

## Nonlinear electrical conductivity response of shaly-sand reservoir – physical explanation

We would like to respond to the ‘opinion’ expressed by Glover<sup>1</sup> in a scientific correspondence that refers to our paper<sup>2</sup>. We take serious objection to the word ‘invalid’ used by Glover<sup>1</sup>. It is apparent that he has appointed himself as judge, pleaded his case and pronounced judgement without listening to the defendant.

We are confident of the main conclusion of our modelling result that the parallel conductor model as an extension of Archie’s equation<sup>3</sup> is a failure in modelling the electrical response of freshwater reservoir including Archie’s equation. We agree that the equation in Glover *et al.*<sup>4</sup> is linear, but Glover<sup>1</sup> is wrong in pronouncing that the Korvin<sup>5</sup> equation is also linear taking a narrow mathematical view. For a linear function, only a first derivative exists, whereas for a nonlinear function higher derivative terms also exist. This is the case with Korvin<sup>5</sup>, especially for value  $m > 1$ . For example, the equation for  $m = 2$  can be written as

$$\sigma_0 = \sigma_w \phi^2 + \sigma_s (1 - \phi)^2 + 2\sigma_w^{1/2} \sigma_s^{1/2} \phi(1 - \phi).$$

Since the model by Glover *et al.*<sup>4</sup> yields similar results as those of the Korvin<sup>5</sup> model, we gave the nonlinear statement wrongly. What Glover<sup>1</sup> claims as strength of the equations of Korvin<sup>5</sup> and Glover *et al.*<sup>4</sup> and weakness of the Bussian equation<sup>6</sup> in modelling the freshwater reservoir is obsession with his model and self-righteous thinking. Interpretation of the modelling results of the three equations in the light of known physical framework concludes just the opposite to that expressed by Glover<sup>1</sup>.

We never claimed that our conclusion of ‘the Bussian equation<sup>6</sup> simulates the effective conductivity curves for all ranges of porosity and water conductivity’ is revolutionary, but it is certainly a statement of fact based on in-depth study of *in situ* field observations and mathematical modelling in the light of known geological and physical framework. It was not necessary to include those discussions in our paper<sup>2</sup>; however we are constrained to include them here to broaden the horizon of knowledge expressed in Glover<sup>1</sup>. It appears that Glover believes only in laboratory data, where it is extre-

mely difficult to simulate *in situ* reservoir condition at very low-conductivity, water-saturating, shaly-sand samples. However, there are plenty of direct and indirect evidences in the literature to support our main conclusions<sup>2</sup>.

The equation of Glover *et al.*<sup>4</sup> is a recent addition to the already existing parallel conductor-based linear models like Waxman and Smits<sup>7</sup>, and Clavier *et al.*<sup>8</sup>, with manipulative additions. However, it suffers from the weaknesses of those models for their inability to simulate the electrical response of freshwater shaly-sand reservoir, explaining relevant physical conditions. It appears that Glover<sup>1</sup> was so rattled by being pointed out by us<sup>2</sup> about the obvious limitation of his model that he ignored to study the theoretical modelling framework of Maxwell<sup>9,10</sup> given in the appendix of our paper<sup>2</sup>. This could have provided him with the physical limitation of their models. How far Glover is obsessed with his model given in Glover *et al.*<sup>4</sup> is comprehensively exposed in the interpretation of figure 1 in Glover<sup>1</sup>. This figure represents the computed bulk conductivity for the four models, viz. Glover *et al.*<sup>4</sup>, Korvin<sup>5</sup>, Bussian<sup>6</sup> and Archie<sup>3</sup> for  $m = 1.5$ ,  $\phi = 0.439$  and  $\sigma_s = 0.22$ . The statement of Glover<sup>1</sup> with respect to his figure 1 – ‘we can see that the value of the effective conductivity as  $\sigma_w \rightarrow 0$  stabilizes at 0.092 and 0.156 S/m for Korvin<sup>5</sup> and Glover *et al.*<sup>4</sup> models respectively. This indicates that the conductivity is dominated by the fixed matrix component (with a slightly different value provided by each model). In contrast, the bulk conductivity from the Bussian<sup>6</sup> model is still controlled by the conductivity of the fluid, and increasing with fluid conductivity at the same rate as the classical Archie’s<sup>3</sup> law’ is simply the qualitative explanation of the curves. However, this statement is wrong in the light of physics of the problem. Glover<sup>1</sup> should appreciate the fact that a reservoir is not always necessarily conducting; it may be resistive as well depending on the conductivity of the saturating fluid. The major contributor to the bulk conductivity (or bulk resistivity) is volume conduction and not the surface conduction. Surface conduction in a shaly-sand

