indicates that the rupture process has no significant bearing on the frequency–magnitude relation. However, a significant finding of this study is the $b$-value for zone 4, which coincides with the Andaman spreading centre. The $b$-value was estimated as $1.15 \pm 0.04$, which is in variance with the values of the other three zones. A plausible explanation for these high $b$-values can be given in terms of the variation in focal mechanism or due to the presence of swarm activity. The dominant focal mechanisms in the Andaman spreading ridge zone 4 are normal and strike slip, which are in concurrence with the tectonic regime and hence, have higher $b$-values in comparison to the thrust-type focal mechanisms observed[19] in zones 1–3. The reason for the $b$-values being higher in the normal-type environment in comparison to thrust-type environment is that the constant overburden equals the least and greatest principal stresses in these two cases respectively, and the difference in the $b$-value is due to the difference in ambient stress level[8]. We further examined to understand whether the focal mechanism is the only cause for the variation in the $b$-values. It is known that zone 4 is associated with one of the most energetic swarms recorded till date. This swarm activity was associated with 600 earthquakes recorded during 26–31 January 2005. The estimated $b$-value for this swarm was $1.55 \pm 0.09$, which resulted in a higher $b$-value for zone 4. The $b$-values of earthquake swarms are often much larger than 1 and sometimes approach a value of 2.5, especially in volcanic areas[20] and are also not comprised of the main shock. In conclusion, this study shows that the spatial variations of $b$-values amongst the three rupture segments are negligible and are similar to the $p$-value, indicating that neither the rupture characteristics nor the crustal heterogeneity straddling the rupture zone is significant enough to influence the $b$-values.


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