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EDITORIAL

Mathematics and Mathematicians

Mathematics can be a formidable and intimidating subject for most people. In the school years, a distinction can be very quickly made between the talented minority who are mathematically gifted and the majority who look forward to the day when they will forever sever their connections to the subject. For students who go on to courses in engineering or some disciplines of science, primarily physics and chemistry, mathematics can continue to cast a long shadow even in the college years. I must confess it was a great relief to drift away into disciplines that required little by way of explicit mathematical skills. The subject returned to haunt me a few weeks ago when a persuasive colleague, involved in organizing the International Congress of Mathematicians (ICM) in Hyderabad, charged me with the task of producing an editorial essay that centres around mathematics. The ICM, like most quadrennial international congresses, is a large affair that moves around the world, much like the Olympic games (and the many more restricted events like the Asian and Commonwealth games). Countries bid to host Congresses, lobby hard, international committees politic and vote; eventually a location emerges. The 2010 ICM in Hyderabad brings the event to India for the first time in its long history, dating back to the first meeting in 1897 in Zurich. The last Congress (Spain 2006) attracted as many as 4000 participants, suggesting that Hyderabad may witness an unusual spectacle when the city hosts such a large congregation of mathematicians. Scientists, unlike sportsmen, require little by way of infrastructure when they assemble for these mega events; the ICM may therefore impose on the organizers a lesser burden than the forthcoming Commonwealth Games in Delhi. The ICM will have its public moments when the Fields medallists and winners of other major awards are announced. In Spain, the drama was provided by the refusal of the reclusive Russian mathematician, Grigori Perelman to accept the Fields Medal. The air of mystery that has so often been associated with the greatest of mathematicians was reinforced when Perelman turned down the million dollar Clay prize. The Perelman phenomenon brought mathematics to the front pages of newspapers, reminding ordinary readers that genius can defy interpretation.

Prodigies and geniuses seem so often associated with mathematics; born with intrinsic gifts that are hard to comprehend. The Ramanujan story, so wonderfully told by Robert Kanigel (*The Man Who Knew Infinity: A Life of the Genius Ramanujan*, Crown, 1991) has formed the basis for a recent novel: *The Indian Clerk* by David Leavitt, Bloomsbury, 2007. This is a fictional account, more about G. H. Hardy and the Cambridge ethos of the early years of the 20th century. The book begins with Hardy on the stage at Harvard in 1936 about to receive an honorary degree 'on the occasion of the university's tercentenary'. In Leavitt's words: 'Unlike most of the visitors however he was not here – nor, he sensed, had he been invited – to speak about his own work or his own life. That would have disappointed his listeners. They wanted to hear about Ramanujan'. In his classic preface to the 1967 edition of Hardy's *A Mathematician's Apology* (Cambridge University Press, 1940), C. P. Snow calls the Ramanujan discovery as 'the one romantic incident' in Hardy's life; 'an admirable story and one, which showers credit on nearly everyone (with two exceptions) in it'. The oft told story of Hardy and Littlewood poring over Ramanujan's 'sheets of paper by no means fresh' never pales in the retelling. Hardy was to later conclude 'that Ramanujan was, in terms of *natural* mathematical genius in the class of Gauss and Euler'. Was this an assessment that was easy to make? Snow says it best: 'It all sounds easy, the kind of judgement great mathematicians should have been able to make. But I mentioned that there were two persons who do not come out of the story with credit. Out of chivalry Hardy concealed this in all that he said or wrote about Ramanujan. . . . It is simple. Hardy was not the first eminent mathematician to be sent the Ramanujan manuscripts. There had been two before him, both English, both of the highest professional standard. They had each returned the manuscripts without comment. I don't think history relates what they said, if anything when Ramanujan became famous. Anyone who has been sent unsolicited material will have a sneaking sympathy with them' (Canto, 2000, pp. 33–34). While Snow's long Foreword (50 pages) is a masterpiece of simple prose and a deeply personal assessment of Hardy, *Apology* is an extraordinary piece of writing; reflective, brutally honest and, at times, hauntingly sad. When the short book appeared in 1940 no greater tribute could have been paid than Graham Greene, a writer of uncommon ability, comparing Hardy's writing to Henry James, praising the work as demonstrating 'with an absence of fuss, the excitement of the creative artist' (*Spectator*, December 1940). The reviews of Hardy's *Apology* were not all laudatory. An analysis by Steve Whittle compares a variety of reactions to Hardy's reflective essay (http://www.aug.edu/dvskel/Whittle2002.htm). Frederick Soddy, the Nobel Prize winning discoverer of isotopes and a contemporary of Hardy, launched a blistering attack; provoked undoubtedly by Hardy's pacificism and his dismissal of 'applied mathematics' (*Nature*, 1941, **147**, 3). Whittle paraphrases Soddy, noting 'that counting and calculation, like art and music, are capable of giving pleasure to those who have the right kind of mind. And in the words of Soddy such skills are "the bloody masters" in a world that is ruled by counting tables'.