reservoir only facilitates to maintain flow of current in the reservoir. However, the current has to overcome the resistance offered by resistive saturating water. This will reflect in the bulk electrical response of the reservoir which the Bussian model<sup>6</sup> shows more appropriately in comparison to those of Korvin<sup>5</sup> and Glover *et al.*<sup>4</sup>. Surface conduction has no capacity to convert the reservoir saturated with low-conductivity water in a conducting zone. Merely expressing the electrical property by conductivity does not make it electrically conducting. The parallel conductor model is suitable as long as currents are flowing in a predominantly horizontal direction and that is possible only in a perfectly and to some extent a highly conducting medium. As a reservoir possesses resistive environment, current lines are forced towards vertical direction and hence the parallel conductor model starts losing its significance. To make the physics part more clear, reference can be made to a classic publication of Maillet<sup>11</sup> stating that in a geological sequence where resistivity  $\rho(z)$  or conductivity  $\sigma(z)$  is a continuous function of depth  $z$ , one can consider the integrals

$$R(z) = \int_0^z \rho(z) dz,$$

termed as transverse unit resistance  $R$  and

$$C(z) = \int_0^z \sigma(z) dz,$$

termed as longitudinal unit conductance  $C$ , observing that if we consider a geological column built on a square unit,  $R$  is the resistance to the lines of current perpendicular to the strata, and  $C$  is the conductance offered to the lines of current parallel to it. Now apply this fundamental concept to a conceptual model of an aquifer in Figure 1. The electrical condition of the medium is characterized by  $R$  and  $C$ , which are linearly independent only in case of two extremes of a perfectly conducting or an insulating medium. For intermediate range they are

dependent on each other with a dividing line,  $C = R$ , when both are equal (conductivity of 1 S/m). Above this zone the medium can be classified as conducting and below the zone it is resistive. However, from a practical point of view, a small zone in the vicinity of conductivity of 1 S/m may be classified as moderately conducting. Keeping in view the definition of Maillet<sup>11</sup>, the current lines in a conducting zone are dominantly in the horizontal direction and in a resistive zone are dominantly in the vertical direction. Therefore, the freshwater-saturated shaly-sand reservoir can be modelled by a series resistor model, as evident from Figure 2 (for same parameters used by Glover<sup>1</sup>). In Figure 2, in case of the Maxwell model<sup>9,10</sup> (Mx2 model given in appendix of Sri Niwas *et al.*<sup>2</sup>), we have used series resistor model in conductivity range 0.00001–0.1 S/m, and parallel conductor model in conductivity range 1–10 S/m. Due to nonlinear behaviour we have not generated data in the conductivity range 0.1–1 S/m. The bulk conductivity computed using parallel conductor approach, and parallel conductor and series resistor approach is compared with values obtained from Archie's and Mx2 equations in Figure 2.

The Bussian model<sup>6</sup> also encompasses these physical demands (Figure 3) more appropriately than the models of Glover *et al.*<sup>4</sup> and Korvin<sup>5</sup>. Conductivity range 10–0.00001 S/m used in Figure 2 is large and can be tentatively divided into three major physical groups: conducting ( $> 1$  S/m) range, moderately conducting (between 0.1–1 S/m) range, and resistive ( $< 0.1$  S/m) range. Bulk conductivity function is approximately linear in the conducting range, nonlinear in the moderately conducting range, and approximately linear in the resistive range. It is clear from figure 1 of Glover<sup>1</sup> that only the Bussian model<sup>6</sup> represents the physics of the medium for the entire range of water conductivity. We cannot negate the Bussian model<sup>6</sup> simply on the basis that it functionally follows the trend of Archie<sup>3</sup> as water conductivity decreases. In fact, it points out the fact that the resistive medium can roughly be modelled with an Archie-type equation which gives a conductivity jump over the conducting range equation (Figure 4:  $\phi = 0.2$ ,  $\sigma_s = 0.1$ ,  $m = 1.5$ ). Hope Glover<sup>1</sup> learns to appreciate the modelling results deduced from the Bussian model<sup>6</sup>. Further modelling results based on the Bus-

sian model<sup>6</sup> can be found in Sri Niwas *et al.*<sup>12</sup>, with the main conclusion that the Bussian<sup>6</sup> nonlinear equation is bounded by two Archie-type asymptotic equations given by:  $\sigma_0 = \sigma_w \phi^m$ , as  $\sigma_w \rightarrow \infty$  and  $\sigma_0 = \sigma_w \phi^{-m}$ , as  $\sigma_w \rightarrow 0$ . No one can

overlook the fact that the Bussian<sup>6</sup> and Mx2 equations are derived theoretically, whereas the models of Glover *et al.*<sup>4</sup>, Korvin<sup>5</sup>, Waxman and Smits<sup>7</sup>, and Winsaur and McCardell<sup>13</sup> (Figure 4) are empirically derived.

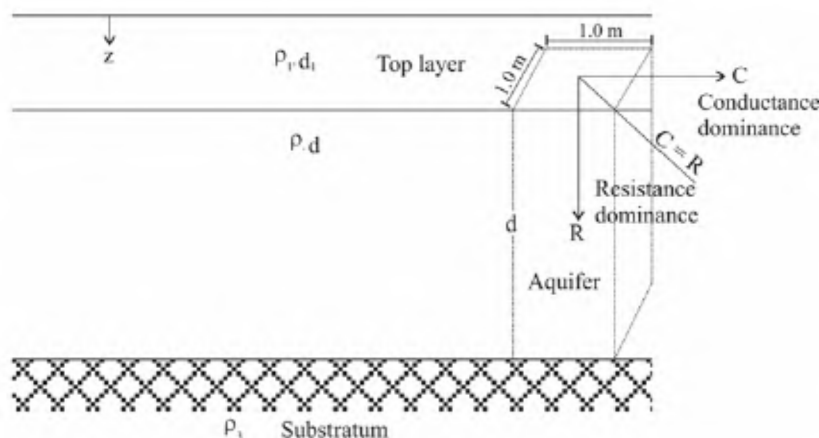


Figure 1. Aquifer (reservoir) model and a prism of unit square shown within it.