In mathematics, more than in any field of science, the superiority of 'pure' over 'applied' work seems implicit in the behaviour of the practitioners. Hardy is emphatic: 'There are two mathematics. There is the real mathematics of the mathematicians, and, there is what I will call the "trivial" mathematics, for want of a better word. The trivial mathematics may be justified, . . . but there is no such defence for the real mathematics, which must be justified as art if it can be justified at all' (p. 139). Hardy notes that Whitehead has spoken of 'the tremendous effect of mathematical knowledge on the lives of men, on their daily avocations, on the organisation of society'. His dismissal of these arguments is magisterial: 'The mathematics which can be used "for ordinary purposes by ordinary men" is negligible, and that which can be used by economists or sociologists hardly rises to scholarship standards' (p. 138). The results or, more appropriately, constructs of 'pure' mathematics have provided the mathematical framework of physics. Half a century ago Eugene Wigner wrote about 'The unreasonable effectiveness of mathematics in the natural sciences' (*Communications in Pure and Applied Mathematics*, 1960, vol. 13). He defined mathematics as 'the science of skillful operations with concepts and rules invented for this purpose'. Physics in Wigner's words is concerned with 'discovering the laws of inanimate nature'. He was cautious about 'theories' of biological phenomena. Wigner's conclusion may still be true: 'The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure, even though perhaps also to our bafflement, to wide branches of learning. Twenty years after Wigner, Richard Hamming returned to the same theme. His discussion centres on examples closer to engineering. Hamming's conclusions seem appealing: 'Mathematics has been made by man and therefore is apt to be continuously altered by him. Perhaps the original sources of mathematics were forced upon us . . . but we have made choices for the extensions that were only partly controlled by necessity and often, it seems to me, more by aesthetics. We have tried to make mathematics a consistent, beautiful thing, and by so doing we have had an amazing number of successful applications to the real world' (*The American Mathematical Monthly*, 1980, vol. 87). Twenty five years later Sundar Sarukkai revisited the same theme, suggesting that mathematics be viewed as a language that may be applied 'to some descriptions of the world'. He argues that 'the use of a language like English to describe the world is itself "unreasonably effective" and the puzzle with mathematics is just one reflection of this larger mystery of the relation between language and the world' (*Current Science*, 2005, **88**, 415).

I have tried, admittedly with considerable difficulty, to devote this column to general reflections on mathematics, a subject that has always seemed intimidating. In my years in college, now a rapidly receding memory, the names of D'Alembert, Lagrange and Euler seemed to intrude in mathematics classes. As I surfed the internet hoping to find material, relevant to the task at hand, I chanced upon an engaging historical account, with the inviting title, 'Frederick the Great on mathematics and mathematicians' (Cajori, F., *The American Mathematical Monthly*, 1927, **34**, 122). The article begins in compelling fashion: 'Frederick William I of Prussia ordered that his son, later known as Frederick the Great, should "learn no Latin"; "let him learn arithmetic, mathematics, artillery, – economy to the very bottom" '. The old king allowed the Berlin 'Society of Sciences', the favourite child of Leibniz, to languish and almost to pass away. His son, Frederick, on the other hand, secretly acquired some Latin, shunned the study of mathematics beyond its rudiments and brought the Berlin 'Academy' to great splendor. Frederick wrote to Voltaire in 1738: 'As for mathematics, I confess to you that I dislike it; it dries up the mind. We Germans have it only too dry; it is a sterile field which must be cultivated and watered constantly, that it may produce'. Despite this apparent dislike, Frederick cultivated mathematicians bringing to Berlin, among others, Euler and Lagrange. He tried hard to get D'Alembert to leave France, exchanging a great deal of correspondence. Cajori's 1927 article notes: 'D'Alembert once wrote to Frederick the Great: "It is the destiny of your majesty to be always at war; in summer with the Austrians, in winter with mathematics" '. Cajori concludes: 'Frederick the Great was desperately in love with poetry and philosophy, and wholly unsympathetic with mathematics. Yet, by his patronage at the Berlin Academy he contributed nothing substantial to poetry and philosophy, but achieved marvels in the advancement of mathematics. . . Frederick the Great's controlling motive for his academy was splendor. This is expressed in the form of the invitation he is said to have sent to Lagrange at Turin: "The greatest King of Europe" wishes to have "the greatest mathematician" at his court'. Both mathematics and recruitment practices have surely changed over the centuries.

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