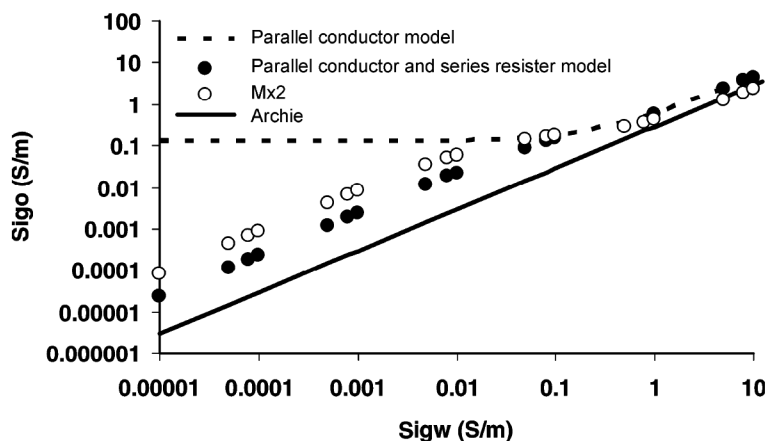


Figure 2. Validity of parallel conductor and a combination of parallel conductor and series resistor model for representing bulk conductivity ( $\sigma_0$ ) for Maxwell-2 phase (Mx2) as a function of water conductivity ( $\sigma_w$ ) for a reservoir with  $\phi = 0.439$ ,  $\sigma_s = 0.22$  and  $m = 1$ .

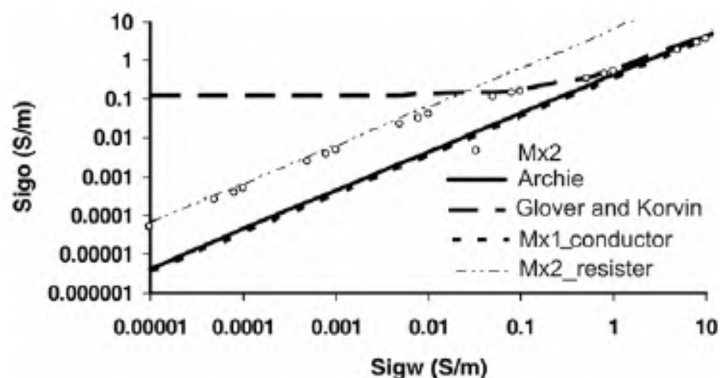


Figure 3. Numerical values obtained from eqs (1), (2) and (4) for a reservoir with  $\phi = 0.439$ ,  $\sigma_s = 0.22$  and  $m = 1$  and comparison with response of Glover, Korvin, Mx1\_Conductor and Mx1\_Resistor.

Numerous models proposed for the computation of bulk electrical response of heterogeneous porous medium are based either on theoretical or empirical approaches. The theoretical approaches are based on broad proven principles of mathematical physics in the form of partial differential equations, whereas the empirical approaches intend to fit a particular set of experimental or field data. In view of its rigorous deduction, a theoretical approach is always more desirable and it has a wider range of applications than an empirical one. The implementation of theoretical equations becomes difficult when the pore geometry is complex. However, a simplified geometry can always be assumed to exploit the advantages of the theoretical approach over the empirical approach. Maxwell<sup>9,10</sup> initiated the rigorous theoretical treatment for flow of electricity in a dilute system where the symmetrically shaped particles comprising the dispersed phase are regularly distributed. The bulk electrical conductivity ( $\sigma_0$ ), of a two-phase medium of uniform conducting spheres (referred as Mx2) of conductivity  $\sigma_s$  immersed in a continuum of conductivity  $\sigma_w$ , and porosity  $\phi$  is given by (neglecting the interaction between two spheres),

$$\sigma_0 = \sigma_w \frac{3\sigma_s + 2\phi(\sigma_w - \sigma_s)}{3\sigma_w - \phi(\sigma_w - \sigma_s)}, \quad (1)$$

that reduces to

$$\sigma_0 = \sigma_w \frac{2\phi}{3 - \phi}, \quad (2)$$

in the case of single-phase conductor when uniform spheres are non-conducting ( $\sigma_s = 0$ ); and referred as Mx1. Lynch<sup>14</sup> derived a theoretical equation for bulk conductivity  $\sigma_0$ , of a clay-free granular sandstone block (with porosity  $\phi$ , length  $L$  and area of cross-section  $A$ ) as,

$$\sigma_0 = a\sigma_w\phi, \quad (3)$$

where  $a$  is the tortuosity. In this background the electrical conduction in clean sands studied by Archie<sup>3</sup> (also see the historical review in Thomas<sup>15</sup>) through the equation

$$\sigma_0 = a\sigma_w\phi^\phi, \quad (4)$$

must be considered, for modelling the clay-free sand reservoir saturated with

brine. Archie<sup>3</sup> observed that for the purpose of modelling *in situ* conditions, the porosity must be reduced by the exponent  $m$  with a value of about 1.3 for unconsolidated sands and a range between 1.8 and 2.0 for consolidated sandstones. Hence  $m$  is called the 'cementation factor'. The value of  $a$  estimated is always nearly unity. Doveton<sup>16</sup> presented the following values for  $m$ .

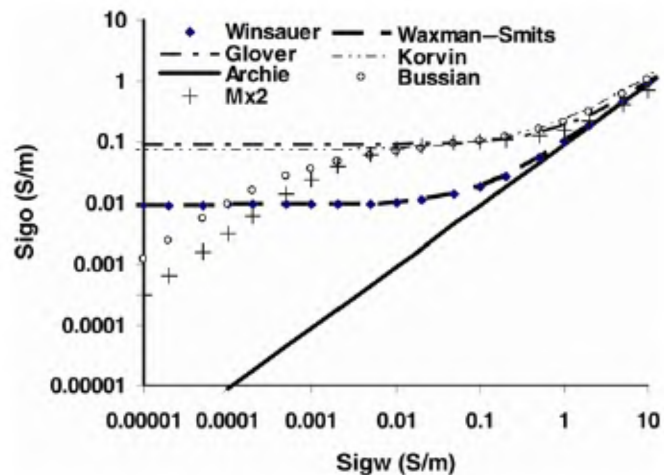
Unconsolidated sand,  $m = 1.3$ ; very slightly cemented sandstone,  $m = 1.4$ – $1.5$ ; slightly cemented sandstone,  $m = 1.5$ – $1.7$ ; moderately cemented sandstone;  $m = 1.8$ – $1.9$ ; highly cemented sandstone,  $m = 2.0$ – $2.2$ .

Theoretical models describing eqs (1)–(3) assume particle shape of uniform spheres; however, the actual shape may not be spherical. The effect of particle shape on the formation factor ( $F$ ) versus porosity relationship,  $F = \sigma_w/\sigma_0 = \phi^{-m}$  is given by Atkins and Smith<sup>17</sup> and Jackson *et al.*<sup>18</sup>. The exponent  $m$  correlates with the sphericity  $S$  of the sediment grains as

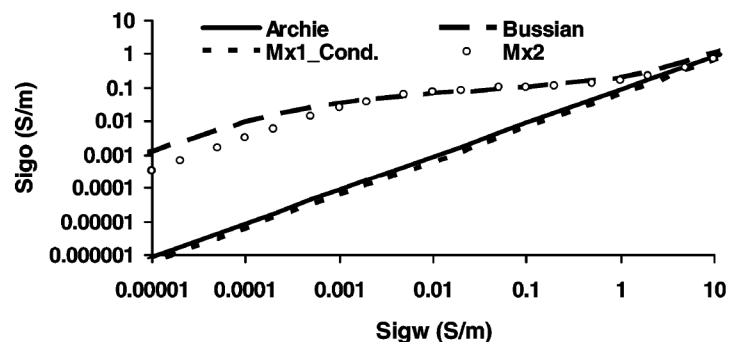
$$m = 2.9 - 1.8S. \quad (5)$$

Figure 3 presents the numerical results using the same model parameter used by Glover<sup>1</sup> in his figure 1, except a simple case of  $m = 1$  (then the models of Glover *et al.*<sup>4</sup> and Korvin<sup>5</sup> as well as Lynch and Archie are identical) for comparing the functional nature of these models with Maxwell models (Mx2 and Mx1). Figures 3 and 4 confirm that although Mx2 is theoretically derived following different mathematical procedure than the Bussian model<sup>6</sup>, the functional behaviour is similar in all the three conductivity ranges. This functional similarity is due to simulation of the same physical framework and not by coincidence. It is also observed that Archie's model<sup>3</sup> is functionally similar to Mx1, providing theoretical support to model reservoir saturated with water having conductivity lying in the conducting range.

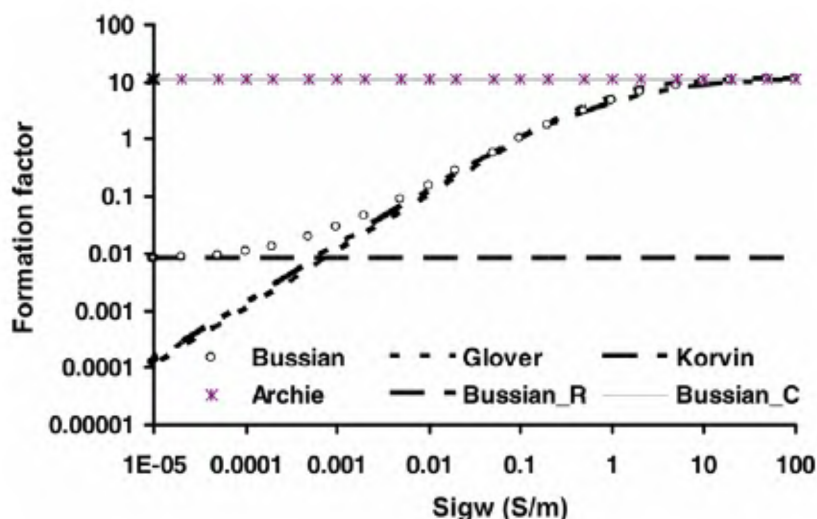
By replacing  $\phi$  with  $\phi_e = \phi^m$  in eq. (1), with the objective of including a medium



**Figure 4.** Comparison of some existing models showing variation of bulk conductivity ( $\sigma_0$ ) as a function of water conductivity ( $\sigma_w$ ) for a reservoir with  $\phi = 0.20$ ,  $\sigma_s = 0.10$  and  $m = 1.5$ .



**Figure 5.** The Bussian and Mx2 models are functionally similar using reservoir model parameters as  $\phi = 0.20$ ,  $\sigma_s = 0.10$  and  $m = 1.5$ .



**Figure 6.** Two limiting formation factors of apparent formation, factor variation with water conductivity for a reservoir with model parameters as  $\phi = 0.20$ ,  $\sigma_s = 0.10$  and  $m = 1.5$ , in case of the Bussian model, whereas only one limiting formation factor in case of the Glover and Korvin models.

having particles approximately spherical in the Mx2 model, we have computed the bulk conductivity for Bussian<sup>6</sup>, Mx2, Mx1 conductor and Archie's models using  $m = 1.5$ ,  $\phi = 0.2$  and  $\sigma_s = 0.1$  (Figure 5). Figures 3 and 5 confirm that the Bussian<sup>6</sup> and Mx2 models are functionally similar, and Glover *et al.*<sup>4</sup>, Korvin<sup>5</sup>, Waxman–Smits<sup>7</sup> and Winsaur–McCardell<sup>11</sup> models are also functionally similar.

Major usage of electrical response modelling of saturated porous medium is in the evaluation of a reservoir. Yes the mathematical and numerical modelling results based on the Bussian equation<sup>6</sup> brought out by Sri Niwas *et al.*<sup>12</sup> is 'revolutionary', a word sarcastically mentioned in Glover<sup>1</sup>. Because only these results can resolve the existing paradox due to empirically observed direct and inverse resistivity–permeability relationship reported by Worthington<sup>19</sup>. Archie's model<sup>3</sup> demands that intrinsic formation factor ( $F$ ) should have only negative relationship with permeability  $k$ . Pfannkuch<sup>20</sup> observed that this approximation seems to break down whenever clays or 'conductive' solids are present in the reservoir due to surface conduction phenomenon occurring within the electrical double-layer developed at charged solid–liquid interface. He defined an apparent formation factor ( $F_a$ ) and concluded that the surface conduction effects can be neglected, so that,  $F_a = F$ , only in case of clean sands saturated with brine. By analysing the field examples

from various sources, Worthington<sup>19</sup> proposed qualitatively three discrete arbitrary classes (A, B and C) of resistivity–permeability behaviour. Class A represents the negative relationship between  $F_a$  and  $k$  when less than 33% of the measured  $F_a$  is attributable to surface conduction. Class B represents the ill-defined relationship between  $F_a$  and  $k$  when neither the clean sand term nor the surface conduction accounts for more than 67% of the measured  $F_a$ . Finally, class C represents the positive relationship between  $F_a$  and  $k$ , where more than 67% of the measured  $F_a$  is attributable to the surface conduction. Purvance and Andricevic<sup>21</sup> also broadly supported this classification. These classifications can be theoretically supported by the modelling results of Sri Niwas *et al.*<sup>12</sup>, with suitable modification. For this purpose we have computed the apparent and intrinsic formation factors for few models based on the data of Figure 4. These are given in Figure 6 as a function of water conductivity. It may be clarified that the intrinsic formation factor ( $F$ ) is constant and the apparent formation factor ( $F_a$ ) is a function of water conductivity. It can be seen from Figure 6 that only in the Bussian model<sup>6</sup>,  $F_a \rightarrow F$  (conducting) in the conducting range,  $F_a \rightarrow F$  (resistive) in resistive range and remains a function in the moderately conducting zone. Thus the relation between  $F_a$  ( $\approx F$  (conducting)) and  $k$ , will be negative in the conducting range, positive ( $F_a \approx F$  (resistive)) in resistive range and ill-

defined in the moderately conducting range. Other models can obtain only negative relation.

These extensive modelling results are supported by field data and on the strength of above discussion we stress that:

- The Bussian equation<sup>6</sup> simulates the effective conductivity curves for all ranges of water conductivity<sup>5</sup>, whereas the Glover *et al.*<sup>4</sup> and Korvin models do not.
- The Bussian equation<sup>6</sup> is more consistent with the physics of the shaly-sand reservoir and reduces to the linear model in the extreme physical situations of conducting or resistive environment.
- The nonlinear nature of effective conductivity in the moderately conducting range can only be simulated with the Bussian model<sup>6</sup>.
- The models of Glover *et al.*<sup>4</sup> and Korvin<sup>5</sup> are unable to simulate resistive range of conductivity. Thus the Bussian model<sup>6</sup> is superior to these two models.
- The model of Glover *et al.*<sup>4</sup> is linear and the effective conductivity reduces to water conductivity when  $\phi = 1$ .

We agree with the recommendation made by Glover<sup>1</sup> that a full and high quality review of all the models be carried out so that we can better understand them. We further add that the review must be in respect of a benchmark situation set by Worthington<sup>19</sup> on the basis of *in situ* data and not experimental data.

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## Mineralogy of disseminated sulphides from the volcanics of Andaman Island

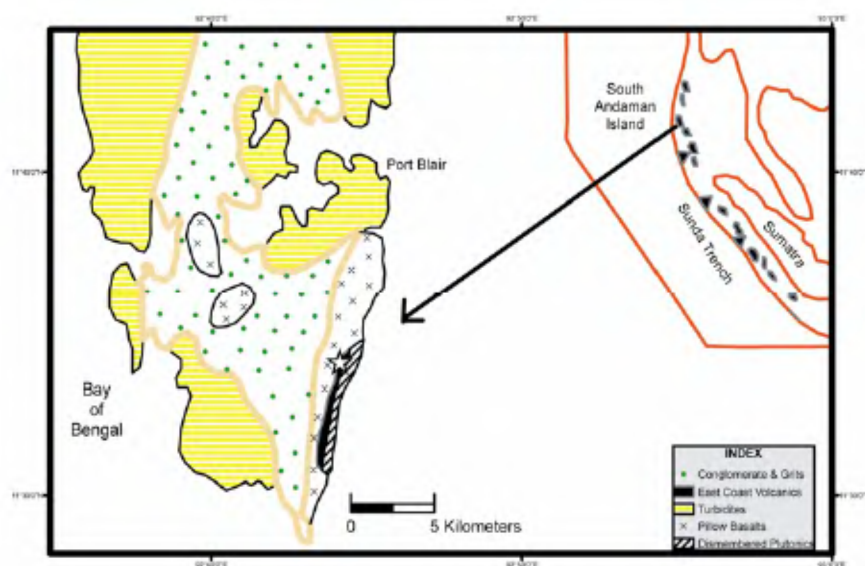
Hydrothermal sulphide mineralization in modern oceanic-spreading and back-arc environments has been a topic of increasing academic research ever since its first reported occurrence in the Galapagos spreading centre in 1979 (refs 1–3). Though the fundamental processes of sulphide mineralization are more or less similar for the mid-ocean ridge (MOR) spreading-axis and back-arc spreading centres, the host-rock association and mineralogy of the sulphides in these two settings varies significantly. For instance, in MOR, the sulphides are hosted within mafic and ultramafic rocks, whereas felsic hosts are more common in back-arc environment<sup>4</sup>. Similarly, the sulphides precipitating in back-arc systems typically have lower Fe, higher  $fO_2$  and high Au contents<sup>5</sup>.

In contrast to the extensive studies on sulphide mineralization in back-arc hydrothermal systems along the active convergent plate boundaries of the western Pacific<sup>4</sup>, reported occurrences of hydrothermal activity and sulphide mineralization in the Andaman back-arc setting are sparse<sup>6</sup>. Here we report the mineralogy and chemical composition of sulphide mineralization associated with the late quartz vein within the volcanic suite. The volcanic rocks mainly include altered basalts of East coast volcanic group as described by Ray *et al.*<sup>7</sup>. Our studies suggest that the chalcopyrite–pyrite association probably co-precipitated within the quartz vein, originally formed at a marginal-basin setting. High Co and Mo content within the sulphides further supports their formation from the circulation of hot fluids.

The Andaman–Nicobar group of islands, located roughly in the central part of Burma–Sunda–Java subduction double-chain arc system, represent part of the outer sedimentary arc<sup>8</sup>. The dominant rock types of the islands are sandstone, grey shale, conglomerate and limestone forming part of the Flysch and Archipelago group of sediments. Ophiolite, closely associated with the Andaman flysch, occurred as rootless, thrust-bound slices, occupying parts of the South Andaman area<sup>9</sup>. Igneous rocks mostly associated with the ophiolites are peridotite/serpentine, pyroxenite, gabbro, anorthositic gabbro, plagiogranite, basaltic dykes, pillow basalts and East coast volcanics<sup>9</sup>. The East coast volcanic rocks range in

composition from basalt, basaltic andesite to acid differentiates<sup>7</sup>. Occurrences of plagiogranite within cumulate gabbro and East coast volcanics have also been reported<sup>9</sup>. The occurrence of pillow basalts has been attributed to their formation in a slow-spreading environment<sup>7</sup>. Geodynamic evolutionary studies of the Andamans reveal that there was a subduction event (during Eocene–Oligocene) prior to the currently active subduction processes initiated at Late Miocene<sup>10</sup>. The manifestation of these two events is exemplified as ophiolite obduction and inner-arc volcanism respectively.

Samples of sulphide hosted volcanics were collected from near Bedanabad village, South Andaman (11°56.912'N/



**Figure 1.** Generalized geological map of South Andaman (modified after Ray *et al.*<sup>7</sup> and Jafri *et al.*<sup>9</sup>).